Nano-PCM filled energy storage system for solar-thermal applications

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Abstract

In this paper, a nano-PCM filled enclosure, which is a representative geometry of a thermal energy storage (TES) system, is investigated using scale analysis, numerical simulation, and experimental analysis. The enclosure is assumed to be square in shape. It is also assumed that one vertical wall of the enclosure is actively participating in absorbing energy from a source while the remaining walls are insulated. The thermal boundary condition at the active wall is treated as 'constant heat flux boundary condition' in this paper. The energy absorbing material, i.e., the nano-PCM, is CuO nanoparticles dispersed in coconut oil PCM. The influence of the volume fraction of nanoparticles (0 ≤ φ ≤ 5%) is investigated on the flow and thermal fields, heat transfer rate, energy stored and liquid fraction during the melting process of nano-PCM at different values of Rayleigh number based on base PCM (10^4 ≤ Ra_{nano} ≤ 10^8). The Rayleigh number is adjusted by adjusting the size of the enclosure (i.e., higher Ra represents the larger enclosure). In addition to the isothermal lines and velocity vectors, heatlines are utilized to exhibit the energy flow patterns inside the enclosure during the melting process. Besides the numerical calculations, scale analysis is presented to demonstrate the different stages of melting process of nano-PCM. The detailed scale analysis assists to identify relationship of Nusselt number and solid-liquid interface location as a function of well established dimensionless numbers: Stefan number (Ste), Fourier number (Fo), and Rayleigh number (Ra_{nano}). Finally, an experimental setup is developed to visualize the melting process of nano-PCM inside a prototype enclosure. Experiments are conducted to illustrate the impact of adding nanoparticles into PCM on the melting process. The numerical and experimental results show the significant improvement of the melting process by adding nanoparticles to PCM.

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1. Introduction

Buildings utilize approximately 40% of the total energy consumed in North America [1]. Therefore, there is an increasing need for saving energy consumed by buildings. Large portions of building materials have relatively low thermal conductivities and high heat capacities. Therefore, the building structures have significant potential to be used as temporary thermal energy storage (TES) systems [2]. Additionally, the large surface areas of structures also assist proper distributions of thermal energy. Although the central idea of thermal management of a building was originated to reduce incoming heat during the summer months (to lower the cooling load) and outgoing heat during the winter months (to lower the heating load), the concept of thermal management have been evolved over the time further with the introduction and advancement of energy efficient buildings and structures, such as, ‘low energy buildings’, ‘net-zero buildings’, and ‘green buildings’. Therefore, more and more emerging energy conversion and storage technologies exhibit potential to be incorporated into the building as an integral part of it or as retrofit to the existing building components. Storing energy in the building components once considered as an accessorial function, however, growing number of technical articles related to building [3–5] have suggested a gradual shift of accessorial need to a necessity towards achieving sustainable or high performance building.

Typically, a moderate temperature difference is required for the thermal management of the building. Therefore, low grade energy sources (e.g., solar, waste heat, and district energy) can be effectively utilized for thermal management of buildings [3–5]. The performance of the structural component based TES system can be improved further by incorporating phase change material (PCM) which can be used in encapsulated form to store energy. PCM is an attractive choice for latent heat thermal energy storage (LHTES)
systems.

PCMs usually refer to the materials that witness a phase transition, within a small temperature variation, from the solid to liquid (or liquid to solid) phases while absorbing (or releasing) thermal energy [6]. PCM is the core component of latent heat thermal energy storage system (LHTES). The amount of thermal energy stored per unit volume in a LHITS system is typically 5 to 14 times more than the sensible heat thermal energy storage system (SHTES) [7]. PCMs have been widely applied to solar thermal systems [8,9], electronic cooling [10], heating and cooling of buildings [11], and food storage [12]. However, PCM suffers from poor thermal conductivity which reduces the performance of TES systems during energy transport [13]. Dispersing high thermal conductivity nanoparticles into PCM (also known as nano-PCMs) [14] is one of the methods of improving the thermal conductivity of PCM. Other methods include use of porous matrix [15] and fin [16] which reduce the effective volume of PCM in a TES.

Dhaidan et al. [17] experimentally and numerically investigated the melting process of nano-PCM (CuO nano-particles dispersed in n-octadecane PCM) in a square enclosure. One vertical wall of the enclosure was exposed to a constant heat flux, while other walls were thermally insulated. The authors found that utilizing nanoparticles increase the thermal conductivity of the nano-PCM and subsequently heat transfer rate increases causing faster melting. The melting process improves as the nanoparticle volume fraction increases. However, higher volume fractions of nanoparticles could cause agglomeration and precipitation. Ho and Gao [18] conducted an experimental investigation of melting of nano-PCM (Al2O3 nanoparticles dispersed in n-octadecane PCM) in a square enclosure heated from left with constant heat flux and cooled from right at constant temperature. The observed results indicated that adding nanoparticles to PCM reduces the effect of natural convection heat transfer. The authors observed a reduction in the Nusselt number (defined based on the pure PCM’s properties) and total energy passes through the hot wall with increasing mass fractions of nanoparticles. In a later work, Dhaidan et al. [19] experimentally and numerically investigated the melting of nano-PCM (CuO nanoparticles dispersed in n-octadecane PCM) filled cylindrical annulus under constant heat flux condition at the inner cylinder and adiabatic condition at the outer cylinder. The reported results revealed that at lower nanoparticles mass fractions the melting process is faster than at higher mass fractions. The enhancement of thermal conductivity due to adding nanoparticles is compensated with increasing the viscosity. Dhaidan et al. [20] further experimentally and numerically investigated the melting of nano-PCM (CuO nano-particles dispersed in n-octadecane PCM) inside a horizontal tube exposed to a constant heat flux boundary condition. The authors observed that the melting process enhanced after incorporation of nanoparticles in PCM.

The solidification process of nano-PCM (Cu nano-particles dispersed in water PCM) inside a differentially-heated square enclosure was numerically studied by Khodadadi and Hosseinizadeh [21]. The authors observed increment in the rate of the released heat throughout the freezing process. Khodadadi and Hosseinizadeh [21] referred that the increment is due to higher thermal conductivity and lower latent heat of fusion because of adding nanoparticles to the PCM.

Using lattice Boltzmann method, Feng et al. [22] numerically investigated the melting of nano-PCM (Cu nano-particles dispersed in water PCM) in a rectangular enclosure. The enclosure was heated from the bottom at a constant temperature while the top part of the enclosure was maintained at the melting temperature of the PCM. The numerical results indicated that the improvement in the heat transfer and the melting rates besides higher stored energy due to the dispersion of nanoparticles in the PCM.

Sharma et al. [23] numerically studied the effect of adding nanoparticles to PCM (Cu nano-particles dispersed in water PCM) on the freezing process of the nano-PCM that filled an isosceles trapezoidal enclosure. The side walls of the enclosure were differentially heated at constant temperatures. The authors concluded that use of nanoparticles decreases the freezing time of the nano-PCM.

Ranjab et al. [24] numerically investigated the solidifications of nano-PCM (Cu nano-particles dispersed in water) in a rectangular enclosure. The enclosure was differentially heated at constant temperature from the vertical walls, while the horizontal walls were thermally insulated. The results showed that adding nano-PCM improves the heat transfer rate. Also with higher volume fractions of nanoparticles, the heat transfer rate improves more.

Fan and Khodadadi [25] theoretically (combination of analytical and integral methods) studied 1-D solidification problem of nano-PCM (CuO nano-particles dispersed in cyclohexane PCM). One vertical wall was cooled at constant temperature, while the opposite wall was thermally insulated. The authors found that the solidification time is reduced by adding nanoparticles.

Hosseinizadeh et al. [26] numerically studied the melting of nano-PCM (Cu nano-particles dispersed in RT27 PCM) inside a spherical container. The spherical container is heated at constant temperature. The authors found that the nanoparticles addition improves the thermal conductivity and reduces the latent heat of the nano-PCM in comparison with base PCM.

Kashani et al. [27] numerically investigated the solidification of nano-PCM (Cu nano-particles dispersed in water) in a vertical wavy enclosure. Constant temperatures were applied on the side walls to heat the left wall and cool the right wall. The authors concluded that adding nanoparticles into the PCM accelerates process of the nano-PCM.

Numerical investigation was conducted by Kashani et al. [28] to investigate the solidification of nano-PCM (Cu nano-particles dispersed in n-hexadecane PCM) in a vertical enclosure. The left vertical wall was heated and the right vertical wall was cooled at constant temperature. The results showed that the solidification process of the nano-PCM is accelerated when nanoparticles are added to the base PCM.

Several articles presented excellent reviews on the fundamental and applied research works on PCM. For example, excellent reviews on the recent progresses on the melting and solidification processes inside different shaped cavities using PCM and nano-PCM are presented by different researchers [e.g., 29–32], phase change processes inside different shaped containers using regular PCM by Dhaidan and Khodadadi [30], thermal conductivity enhancement of PCM using colloidal suspension by Khodadadi et al. [31], thermophysical properties of nano-PCM by Kibria et al. [32]. Thermal analysis of PCM and applications to TES by Zalba et al. [33], and PCM materials and their applications by Farid et al. [34]. Additionally, fundamental and applied research works on pure PCM were discussed in numbers of articles [35–38].

A good number of articles in existing literature are devoted to understand the melting and/or solidification characteristics of wide range of pure PCMs inside different shaped geometries subjected to different initial/boundary conditions [e.g., 29, 36]. In contrast, a limited number of articles have been devoted for nano-PCM inside enclosed spaces of rectangular, cylindrical, and spherical shapes [e.g., 27, 28, 39, 40]. Correlations are proposed as well to determine melt fraction and/or Nusselt number based on experimental analyses [18,36] and numerical predictions. A few articles [e.g., 41] carried out extensive scale analyses using pure PCM inside enclosed space with isothermal boundary condition to determine complete functional relationships between Nusselt number, melt fraction, and dimensionless time. Nevertheless, a rigorous scale
analysis for the phase change processes involving nano-PCM is not reported in the literature, specifically for constant heat flux boundary condition, which is essential to establish functional relationship between important parameters with single correlations covering the complex phase change processes in all regimes of melting. This paper is the first one to report a detailed scale analysis on the melting process of nano-PCM inside an enclosed space under constant heat flux boundary condition.

Kimura and Bejan [42] proposed the concept of heatlines as an effective technique to visualize the energy flow as isotherms and streamlines provide typically inadequate pictures of energy flow in convection heat transfer problems. Although the heatline technique [42] has been extensively used in the literature for single phase fluid, the use of this technique for melting/solidification problems is extremely limited. For example, Dhar et al. [43] visualized the heatlines throughout the solidification of a eutectic solution. More recently, heatlines are used to describe a convection problem involving single phase CuO-water nanofluid [44]. The present paper is the first one to incorporate a heat line visualization and analysis during melting of nano-PCM inside an enclosed space under constant heat flux boundary condition.

Furthermore, paraffin based PCMs are very popular among the researcher. Specifically, a large number of research articles [e.g., 18,36] used research grade n-octadecane which is an ideal PCM having melting temperature \( \approx 28^\circ \text{C} \) several degrees above a usual room/lab temperature (22–23°C) and has excellent temperature-time transition characteristics. Different types of nanoparticles were incorporated into n-octadecane to prepare nano-PCMs (e.g. [18]) and a moderate stability with sedimentation was reported in the literature. The relatively inexpensive coconut oil PCM (used in this work) is edible, having melting temperature (\( \approx 24^\circ \text{C} \)) close to the typical room/lab temperature (22–23°C), and exhibits excellent stability and very small sedimentation for longer uses once nanoparticles are incorporated (tested in the lab). However, the temperature-time transition characteristics of coconut oil is not as ideal/smooth as n-octadecane, which probably makes it less used PCM in research application.

In summary, in the present article the authors first time studied the melting process of edible coconut oil PCM (\( T_m = 24^\circ \text{C} \)) with CuO nanoparticles inside a rectangular enclosure with one isoflux wall and remaining adiabatic walls. Properties of the nano–PCM were measured and compared with the modeled equations, a maximum discrepancy of \( \pm 5\% \) was observed for the thermal conductivity of nano-PCM. Furthermore, a detailed scale analysis is reported and functional relationships between melt fraction, Nusselt number, and dimensionless time were established for nano-PCM inside rectangular enclosure with isoflux boundary condition. Heatline visualization technique was successfully applied to understand the energy flow pattern inside the enclosure with nano-PCM during the melting process.

### 2. Problem description, physical model, and mathematical approximation

Active wall thermal energy storage (AW-TES) systems are typically structural components of energy efficient buildings [45]. Such components are capable of storing energy from low potential sources (e.g., solar and waste heat) and releasing the stored energy when required. A schematic diagram of an AW-TES system is shown in Fig. 1(a) which contains multiple PCM filled enclosures. In this paper, the enclosures of the AW-TES system are assumed packed with nano-PCM.

One enclosure, which is further approximated by a two-dimensional enclosure (Fig. 1(b)), is investigated under constant heat flux boundary condition. The enclosure is initially occupied by a solid form of a nano-PCM. Constant heat flux thermal source is suddenly exposed to one of the vertical walls of the enclosure to initiate the melting process. The remaining three walls are assumed properly insulated by an insulation liner or surrounded by a primary insulating material (e.g., glass fiber, polyurethane foam, and urethane rigid foam) having thermal conductivity much lower than the conductivity of the PCM itself. The main focus of this paper is on the unit enclosure (Fig. 1(b)). However, a detailed strength and structural analyses are required to perform on the entire system (Fig. 1(a)) to identify its structural integrity, which is beyond the scope of this paper.

Once the melting process is initiated the liquid-solid interface of nano-PCM will advance as time increases. It is further assumed that the liquid nano-PCM is incompressible and Newtonian fluid so that the thermophysical properties will remain constant except the density term in the vertical momentum equation. To approximate the density variation with temperature, the Boussinesq approximation is used.

The conservation of mass, momentum, and energy can be expressed by the following differential equations:

**Mass Conservation (liquid):**

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0
\]  

\[(1)\]

**Momentum Conservation (liquid):**

\[
\frac{1}{\rho_{nf}} \left[ - \frac{\partial P}{\partial x} + \mu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \right]
\]

\[
= \frac{1}{\rho_{nf}} \left[ - \frac{\partial P}{\partial y} + \mu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \right]
\]

\[+ g(\rho_b)(T - T_m)\]

\[(2)\]

**Energy Conservation (liquid):**

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]
\]

\[(4)\]

**Energy Conservation (solid):**

\[
\frac{\partial T}{\partial t} = \frac{k_f}{(\rho c_p)_{nf}} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]
\]

\[= \frac{k_f}{(\rho c_p)_{nf}} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]
\]

\[= (1 - \varphi)(\rho c_p)_{nf} + \varphi(\rho c_p)_{nf}\]

\[= (1 - \varphi)(\rho c_p)_{nf} + \varphi(\rho c_p)_{nf}\]

\[(5)\]

**where**

\( u, v, t, \rho, \mu, \beta, T, T_m, k_{nf}, \) and \( c_{pmf} \) are the velocity component in the x-direction, velocity component in the y-direction, time, density of the nano-PCM, pressure, dynamic viscosity of nano-PCM, gravitational acceleration, coefficient of thermal expansion of the liquid nano-PCM, temperature, melting temperature of the nano-PCM, thermal conductivity of the nano-PCM, and specific heat at constant pressure of the nano-PCM, respectively.

The thermophysical properties of the liquid and solid forms of nano-PCM, available in Eqs. (1)–(5), can be obtained from the following equations [26,46]:

**density of nano – PCM:**

\( \rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_n. \)

\[(6)\]

**heat capacity of nano – PCM:**

\( (\rho c_p)_{nf} = (1 - \varphi)(\rho c_p)_f + \varphi(\rho c_p)_n. \)

\[(7)\]
expansion coefficient of nano–PCM: 
\[ (\rho \beta)_{nf} = (1 - \varphi)(\rho \beta)_f + \varphi(\rho \beta)_n, \]  
\[ (8) \]

latent heat of nano–PCM: 
\[ (\rho h_f)_{nf} = (1 - \varphi)(\rho h_f)_f, \]  
\[ (9) \]

viscosity of liquid nano–PCM: 
\[ \mu_{nf} = \frac{\mu_f}{(1 - \varphi)^2}, \]  
\[ (10) \]

and

thermal conductivity of nano–PCM: 
\[ k_{nf} = \frac{k_n + 2k_f - 2\varphi(k_f - k_n)}{k_n + 2k_f + \varphi(k_f - k_n)}k_f. \]  
\[ (11) \]

in Eqs. (6)–(11), \( \varphi, \rho_f, \rho_n, k_f, \) and \( k_n \) the volume fraction of the nanoparticles, density of the base-PCM, density of the nanoparticles, thermal conductivity of the base-PCM, and thermal conductivity of the nanoparticles, respectively.

The thermophysical properties of the coconut oil and CuO nanoparticles [47] are listed in Table 1 for some selected temperatures. To validate the accuracy and uncertainty involved for using the thermophysical property models (Eqs. (6)–(11)), properties were directly measured in our facilities at the Advanced Energy Conversion and Control Lab, Bio-Innovation Research Lab, and Food Research Lab for a temperature range of 10–40 °C. Measurements were executed for \( \varphi = 0–1\% \) as well. The thermal conductivity was measured using KD2 Pro and TPS-500 which showed a maximum discrepancy of \( \pm 5\% \) with the modeled equation. Viscosity was measured using a Rotational Viscometer and a Vibration Viscometer which showed a maximum discrepancy of \( \pm 2.5\% \) with the modeled equation. The remaining properties (i.e., density, specific heat, and heat of fusion) showed discrepancies within \( \pm 1\% \) with the modeled equations. Density was measured directly and using a DMA 35 Density meter, while specific heat and heat of fusion were measured using a Differential Scanning Calorimeter. The stability of the nano-PCM was tested by periodically melting and solidifying some samples in a temperature controlled bath. Additionally, selected samples of the nano-PCM were stored and tested for approximately six months in the lab environment and negligible sedimentation was identified.

In subsequent sections, scale analysis and numerical methods are applied to solve Eqs. (1)–(5) using the following boundary and initial conditions:
horizontal walls (top and bottom): $\frac{\partial T(x, 0, t)}{\partial y} = \frac{\partial T(x, L, t)}{\partial y} = 0, u = v = 0$,
left wall vertical wall: $-k_{nf} \frac{\partial T(0, y, t)}{\partial x} = q^\ast$,
right wall vertical wall $\frac{\partial T(L, y, t)}{\partial x} = 0, u = v = 0$,
interface condition: $T(D, y, t) = T_m, q^\ast = \rho_{nf} h_{nf} \frac{\partial D}{\partial t}$,
and initial condition: $T(x, y, 0) = T_0, u = v = 0$.

In Eq. (12), $L, h_{fr}$, and $D$ represent the height of the enclosure, latent heat of fusion of the nano-PCM, and position of the liquid-solid interface from the left wall, respectively.

Finite element method is used to numerically solve the governing equations (i.e., Eqs. (1)–(5)) with the initial and boundary conditions (Eq. (12)). A grid system of 26484 triangular and quadrilateral elements was formed for a time step of 0.1 s from the beginning of the simulation until 9 s. It is from 10 s to 99 s, and 5 s from 100 s to the end of the melting process. The solution algorithm started by assuming initial values for the temperature, velocity, and the pressure of the nano-PCM. Then the new values of the temperature, velocity, and the pressure are re-calculated. If the difference between the previous and recent values is higher than the convergence criteria, the calculations were repeated; otherwise, the values will be confirmed. The selected convergence criteria were $10^{-3}$. The detailed description of the numerical methods is available elsewhere [48] and is not repeated here.

3. Scale analysis

In this section, a scale analysis is performed to identify appropriate dimensionless numbers and establish functional relationships between these numbers to understand the melting process of nano-PCM better. The melting process passes through four qualitatively predictable regimes as shown in Fig. 1(c)–(f): (i) conduction regime (Fig. 1(c)), (ii) mixed (conduction + convection) regime (Fig. 1(d)), (ii) pure convection regime (Fig. 1(e)), and (iv) shrinking solid (or variable height) regime (Fig. 1(f)) [41]. The following subsections detail the scale analysis for these regimes.

3.1. Conduction regime

Conduction is the dominant mode of heat transfer at the beginning of the melting process. Therefore, one can balance the amount of heat transfer flux $q^\ast$ through the interface with the rate of enthalpy absorbed using

$$q^\ast = q^\ast_{ext} = \rho_{nf} h_{fr} \frac{dx}{dt}, \quad (13)$$

where $q^\ast$ is the rate of the enthalpy absorbed at the interface per unit area and $h_{fr}$ is the latent heat of fusion of the nano-PCM. One can substitute the scaling parameter $x \sim D$ into Eq. (13) to obtain

$$q^\ast \sim \rho_{nf} h_{fr} \frac{D}{T} \quad \text{(14)}$$

The thickness of the melted nano-PCM $D$ can be expressed, after re-arranging Eq. (14), as

$$D \sim L (Stef \times Fo) \quad \text{(15)}$$

where Ste is the Stefan number ($Ste = \frac{q^* c_{pd} L}{k_{nf} h_{fr}}$) and $Fo$ is the Fourier number, ($Fo = \frac{k_{nf} t}{\rho_{nf} C_{pd} L^2}$), respectively. The term $Stef \times Fo$ is called time group [41]. Equation (15) concludes that the thickness of the melted portion of nano-PCM in conduction regime is proportional to $L$ and $(Stef \times Fo)$. The nanoparticle volume fraction $(\varphi)$ appears inside $Stef \times Fo$ through the thermophysical property definitions. Therefore, any change in $\varphi$ directly influences the value of $Stef \times Fo$, as well as, the interface thickness $D$. For example, specific heat and the latent heat of fusion decreases with increasing $\varphi$, while density and thermal conductivity decreases. However, reduction in the specific heat is smaller in comparison with the reduction in the latent heat of fusion, which leads to higher Ste. Similar, conclusion can be achieved for $Fo$. In conclusion, $Stef \times Fo$ increases with increasing $\varphi$ which leads to an increase in the thickness $D$.

$Nu$ in conduction regime can be defined as [49]:

$$Nu = \frac{q^\ast L}{k_f (T_h - T_m)} \quad \text{(16)}$$

where $q^\ast$ can be expressed as,

$$q^\ast = -k_{nf} \frac{\partial T}{\partial x} \bigg|_{x=0} \quad \text{(17)}$$

Using the scaling parameters, $T \sim (T_h - T_m), x \sim D$, Eq. (17) becomes

$$q^\ast \sim k_{nf} (T_h - T_m) \frac{D}{D} \quad \text{(18)}$$

Using Eqs. (15), (16) and (18) and after simplifications, the scale of $Nu$ can be expressed as

$$Nu \sim \frac{k_{nf}}{k_f} (Stef \times Fo)^{-1} \quad \text{(19)}$$

As can be observed from Eq. (19), $Nu$ decreases with advancing time in the conduction regime. Also, Eq. (19) reveals the increment in $Nu$ due to the increase in the thermal conductivity of the nano-PCM.
3.2. The mixed (conduction + convection) regime

With the advancement of time during the melting process, the upper part of the liquid nano-PCM region starts expanding due to the appearance of convection current, while the lower portion of the liquid nano-PCM is still dominated by conduction. As a result, the liquid-solid interface of the nano-PCM is deformed. The deformation of the interface indicates ending the pure conduction regime.

Applying scaling parameters to the continuity equation (Eq. (1)), \( x \sim \delta_b \) and \( y \sim b \), one can obtain the following relationship

\[
\frac{u}{\delta_b} \sim \frac{v}{b} \quad (20)
\]

Establishing a balance between the friction force and the buoyancy force in the \( y \)-momentum equation and recalling that when \( Pr \gg 1 \) [41] (for coconut oil \( Pr = 394.73 \), Table 1) for the liquid nano-PCM, one obtains after re-arrangements

\[
v \sim \frac{g(\rho\beta)_{nf} q}{k_{nf} \mu_{nf}} b^3 \delta_b^3 \quad b^3 \quad (21)
\]

Similarly, applying the scale analysis to the energy equation in the liquid region of nano-PCM results in

\[
v \sim \frac{\alpha_{nf} b}{\delta_b} \quad (22)
\]

Combining Eqs. (21) and (22), the thermal boundary layer thickness in the upper convection region can be expressed as

\[
\delta_b \sim bR_a^{-1} \quad (23)
\]

where \( R_a \) is the Rayleigh number based on the length scale \( b \)

\[
(R_a = g(\rho\beta)_{nf} q b^4 / k_{nf} \mu_{nf} \alpha_{nf}).
\]

At the lower part of the convection region, the conduction-convection transition level scales as

\[
\delta_b \sim D \quad (24)
\]

At this level, the convection effect disappears and the conduction effect starts. Combining Eqs. (15), (23) and (24) and knowing that \( R_{ab} = (d/L)^4Ra \) result in a scale for the vertical height \( b \) of the upper convection region

\[
b \sim L(Ste \times Fo)^5Ra \quad (25)
\]

where \( Ra \) is the Rayleigh number based on the length scale \( L \)

\[
(Ra = g(\rho\beta)_{nf} q L^4 / k_{nf} \mu_{nf} \alpha_{nf}).
\]

In the mixed regime, the total heat transfer rate per unit length can be calculated using the convection over the height \( b \) and the conduction over \( L - b \) as

\[
\dot{Q} = k_{nf} (L - b) \frac{(T_h - T_m)}{D} + k_{nf} b \frac{(T_h - T_m)}{\delta_b}. \quad (26)
\]

Using Eq. (15) for the conduction region and Eq. (23) for the convection region, a scale of \( Nu \) in the mixed regime can be expressed as

\[
Nu \sim \frac{k_{nf}}{k_f} \left[ (Ste \times Fo)^{-1} + (Ste \times Fo)^4Ra \right]. \quad (27)
\]

The heat transfer behavior in the liquid nano-PCM region in the mixed conduction-convection regime is revealed in Eq. (27).

Equation (27) combines the conduction and convection heat transfer in the mixed conduction-convection regime. At the beginning of the mixed conduction-convection regime, conduction is the dominant mode of heat transfer. Then as time progresses, the effect of conduction inclines and the effect of convection strengthens. The mixed (conduction + convection) regime ends when \( z \sim L \). Eq. (25) suggests that end of the mixed regime occurs at \( (Ste \times Fo)_1 \) such that

\[
(Ste \times Fo)_1 \sim Ra^{-1/4} \quad \text{and} \quad Nu_{min} \sim \frac{k_{nf}}{k_f} Ra^{3/4}. \quad (28)
\]

3.3. The convection regime

When the deformed interface touches the lower surface of the enclosure, it indicates the beginning of pure convection regime. In this stage, the convection currents occupied the whole melted nano-PCM region. The dimensionless Nusselt number in the convection regime can be defined by the following equation assuming \( Pr > 1 \)

\[
Nu = \frac{hL}{k_f}. \quad (29)
\]

By using a simple energy balance, the scale for the convection heat transfer coefficient \( h \) can be expressed in terms of thermal conductivity and thermal boundary layer thickness \( (\delta_T) \) as

\[
h \sim \frac{k_{nf}}{\delta_T}, \quad (30)
\]

which gives a scale for \( Nu \) in the convection regime

\[
Nu \sim \frac{k_{nf}}{k_f} Ra^{3/4} \quad (31)
\]

Next, we focus on the average interface distance \( (D_{av}) \) in the convection regime as the interface remains as a curvature throughout this regime. The total heat transfer rate at the interface is

\[
\dot{Q}^* \sim \rho_{nf} h_{nf} \frac{D_{av}}{dt}. \quad (32)
\]

After using Eqs. (16), (31) and (32), the scale of the average melted nano-PCM thickness \( (D_{av}) \) becomes

\[
D_{av} \sim L(Ste \times Fo)Ra^{3/4} \quad (33)
\]

When \( D_{av} \sim L \), the convection regime ends. This event occurs as suggested by Eq. (33) at time of order

\[
(Ste \times Fo)^2 \sim Ra^{-1/4} \quad (34)
\]

3.4. The shrinking solid (or variable height) regime

The fourth stage of melting (i.e., the shrinking solid or variable height regime [41]) starts after the liquid-solid interface reaches the top right corner of the enclosure, as shown in Fig. 2(d). The heat transfer and melting rates affect the amount of remaining solid nano-PCM. It is assumed that the cross section \( (lz) \) of the remaining solid nano-PCM takes the form of a triangle. The hypotenuse of the triangle represents the liquid-solid interface of the nano-PCM. The
base of the triangle \( l \) is assumed to be fixed. Therefore, the dimensionless height of the remaining solid \( (z = z/l) \) can be calculated as

\[
2^5 \sim 1 - \frac{L}{l} \text{Ra}^\frac{1}{2} \left[ (\text{Ste} \times \text{Fo}) - (\text{Ste} \times \text{Fo})_2 \right]
\]

Equation (35) suggests that the height of the remaining solid nano-PCM decreases beyond the time of scale \( (\text{Ste} \times \text{Fo})_2 \). If \( z/l < 1 \), the melting process completes at time of order \( (\text{Ste} \times \text{Fo})_3 \). Equation (35) can be written as

\[
\left( \text{Ste} \times \text{Fo} \right)_3 - \left( \text{Ste} \times \text{Fo} \right)_2 = \frac{L}{l} \text{Ra}^{-\frac{1}{2}}
\]

(36)

During the fourth stage of the melting process in some stages, a similar reduction rate of \( l \) and \( z \) is assumed. In this case, the dimensionless height of the remaining solid nano-PCM \( z \) is expressed as

\[
z \sim \left[ 1 - \frac{L}{l} \text{Ra}^\frac{1}{2} \left( \text{Ste} \times \text{Fo} \right) - \left( \text{Ste} \times \text{Fo} \right)_2 \right]^\frac{1}{2}
\]

(37)

The full melting stage, Eq. (37) can be written as

\[
\left( \text{Ste} \times \text{Fo} \right)_3 - \left( \text{Ste} \times \text{Fo} \right)_2 \sim \text{Ra}^{-\frac{1}{2}}
\]

(38)

For the shrinking solid nano-PCM regime, \( Nu \) scale can be expressed as

\[
\text{Nu} \sim \frac{k_{nf}}{k_f} \left[ 1 - \text{Ra}^\frac{1}{2} \left( \text{Ste} \times \text{Fo} \right) - \left( \text{Ste} \times \text{Fo} \right)_2 \right]^\frac{1}{2} \text{Ra}^\frac{1}{2}
\]

(39)

Equation (39) suggests that \( Nu \) drops with increasing \( R_a \) as the liquid-solid interface touches the right wall.

A summary of the equations, derived from the scale analysis, is reported in Table 2.
4. Results and discussion

In the previous sections, the dimensionless Nusselt numbers and interface distances are identified by applying scale analysis on the governing mass, momentum, and energy equations. This section is devoted to present the results obtained from numerical solutions of the governing equations. Numerical technique is applied to investigate detail transport characteristics of nano-PCM (CuO nanoparticles disperse in coconut oil PCM) during the melting process inside an enclosed space. A detailed description of the numerical technique is presented in Mahmud and Pop [48].

Constant heat flux boundary condition is applied to the left vertical wall of the enclosed space, while the remaining walls are kept thermally insulated. Numerical simulation is carried out for nanoparticle volume fraction range 0% ≤ φ ≤ 4% and Rayleigh number range $10^4 \leq Ra_{φ=0.06} \leq 10^8$.

It should be noted that two definitions of $(Ste \times Fo)$ and $Ra_{φ=0.06}$ are used in the paper based on the thermophysical properties used for the definitions:

(i) $(Ste \times Fo)_{φ=0.06}$ and $Ra_{φ=0.06}$ are based on the thermophysical properties of base-PCM (i.e., thermophysical properties when no nanoparticles are added to the PCM).

(ii) $(Ste \times Fo)_{φ}$ and $Ra_{φ}$ are based on the thermophysical properties of nano-PCM. For example, the definition of $Ra_{φ=2%}$ includes the thermophysical properties of nano-PCM at φ = 2% loading.

4.1. Progression of the melting process at $Ra_{φ=0.06} = 10^5$ and φ = 2%

The progression of the melting process of nano-PCM over the time using flow and thermal fields plots at $Ra_{φ=0.06} = 10^5$ and volume fraction of 2% is presented in Fig. 2(a)–(f). The flow field in the liquid nano-PCM region is represented by velocity vectors. While the thermofluids in liquid and solid regions are represented by isothermal flood-contour plots. The blue region represents solid nano-PCM, while the colored region represents liquid nano-PCM. It should be noted that, a change in φ alters the magnitudes of thermophysical properties (see Eqs. (6)–(11)) of nano-PCM. Such changes in the properties subsequently alter the values of Rayleigh numbers even the thermophysical properties of base-PCM remain constants. It is observed that the value of $Ra_{φ=0.06}$ reduces with increasing φ. For example, $Ra_{φ=0.06} = 10^5$ when $φ = 0\%$, $Ra_{φ=3°} = 78967$ (21.03% reduction) when $φ = 3\%$, and $Ra_{φ=5°} = 67463$ (32.54% reduction) when $φ = 5\%$. Therefore, a 6.08% increase in the dimension of the enclosure when $φ = 3\%$ and a 10.34% increase in the dimension when $φ = 5\%$ are required to maintain $Ra_{φ=0.06} = 10^5$. To handle this issue, dimension of the enclosure is adjusted to achieve same Rayleigh number at higher values of φ.

Additionally, the magnitude of the liquid fraction is presented in each individual image title. Initial and boundary conditions are given in Equation (12). The phase change material used in this paper has a melting point of $T_m = 24^\circ C$ with a temperature transition of $ΔT_m = ±1^\circ C$ (approximately) during melting. Therefore, a mushy zone has appeared for approximately a 2°C temperature difference (i.e., 23°C–25°C). The liquid fraction (LF) in this paper is calculated by applying an area integration technique over the total domain of the enclosure using the following temperature constraint $T ≥ (T_m + ΔT_m)$. Therefore, the calculated areas will be zero for the elements where $T < (T_m + ΔT_m)$ (i.e., area representing solid or semi-solid) or non-zero where $T ≥ (T_m + ΔT_m)$ (i.e., area representing liquid).

The solid nano-PCM is assumed initially at its melting temperature (i.e., no sub-cooling) inside the enclosure. Melting process starts after a constant heat flux thermal source is applied to the left wall of the enclosure. Shortly after the initiation of the melting process, the liquid-solid interface appears, and it is parallel to the vertical wall of the enclosure as can be seen from Fig. 2(a) for $(Ste \times Fo)_{φ=2°} = 0.05$. Such interface indicates that conduction is the dominant mode of heat transfer within the liquid nano-PCM region. As time advances the liquid-solid interface starts to deform and the liquid nano-PCM region becomes wider at its upper part as can be seen in Fig. 2(b) for $(Ste \times Fo)_{φ=2°} = 0.13$. The deformation of the interface at the top and the semi-straightness at the lower part of the liquid-solid interface reveals the existence of the mixed conduction-convective regime. As energy transfer at the left wall continued, the deformation of the liquid-solid interface increases further as shown in Fig. 2(c). The relatively warm liquid nano-PCM, adjacent to the hot vertical wall, moves upward direction and then it moves horizontally along the upper adiabatic wall of the enclosure. The thermal energy, carried by the hot liquid nano-PCM, penetrates gradually into the solid nano-PCM through the interface during the downward motion of the liquid nano-PCM. Because of the thermal stratification effect, the warmest liquid nano-PCM occupies the upper portion of the liquid nano-PCM region. Therefore, the interface energy transfer rate gradually decreases towards the downwards direction along the interface. Therefore, the melting rate is higher at the upper portion and gradually slows down along the downward direction of the interface which results in a wider liquid region at the top of liquid nano-PCM region as shown in Fig. 2(c). As heating continued further, the liquid-solid interface moves until it reaches to the top right corner of the enclosure as shown in Fig. 2(d). Beyond this point, the height of the solid nano-PCM will not remain fixed during subsequent melting process. This last stage of melting is typically termed as variable height or shrinking solid regime [41]. Fig. 2(e) reveals a case of variable height melting regime where the melting rate of the remaining solid nano-PCM is higher at the top of the liquid-solid interface than at its bottom. At a particular time, the melting process will be entirely completed (see Fig. 2(f)) and the remaining liquid nano-PCM will be heated up sensibly until it reaches to a thermal equilibrium.

The transient temperature variations at eleven specified

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Summary of the scale analysis equations for the four melting stages of the nano-PCM.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting stage</td>
<td>Average interface distance and height</td>
</tr>
<tr>
<td>Conduction limit</td>
<td>$D \sim (Ste \times Fo)$</td>
</tr>
<tr>
<td>The mixed regime</td>
<td>$b \sim (Ste \times Fo)^5 Ra$</td>
</tr>
<tr>
<td>The convection regime</td>
<td>$D_{mf} \sim (Ste \times Fo)Ra^2$</td>
</tr>
<tr>
<td>Variable height region</td>
<td>$z \sim \left[1 + \frac{1}{4}Ra^2((-Ste \times Fo) - (Ste \times Fo)^2)\right]^2$</td>
</tr>
</tbody>
</table>
locations in the hot vertical wall are plotted in Fig. 3 for $Ra_{4,0\%} = 10^5$ and $\phi = 2\%$. In the same graph, the average temperature of the hot vertical wall is plotted as a function of time. In the conduction regime, the magnitudes of the hot wall temperatures are same at all specified vertical locations and increase linearly with time as heat input continued. Gradual appearance of boundary layers alters this pattern in the transition regime of melting where the linear variation in temperature with time is still visible however the magnitude of temperature is higher at higher locations at the hot wall. Throughout the convection regime, where boundary layers extend from the top wall to the bottom wall, insignificant variations in the hot wall temperatures with time are observed. This trend continues in the shrinking solid regime. Finally, temperatures at all locations at the hot wall increase linearly when entire enclosure is occupied with liquid nano-PCM.

4.2. The effect of the volume fraction ($\phi$) on melting process at $Ra_{4,0\%} = 10^4$

The effect of the nanoparticles volume fractions on the melting process of nano-PCM inside the enclosure is presented in Fig. 4(a)–(c) at $Ra_{4,0\%} = 10^4$. Three arbitrarily selected volume fractions ($\phi$=0%, 2%, and 4%) were studied for three selected dimensionless times ($Ste \times Fo$) = 0.08, 0.27, and 0.48. The idea behind selecting these times is to cover all melting regimes. Again, velocity vectors are selected to present the flow filed inside the liquid nano-PCM region, while isothermal floods are used to describe the thermal fields inside both liquid and solid nano-PCM regions. The flow and thermal fields progression with time at a specific volume fraction are exhibited from left to right in the graph, while the impact of the volume fraction at a specific time are exhibited from the top to bottom of the graph in Fig. 4. The physics behind melting process and developing different melting regimes is described elaborately in previous section and will not repeated again while describing Fig. 4. Dimension of the enclosure is adjusted to keep $Ra_{4,0\%} = 10^4$ at all considered $\phi$s in Fig. 4 (see discussion in the previous section). Due to this dimension adjustment, volume of the enclosure increases to 3.09% and 6.97% for $\phi = 2\%$ and $\phi = 4\%$, respectively.

At the early stage of the melting process (i.e., $(Ste \times Fo)_{\phi} = 0.08$), the appearance of the vertical liquid-solid interface indicates the existence of the conduction regime of melting for three selected $\phi$s. Temperature drops linearly from the isoflux wall to the solid-liquid interface while maintaining a uniform heat transfer rate from the top to the bottom of the enclosure. Dispersing more nanoparticles has small impact on the melting process in this regime which can be observed from the small improvement in the liquid fraction available in Fig. 4(a). Such enhancement in the melting rate at this stage is due to the higher thermal conductivity and lower latent heat of fusion at higher $\phi$.

At the middle stage of the melting process (i.e., $(Ste \times Fo)_{\phi} = 0.27$) convection dominates. Appearance of boundary layers and circulatory motion of liquid nano-PCM is clearly visible for all three selected $\phi$s. A considerable improvement in the melting process is observed at higher $\phi$. Dispersion of more nanoparticles increases the viscosity, density, and specific heat while reduce the thermal diffusivity of the nano-PCM. However, increase in viscosity is comparable to the increase in thermal conductivity. For example, adding 2% nanoparticles increase the viscosity and thermal conductivity by 4.9% and 5.6%, respectively. Therefore, an increase in density, thermal conductivity, and a decrease in thermal diffusivity of nano-PCM enhance natural convection heat transfer during the melting process of nano-PCM when melting is dominated by convection.

Variable height (or shrinking solid) melting regime appears at $(Ste \times Fo)_{\phi} = 0.48$ for all three selected $\phi$s. A large amount of liquid nano-PCM, having temperature over its melting point, circulates over small amount of shrinking solid nano-PCM and transfer thermal energy to melt it further. Dispersing more nanoparticles has considerable impact on the melting process in this regime which can be observed from the improvement in the liquid fraction available in Fig. 4(c).

4.3. Heatlines

The heatline visualization technique, pioneered by Kimura and Bejan [42], is an effective way to visualize the energy flow in many convection heat transfer problems. Because, a complete energy flow picture is difficult to achieve directly by observing isothermal lines, velocity vectors, and streamlines. Although the heatline visualization technique has been successfully applied to many convection heat transfer problems, its use in phase change heat transfer problems is extremely limited. Therefore, authors use heatline visualization technique in this paper to describe and clarify further energy flow during the melting process of nano-PCM inside the selected enclosure under isoflux boundary condition. For the present work, heatlines are plotted using the values of heatfunction ($H$) which is obtained by solving the following equation [50]:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = \frac{\partial}{\partial y} \left[ \rho_{nf} c_{nf} u T - k_{nf} \frac{\partial T}{\partial x} \right] - \frac{\partial}{\partial x} \left[ \rho_{nf} c_{nf} u T - k_{nf} \frac{\partial T}{\partial y} \right]$$

$$-E_x$$

$$-E_y$$

where $E_x$ and $E_y$ represent the net energy flow in the x-direction an y-direction, respectively [50]. Note that each energy flow component ($E_x$ or $E_y$) in Eq. (40) is composed of two fluxes: (i) conduction energy flux (i.e., $- k_{nf} \frac{\partial T}{\partial x}$ or $- k_{nf} \frac{\partial T}{\partial y}$) and (ii) convection energy flux (i.e., $\rho_{nf} c_{nf} u T$ or $\rho_{nf} c_{nf} u T$). The following boundary condition [50] is applied to solve the differential equation in Eq. (40):
where $ds_b$ is the differential boundary segment, $\mathbf{n}$ is the surface normal, and $E = E_x \mathbf{i} + E_y \mathbf{j}$ is the energy flux density vector, respectively. The symbols $b$, ref, and $P$ address a value at the boundary, a reference value, and a point at the particular location at the boundary, respectively. Following conditions are applied to ensure the continuity of energy flow at the solid-liquid interface:

\[
H_{P,b} = H_{\text{ref},b} + \int_{\text{ref},b}^{P,b} \mathbf{E} \mathbf{n} \, ds_b, \tag{41}
\]

\[
\left(\frac{\partial H}{\partial n}\right)_{\text{liquid nano-PCM}} = \left(\frac{\partial H}{\partial n}\right)_{\text{solid nano-PCM}} \quad \text{and} \quad (H)_{\text{liquid nano-PCM}} = (H)_{\text{solid nano-PCM}} \tag{42}
\]

Heatlines are presented in Fig. 5(a)–(c) for 3 selected $\left(\text{Ste} \times \text{Fo}\right)_{\psi} (= 0.03, 0.16, \text{and } 0.37)$ and $Ra_{\psi = 0} = 10^6$. Presented results cover all four regimes of melting process of the nano-PCM inside the enclosure. Again, the...
selections of \((Ste \times Fo)_{p=0\%}\), \(\phi\), and \(Ra_{p=0\%}\) are arbitrary. At the beginning of the melting process of the nano-PCM (i.e., \((Ste \times Fo)_{p=0\%} = 0.03\)), the heatlines are originated from the left vertical wall and remain as parallel horizontal lines pattern as observed from Fig. 5(a) for all 3 \(\phi\), crossed the solid-liquid interface, and finally terminated in the solid nano-PCM region which acts as a sink in this case. Conduction energy fluxes significantly dominate over the convection energy fluxes at \((Ste \times Fo)_{p=0\%} = 0.03\) and, because of the three insulated walls other than the left vertical wall, \(-k_{nf} \partial T/\partial x \gg -k_{nf} \partial T/\partial y\). Therefore, the horizontal straight line patterns of heatline appear which reveal again that the conduction is the dominant mode of the heat transfer at the initial period of the melting process. Suspending metallic nanoparticles into the PCM has small impact on the heatline pattern in the conduction regime, as observed from Fig. 5(a). The appearance of the convection regimes for all the studied volume fractions are observed from the appearance of both the non-horizontal and closed loop the heatlines in Fig. 5(b) where heatlines are plotted at \((Ste \times Fo)_{p=0\%} = 0.16\). Note that heatlines can be termed as 'free' if they are originated from a source and terminated in a sink, while it can be termed as ‘trapped’ if they form a closed lopped pattern inside one or multiple regions. At \((Ste \times Fo)_{p=0\%} = 0.16\), dominance of convection energy fluxes is comparable to conduction fluxes inside some parts of the enclosure and, in cases, this dominance can be

![Image of heatlines and thermal fields for nano-PCM](image)

**Fig. 5.** Heatlines and thermal fields for nano-PCM for different times at \(Ra_{p=0\%} = 10^6\) for three values of volume fractions (0%, 2%, and 4%) and \((Ste \times Fo)_{p=0\%} = 0.03, 0.16,\) and 0.37.
significant. All free heatlines are originated from the left vertical wall, curved upward direction inside or near the region where boundary layer formation is distinct and vertical convection heat flux (i.e., \( \rho_l C_p \dot{q}_u \)) is dominant, passed horizontally through a narrow corridor at the top of the liquid nano-PCM under the strong influence of the horizontal convection flux (i.e., \( \rho_l C_p \dot{q}_u \)), and penetrated through the liquid-solid interface. From the interface, the free heatlines further pass thorough the solid nano-PCM unit they are terminated. Beyond the boundary layer regions inside the liquid nano-PCM region, trapped heatlines are observed where both conduction and convection energy fluxes are comparable. Similar, pattern is observed also for \( (St \times Fo)_o = 0.37 \) where melting process is inside its variable height (or shrinking solid) regimes with a larger liquid nano-PCM region compared to the solid nano-PCM region. The only characteristic difference at this stage in comparison with the convection regime is the size of the trapped heatlines which is much larger.

### 4.4. Nusselt number and liquid fraction

The dimensionless Nusselt number \( Nu \) can be used to calculate the heat transfer from the left wall as defined in Eq. (16). Fig. 6 shows the impact of adding nanoparticles on \( Nu \) at \( Ra_{\phi=0\%} = 10^5 \) and \( 10^7 \) for four different volume fractions (\( \phi = 0\%, 2\%, 4\%, \) and \( 5\%) \). The beginning of the melting process, where conduction dominates, is characterized by high \( Nu \) values due to the high temperature gradient between the hot wall and the cold nano-PCM. As the conduction regime thickness increases with ongoing heating, \( Nu \) sharply drops due to decreasing in temperature difference between the hot wall and the adjacent nano-PCM. As melting progress in the mixed conduction and convection regime, \( Nu \) profile is dominated by two competing asymptotes: (i) an increasing profile in the upper convection dominated region which is proportional to \( (St \times Fo)^4 \) and (ii) a decreasing profile in the lower conduction dominated region which is proportional to \( (St \times Fo)^{-1} \). The appearance of the convection in the upper region is a result of the wide liquid region in its upper part comparing to the lower part of the liquid region. In the wider liquid region, the buoyancy forced dominates more over the viscous force thus establishing a motion in the liquid nano-PCM. Therefore, in a narrow region of time, \( Nu \) shows a concave profile with a minimum (\( Nu_{\text{min}} \)). In the convection regime of melting, \( Nu \) maintains a constant value which is independent of time. Finally, in the shrinking solid regime, \( Nu \) gradually drops to a minimum where the melting process is entire completed. An increasing \( \phi \) improves the value of \( Nu \) and, alternatively, higher values of the convection heat transfer coefficients \( \tilde{h} \) as \( \tilde{h} = Nu (k_{nf}/l) \).

Fig. 7 shows the impact of adding nanoparticles on liquid fraction at \( Ra_{\phi=0\%} = 10^5 \) and \( 10^7 \) for four different volume fractions (\( \phi = 0\%, 2\%, 4\%, \) and \( 5\%) \). As discussed in the introduction part of Section 4.1, liquid fraction is very small or zero until the temperature at the hot wall reaches \( T_{m} + \Delta T_m \). Beyond this point, liquid fraction increases monotonously as time advances and the profiles are similar for all volume fractions considered. Note that the size of the enclosure is larger (i.e., more nano-PCM) at higher volume fractions to keep Rayleigh number same (discussed in the introduction part of Section 4.1). Therefore, it requires slightly longer in real time to complete the melting process for the case of higher volume fractions.

The impact of \( Ra_{\phi=0\%} \) on \( Nu \) is shown in Fig. 8(a). Six values of \( Ra_{\phi=0\%} (10^2, 10^4, 10^5, 10^6, 10^7, \) and \( 10^8 \) ) were studied at \( \phi = 2\% \). By increasing \( Ra \), the effect of convection increases due to the increase in the buoyancy force, as a result \( Nu \) increases. The behavior of \( Nu \) is similar to that when changing the volume fraction except at \( Ra_{\phi=0\%} = 10^8 \). After the reduction \( Nu \) in the conduction regime, an increment of \( Nu \) is observed. This increment represents the mixed conduction-convection regime where the impact of convection starts, as predicated by Eq. (27).

Observing the trend of variation of \( Nu \) with \( (St \times Fo) \) and using the method described in Jany and Bejan [41], a general correlation of Nusselt number can be established using the scaling results available in Table 2 for first 3 regimes of melting. This general correlation is shown below:
Nu = \frac{k_{nf}}{k_f}(c_0 \text{Ste} \times \text{Fo})^{-1} + \frac{k_{nf}}{k_f}\left[c_1 \text{Ra}_\phi^{\frac{1}{3}} - (a \text{Ste} \times \text{Fo})^{-1}\right] \left[1 + \left\{c_2 \frac{k_{nf}}{k_f} \text{Ra}_\phi^{\frac{1}{3}}(\text{Ste} \times \text{Fo})^4\right\}^{\frac{1}{3}}\right]^{\frac{1}{2}}
\tag{43}

where \(c_0, c_1, c_2,\) and \(n\) are the correlation coefficients and can be identified by curve fitting. It should be noted that the exponent \(n\) should be a negative number as discussed in Jany and Bejan [41].

For the limiting cases of \((\text{Ste} \times \text{Fo})\), the proposed correlation can be reduced to forms already available in Table 2. For example, in the upper limit, i.e., \((\text{Ste} \times \text{Fo}) \to \infty\), Eq. (43) approaches to the scaling relationship obtained for the third melting regime, i.e.,

\[\text{Nu} = c_1 \frac{k_{nf}}{k_f} \text{Ra}_\phi^{\frac{1}{3}}\]
\[\tag{44}\]

The constant \(c_1\) can be obtained by fitting \(\text{Nu}\) asymptotes in the convection regime with \(\text{Ra}_\phi\). Similarly, in the lower limit, i.e., \((\text{Ste} \times \text{Fo}) \to 0\), Eq. (43) approaches to the scaling relationship obtained for the first melting regime, i.e.,
\[ Nu = \frac{k_{nf} (c_0 \text{Ste} \times \text{Fo})}{k_f} \]  

(45)

The constant \( c_0 \) can be obtained by fitting Nu asymptotes in the conduction regime with \( (\text{Ste} \times \text{Fo}) \). Finally, in the limit of \( (\text{Ste} \times \text{Fo}) \rightarrow (\text{Ste} \times \text{Fo})_1 \sim Ra_{10}^{1/2} \), Eq. (43) approaches to the \( Nu_{\min} \) scaling relationship obtained in Eq. (28). The remaining task is to determine two unknown coefficients \( c_2 \) and \( n \) from curve fitting.

Fig. 8(b) shows the liquid fraction as a function of time at \( \phi = 2\% \) for six different values of \( Ra_4 = 10^3, 10^4, 10^5, 10^6, 10^7, \) and \( 10^8 \). Liquid fraction profiles at all \( Ra_4 \) show a linear increase with time until the end of the melting process where some non-linearity appears. As expected, the magnitude of the liquid fraction is higher at higher values of \( Ra_4 \) due to the stronger convection effect except at \( Ra_4 = 10^3, \) where melting is mostly dominated by conduction.

4.5. Energy stored

The rate of energy stored per unit depth of the enclosure is calculated and presented in Fig. 9 for three different values of the volume fractions of the nanoparticles. The energy stored, \( E \), could be calculated as

\[ E = \int_0^t \dot{Q}(t) \, dt \]  

(46)

where \( \dot{Q}(t) \) could be found from

\[ \dot{Q}(t) = \int_0^L -k_{nf} \frac{\partial T(0, y, t)}{\partial x} \, dy \]  

(47)

In Fig. 9, a Rayleigh number of \( 10^6 \) is selected, while both definitions of Rayleigh numbers (i.e., based on thermophysical properties of base PCM and nano-PCM, \( Ra_{\phi=0} \) and \( Ra_\phi \)) are used for further discussion. For the considered base Rayleigh number (i.e., \( Ra_{\phi=0} = 10^6 \)), transient variation in the stored energy is tested for three volume fractions (i.e., \( \phi = 0, 3, \) and \( 5\% \)). As discussed earlier, the size of the enclosure will remain the same with change in \( \phi \) as the Rayleigh number is defined based on the base properties. It is observed from Fig. 9 that the amount of stored energy increases as time advances during the melting process. As mentioned earlier, the boundary condition applied to the left vertical wall is an isoflux boundary condition, while the remaining walls are insulated. Therefore, the temperature of this wall will increase gradually from its initial value after the introduction of the constant heat flux at \( t = 0 \) s. As a result, a small amount of energy is stored at the very beginning of melting. This can be observed clearly from the magnified view of \( E - t \) plot posted at the upper left corner of Fig. 9. Beyond this point, a gradual increase of energy storing rate is observed. The rate of energy stored is initially high in the conduction dominated melting zone which can be confirmed from the high slope of the profile at this stage. A slight change in the slope of the \( E - t \) curves is an indication of the transition (i.e., the conduction + convection) melting zone. Beyond this point, \( E - t \) curves exhibit an approximately linearly increasing trend as time advances further through subsequent regimes of melting (i.e.,
Overall, increasing $Ra_{4}$ considered range of very small (e.g., a change in $\phi$ from 0% to 5% results in a 0.06% increase in $(\rho_c p)_nf$. Therefore, a very small variation in the $E - t$ profiles is observed with changing $\phi$ in the conduction regime of melting. In the transition and convection with shrinking solid regimes, energy storage rate depends on the relative magnitudes of the specific heat capacities (both $\rho cp$ and $(\rho c p)_nf$ increase with $\phi$) and latent heat capacity ($\rho_l h_{fs}$ decreases with $\phi$) of nano-PCM. Overall, increasing $\phi$ slightly decreases total energy stored inside the enclosure given that the size of the enclosure is unchanged (i.e., Rayleigh number is defined at base properties).

As discussed earlier, the size of the enclosure is adjusted with changing $\phi$ in order to keep Rayleigh number constant when it is defined based on the properties of nano-PCM. Fig. 9 shows two additional $E - t$ profiles at $Ra_{\phi=3\%}$ and $Ra_{\phi=5\%}$. The enclosure at $Ra_{\phi=3\%}$ and $Ra_{\phi=5\%}$ can store more energy than enclosure at $Ra_{\phi=0\%}$ because of the larger size of the enclosure.

The effect of Rayleigh number on $E - t$ profiles is shown in Fig. 10 for a given $\phi$. More energy can be stored at higher Rayleigh numbers which is expected as the size of the enclosure is higher at larger Rayleigh number. The stored energy keeps increasing with time which is more pronounced at the highest Rayleigh number as heat flux increases. In the conduction regime of melting, the difference in energy stored with Rayleigh number is less during the beginning of the melting; as melting reaches in the convection dominated regime, a larger difference is observed due to increased melting with Rayleigh number in convection regime.

5. Experimental work

An experiment investigation was conducted to visualize the melting process of a base PCM (coconut oil) and a nano-PCM (CuO nanoparticles dispersed in coconut oil PCM). The thermophysical properties of the coconut oil and CuO are listed in Table 1. The experimental rig is shown in Fig. 11. A rectangular enclosure was made of acrylic with dimensions of 7.2 cm × 6.2 cm × 5 cm. One side of the enclosure was covered with an aluminum lid to fix a heater on it. A 2.54 cm × 2.54 cm electrical heater (Omega LUX-SRFG-202/10-P), of resistance of 341.7 $\Omega$ was used to supply a constant heat flux. A variable transformer is used to control the supplied electric power to the heater. The heater was fixed on one of the enclosure vertical surfaces. Three J-type thermocouples were fixed inside the aluminum lid. Two of them were 0.266 cm from the left and bottom edges of the lid, each was 0.01 cm from the inner and outer surfaces of the lid, respectively, and 0.043 cm apart. The third J-type thermocouple was fixed 0.266 cm from the right edge of the lid and 0.01 cm from the inner surface of the lid.

For the visualization experiment, 1 g of CuO nanoparticles were dispersed in 250 ml of liquid coconut oil at 30 °C. Selected combination of CuO and coconut oil resulted in 0.4338 wt % and $\phi = 0.0615\%$, respectively. This selection is completely arbitrary as the major intention of this experiment is to observe the melting process of PCM and nano-PCM at different wall heat flux conditions and also to avoid any sedimentation. A magnetic stirrer (Thermo Fisher Scientific, Model No: SP88854100) was used for 8 h continuously to disperse the CuO nanoparticles in the liquid coconut oil. The temperature was fixed at 30 °C to maintain the coconut oil liquid during mixing. The accuracy of the thermocouples is ±2.2 °C, and the relative uncertainty of the temperature measurement is ±4.0% and for the variable transformer is ±5%. For more details see Appendix A.

Note that, the experimental set up has provisions to handle the expanding nano-PCM while melting. This is accomplished by three vents installed at the top of the enclosure. Besides, nano-PCM was poured inside the enclosure layer by layer while allowing each layer to solidify first before pouring another layer. Additionally, precautionary measure was taken during the preparation of the nano-PCM to remove trapped air by vacuum degasification. Nevertheless, sometimes small air bubbles (or small void at the top) were observed which was unavoidable. However, authors repeated selected experiments and the discrepancies between these experiments with identical conditions were identified small.

Fig. 12 shows experimental comparison of the melting processes between the base-PCM and the nano-PCM (0.4338 wt %, $\phi = 0.0615\%$) at three different applied heat fluxes (i.e., $q = 1020, 1814, \text{ and } 2835 \text{ W/m}^2$). Five different times (i.e., 0, 20, 30, 40, and 52 min) were selected to cover the different stages of the melting process. A comparison between the experimental and numerical results is shown in Fig. 12. The numerical results are based on the same boundary and initial conditions. The images represent the experimental results, while the dotted lines represent the numerical results. A fair agreement was observed. It is observed from Fig. 12 that increasing the applied heat flux increases the melting
rate for both base-PCM and nano-PCM. For each case of applied heat flux, melting process of nano-PCM is faster than the base-PCM which qualitatively matches our findings in scale analysis and numerical simulation. The corresponding liquid fractions for both base-PCM and nano-PCM are plotted in Fig. 13. The liquid fraction was calculated using a subsequent image analysis from the experimentally obtained images. The uncertainty of the liquid fraction calculation due to this process is ±0.5%.

Results presented in Fig. 13 also confirm that for the nano-PCM melting rate is faster than the base-PCM for an applied heat flux. An increase in the applied heat flux improves the melting rate further. The observed results confirm the positive impact of the nanoparticles on improving the melting process. The melting process accelerates by enhancing the PCM with nanoparticles.

Fig. 14 shows the temperature of the base-PCM and nano-PCM inside the enclosure on the surface of the lid. Three different heat flux rates were applied ($q'' = 1020, 1814, \text{ and } 2835 \text{ W/m}^2$). For $q'' = 1020$ and $1814 \text{ W/m}^2$, the temperature of the PCM is higher.
during the melting process of nano-PCM. To represent the energy storage geometry, a rectangular enclosure is selected with one isoflux wall and three insulated walls. To form a nano-PCM, CuO nanoparticles are added to coconut oil. The following conclusions could be stated.

- The liquid-solid interface shape signifies the stage of the melting process. The conduction limit is indicated by a vertical flat interface. The mixed conduction-convection regime is indicated by the deformation in the upper part of the interface, while the remaining part of the interface is vertical flat. The convection regime is indicated by the entire deformation of the interface.
- The value of $Ra$ plays a key role to decide the melting stage at a specific time. At low $Ra_{min} = 10^4$ conduction dominates. At higher $Ra_{min}$ values, the buoyancy force strengths which causes the domination of natural convection.
- Scale analysis successfully predicated the numerical and experimental results. This is can be observed by both the melted area thickness behavior throughout the melting process and the trend of $Nu$ for the studied volume fractions and $Ra$.
- In the conduction limit, the heatlines are free, straight lines, because all the supplied thermal energy melts the solid nano-PCM. After the melting occurs and liquid nano-PCM appears, portion of the supplied energy melts the remaining solid nano-PCM, and other portion heats the liquid nano-PCM. The heatlines are free in the first region and trapped in the second region.
- The trend of $Nu$ reflects the stages of the melting process. At the conduction limit, $Nu$ decreases. At the mixed regime, $Nu$ reaches its lowest value. At the convection regime, $Nu$ gains a plateau behavior.
- The experimental results matched the predicted behavior of the PCM using the scale analysis and the numerical calculations. Also, the experimental results show the improvement of the melting process of nano-PCM in comparison with base PCM.

Nomenclature

- $b$ height of the convection dominated liquid region (m)
- $c_p$ specific heat at constant pressure (J/kg K)
- $D$ melting interface position from the left wall (m)
- $Fo$ Fourier number, $Fo = k_{nf} t / (p c_p)_{nf} L^2$
- $E$ energy flux density vector (W/m²)
- $g$ gravitational acceleration (m/s²)
- $H$ Heatfunction defined in Eq. (6)
- $h$ convection heat transfer rate (W/m² K)
- $h_f$ latent heat of fusion (J/kg)
- $k$ thermal conductivity (W/(m K))
- $L$ enclosure height (m)
- $LF$ Liquid fraction
- $l$ height of the remaining solid nano-PCM (m)
- $Nu$ average Nusselt number
- $p$ pressure (Pa)
- $Pr$ Prandtl number, $Pr = \mu_{nf} c_{pf} / \rho_{nf}$
- $Q_c$ total heat transfer rate per unit length (W/m)
- $q$ rate of heat transfer flux (W/m²)
- $Ra^*$ Rayleigh number based on $L$, $Ra^* = g(\rho\beta)_{nf} L^4 \bar{Q} / \kappa_{nf} \mu_{nf} \alpha_{nf}$
- $Ra_b^*$ Rayleigh number based on $b$, $Ra_b^* = (b/L)^4 Ra^*$
- $Ste^*$ Stefan number, $Ste^* = \bar{Q} c_{pf} / L k_{nf} h_f$
- $T$ temperature (°C)
- $\Delta T_m$ temperature transition in the mushy zone (°C)
Appendix A. Uncertainty analysis of experimental results

To evaluate the accuracy of the experimental results, uncertainty analysis is performed in this section. The analysis includes accuracy of the temperature measurements, i.e. thermocouple and DAQ system, and the variable transformer. The accuracy of the thermocouple and the DAQ system are provided by the manufacturer as tabulated in Table A-2.

Table A-2

<table>
<thead>
<tr>
<th>Sensors and equipment</th>
<th>Accuracy, ( U_t ) (°C)</th>
<th>Measured value (°C)</th>
<th>Relative uncertainty (( U_t/x ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-type thermocouples</td>
<td>±2.2°C</td>
<td>( T_{\text{max}} = 50°C )</td>
<td>±4.0%</td>
</tr>
<tr>
<td>DAQ system</td>
<td>±0.05°C</td>
<td>( T_{\text{max}} = 50°C )</td>
<td>±11.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( T_{\text{min}} = 20°C )</td>
<td>±0.1%</td>
</tr>
</tbody>
</table>

The uncertainty of the temperature measuring set can be calculated using the root-sum-squared method [51].

\[
U_t = \sqrt{\left(\frac{U_{\text{ACC}}}{X_t}\right)^2 + \left(\frac{U_{\text{DS}}}{X_d}\right)^2} \tag{A-1}
\]

where \( U_t, U_{\text{ACC}}, U_{\text{DS}}, X_t, \) and \( X_d \) represent the relative uncertainty of the experiment, the accuracy of DAQ system, the accuracy of the thermocouple, the measured value by the DAQ system and the measured value by the thermocouple, respectively. As a result, the uncertainty of the temperature measurement experiment is defined as ± 7.24%.


