Cooperative path planning for multi-AUV in
time-varying ocean flows

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Abstract: For low-speed underwater vehicles, the ocean currents have a great influence on them, and the changes in ocean currents is complex and continuous, thus whose impact must be taken into consideration in the path planning. There are still lack of authoritative indicator and method for the cooperating path planning. The calculation of the voyage time is a difficult problem in the time-varying ocean, for the existing methods of the cooperating path planning, the computation time will increase exponentially as the autonomous underwater vehicle (AUV) counts increase, rendering them unfeasible. A collaborative path planning method is presented for multi-AUV under the influence of time-varying ocean currents based on the dynamic programming algorithm. Each AUV cooperates with the one who has the longest estimated time of sailing, enabling the arrays of AUV to get their common goal in the shortest time with minimum time-difference. At the same time, they could avoid the obstacles along the way to the target. Simulation results show that the proposed method has a promising applicability.

Keywords: dynamic programming, time-varying, cooperate, path planning, autonomous underwater vehicle (AUV).

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1. Introduction

Multi-autonomous underwater vehicle (AUV) path planning[1–3] means that it plans routes for the AUVs simultaneously under the consideration of cooperative constrains, so that they can cooperate to complete the task [4] under conditions of minimum objective function value. For the low-speed underwater vehicle, such as the underwater glider [5], due to its small speed, it could be easily influenced by the ocean currents, especially for the underwater vehicle that has a long time or long distance mission, thus the influence of the ocean currents exerting on it must be taken into consideration in its path planning.

The key problem of the cooperative path planning for multi-AUV in time-varying ocean flows can be decomposed in three aspects: (i) The cooperative path planning algorithm for multi-AUV is hard to design. The computation time of the normal cooperating method will increase exponentially as the AUV counts increases using the existing methods, so it is necessary to design new algorithms to reduce computational complexity. (ii) The ocean flows are typical continuous and time-dependent vector field, so the time cost between any two points would be difficult to calculate. (iii) An evaluate function is needed to estimate collaborative performance.

The commonly used algorithms are evolutionary algorithms [6] such as genetic algorithm (GA), particle swarm optimization (PSO), heuristic search algorithms like A-Star (A*) and ant colony algorithm. PSO and GA are evolutionary iterative algorithms, and A* [7] and PSO [8] were applied in a dynamic current for path planning of single AUV. A* [9] was also applied in steady currents for path planning of single AUV, it is not only carried out simulation, but also used with the actual data to test and verify. In addition, [10] using the factorization machine (FM) algorithm for path planning in two-dimension, which focuses on obstacle avoidance. Reference [11] illustrates that the path planning could be applied in underwater collaborative navigation based on single beacon. The real time cooperative path planning for multi-AUV needs more complicated algorithms and hardware, the AUVs need to communicate with each other and the sensors in the AUV could tell the surrounding to avoid obstacles[12].
References [13] and [14] proposed the solutions for multi-UAV path planning, which can avoid the obstacles and threats detected on the way. On the bases of the above, the algorithm for the multi-AUV cooperative path planning in time-varying environment is still need to be developed.

Cooperative path planning for multi-AUV in time-varying ocean flows can be achieved as follows: the multi-AUV path planning is composed of a group of single AUVs path planning, so, the research for single AUV path planning is needed to be done. This paper reviews the speed calculation in time-varying environment and proposes the time calculation model [7], after that it employs a dynamic programming algorithm to find the path of single AUV. After paths for the series of AUVs are found, the cooperating algorithm is used to make the AUVs arrive the target at the same time. Then, an evaluation function is used to estimate the cooperating effect. The arriving time of three AUVs and the comparison of $D_{time}$ prove that the algorithm archives goals of cooperative path planning at the least computation time. Besides, during the multi-AUV cooperative path planning, the cooperation coefficient of the cooperative algorithm has a great effect on the searching process of the optimal path, so we can adjust the parameters to find the optimal path.

Compared with heuristic algorithms, evolutionary algorithms such as the PSO, are relatively better under the condition of environment and parameters are already known. Meanwhile, they do not fall into local minimum and lost the optimal path.

2. Math model of cooperate path planning

2.1 Dynamic programming model

In order to illustrate multi-AUV path planning, we introduce the single AUV path planning at first, then the dynamic programming algorithm [15] is employed to find the shortest path in the planning space. In this paper, the time consumption is considered to be the weight of the algorithm. The objective function of the algorithm is defined as follows:

$$f(s_{i,j}) = \min_{k \neq j} \left( f(s_{i-1,k}) + w(s_{i-1,k}, s_{i,j}) \right)$$

where $f(S_p)$ represents the shortest time which takes from the point $S_{start}$ to the point $S_p$, and $w(S_{i-1,k}, S_p)$ denotes the time which takes from the point $S_{i-1,k}$ to the point $S_p$. Fig.1 shows the sketch of the model. $f(S_{start})$ is the shortest time from the start point to the end point and it is the optimal path we are looking for.

Fig. 1 Sketch of dynamic programming applied in multistage decision making model

Spatial field rasterization for the planning area is the first step for the dynamic programming algorithm. The more points we designed in the planning space, the more time we need to compute, but on the other hand, the smoother and shorter path we can get.

The pseudo-code could be shown as follows:

**Algorithm 1 Dynamic programming**

for each $i \in n$ do
  for each $j \in m$ do
    for each $k \in m$ do
      if $f(s_{i,j}) > f(s_{i-1,k}) + w(s_{i-1,k}, s_{i,j})$
        then $f(s_{i,j}) = f(s_{i-1,k}) + w(s_{i-1,k}, s_{i,j})$
      end if
    end for
  end for
end for

For multi-AUV path planning, the weight between two points is composed by two parts: The first part is the time to get through from one point to another, and the target is to reach the destination within a smallest period of time. The second part is that those AUVs should arrive at the destination at the same time. Namely the weight of multi-AUV path planning is based on the weighted average of total time-consuming and time difference when all those AUVs arrive at the same search stage.

This paragraph introduces the method for the cooperating path planning of multi-AUV. Firstly, we let the group of AUVs start their path planning on their own start point at the same time, which means that each AUV makes their own path planning, without coordination constraints. This means we will do $x$ times single path planning, where $x$ is the number of the AUVs. Secondly,
we pick up the one who has the longest voyage time, and make it be the one the others need to cooperate with. Thirdly, we make the second path planning start, all of the AUVs cooperate with the one we picked up, and they are under the constraint of cooperation. The objective function can be described as

\[
f(s_{i,j}) = \min \left\{ f_i(s_{i-1,p}) + w(s_{i-1,k}, s_{i,j}) \right\} + k \cdot \text{abs}(f_i(s_{i-1,p}) + w(s_{i-1,k}, s_{i,j}) - f_{\max}(s_i))
\]

where \(k\) represents the coordination coefficient, and \(r\) is the serial number of the AUV. \(f_{\max}(S_i)\) means the time to get to the \(i\)th stage for the one who has the longest voyage time. In this paper, by making all AUV tend to be the longest voyage time, the aim of all the AUV reach the same destination simultaneously can be achieved. In other words, the path that makes the \(f(S_{\text{route}}) = f_{\max}(S_o)\) is the best path, where \(f(S_{\text{route}})\) represents the time of the \(r\)th AUV reach the \(r\)th point of the \(m\)th stage. The percentage of the total time difference to the total time they need to reach the target can be shown as

\[
D_{\text{time}} = \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \text{abs}(f_i - f_{\text{route}}) \sum_{i=1}^{n} f_{\text{route}}.
\]

Thus we can use \(D_{\text{time}}\) to evaluate the cooperate effect.

2.2 Dynamic programming algorithm complexity analysis

It is necessary to study every AUV respectively [16–19] when researching the time complexity of multi-AUV path planning, as the multi-AUV path planning is consisted of a group of individual AUVs path planning. For the single-AUV path planning, as shown in Fig. 1, the assume time complexity of each transfer time is \(O(1)\), then, the time complexity of this article for the dynamic programming algorithm is \(O(mn^2)\). Besides, when Dijkstra is applied for path planning, the time complexity of the algorithm is \(O(mn^2)\), and the amount of computations is much higher than the dynamic programming algorithm.

In the multi-AUV path planning, if we use the brute-force algorithm for every possible solution, the computation time will increase exponentially as the AUV counts increase. In this case, the time complexity is \(O((m^2n^2))\), but the time complexity of the method proposed in this paper for the multi-AUV path planning is \(O(mn^2r)\), implying that the computation time increases linearly as the number of the AUV increases. Finally, based on the analysis of the time complexity, we get the optimal path and the computation time is acceptable. The approach we proposed is practical, especially for the case of a large number of AUVs.

3. Time calculation in time varying currents

In the multi-AUV path planning, the weight is weighted average of total time and time difference between paths, both of them are based on the value of the time-consuming calculation, so, how to get the time between two points is the crucial part of the path planning problem. To facilitate the expressing, this paper uses \(w(v_x, v_y)\) to express \(w(S_{i-1,k}, S_{i,j})\) in this section, and \(w(v_x, v_y)\) can be defined as follows:

\[
w(v_x, v_y) = t_{ij} = s_{ij} / v_{ij}
\]

where \(s_{ij}\) represents the distance between the \(i\)th point and the \(j\)th point, and \(v_{ij}\) represents the velocity from \(i\)th point to the \(j\)th point, and it can be calculated through the following steps:

**Step 1** Assuming that the ocean current between two points is steady, we use the velocity of \(i\)th point \(V_{\text{ocean start}}\) as the velocity, then set the initial simulation time of the start point to zero. After that we can calculate the value of the criterion \(D\) as

\[
D = (V_x V_{\text{ocean start}})^2 + v_{\text{vehicle}}^2 - V_x V_{\text{ocean start}} V_{\text{ocean start}}
\]

where \(V_x\) represents the unit vector from the \(i\)th point to the \(j\)th point, and \(V_{\text{vehicle}}\) is the velocity of the AUV. In this paper, it is assumed that the velocity of the AUV is constant and can be adjusted to any direction. The velocity vector sketch is showed in Fig. 2.

![Fig. 2 Velocity composition sketch](image_url)
Step 3 After that, $V_{route}$ is used to replace $v_{ij}$ in (4) to estimate the rough value of the time it takes from the $i$th point to the $j$th point, and set as $t_{rough}$.

Step 4 Assuming that the change of the ocean currents is already known, then the model of ocean current satisfies the following equation:

$$V_{ocean\_current} = f(x, y, t).$$  \hspace{1cm} (7)

Step 5 Equation (7) is based on the theory that if the position of the point in the path and time are already known, then we could get the size and direction of the ocean currents. Let $t_{start}$ represent the simulation time when the AUV is at the $i$th point. Then the simulation time will be $t_{start} + t_{rough}$ when the AUV arrives at the $j$th point. After that bringing $t_{start} + t_{rough}$ into (7) as the time $t$, we can get the velocity of ocean current $V_{ocean\_end}$ at the $j$th point.

Step 6 Use $(V_{ocean\_start} + V_{ocean\_end})/2$ as the average velocity between the $i$th point and the $j$th point, and let $V_{ocean\_current}$ express it, then bring it into (6) to get the new $V_{route}$, and then bring new $V_{route}$ into (4) to get $w(v_i, v_j)$.

4. Simulation and analysis

The size of the simulation area is 200 m $\times$ 150 m, within the region there are five static eddies. Assuming that there are three AUVs in the path planning simulation, arrows in the figure represent the relative direction and size of the ocean currents. When there are three AUVs for cooperative planning, according to (3), we can

$$D_{time} = \frac{[\text{abs}(f_1 - f_2) + \text{abs}(f_2 - f_3) + \text{abs}(f_3 - f_1)]}{(f_1 + f_2 + f_3)}.$$  \hspace{1cm} (8)

Set the coordinates of start point of the 1th, 2th, 3th AUV (0, 110), (0, 0), (100, 0), respectively, Their coordinates of common goal point is (200, 150), and their start time are zero. Fig. 3-Fig. 12 are the sequence diagrams of ocean currents distribution and paths found by three AUVs when the dynamic programming algorithm is deployed, where the dotted circles point out the major affected area of the changing current flows. The center of the changing flows is updated every certain amount of time, as shown in Table 1. The $t_1$, $t_2$, $t_3$ in the caption of the figures mean the time when three AUVs reach the same stage, which shows the position of three AUVs at the specified point-in-time. The rectangle bar is the obstacle the AUVs must avoid. In Tables 2, 3 and 4, $f_1$ is the time of the AUV whose start point is (0, 110), $f_2$ corresponds to (0, 0), and $f_3$ corresponds to (100, 0). The value of $D_{time}$ shows that there is a significant time-difference among each AUV.
Fig. 7 \( t_1 = 193.29 \) s, \( t_2 = 231.02 \) s, \( t_3 = 206.08 \) s

Fig. 8 \( t_1 = 84.78 \) s, \( t_2 = 85.84 \) s

Fig. 9 \( t_1 = 100.30 \) s, \( t_2 = 101.36 \) s

Fig. 10 \( t_1 = 129.84 \) s, \( t_2 = 129.84 \) s, \( t_3 = 75.11 \) s

Fig. 11 \( t_1 = 175.52 \) s, \( t_2 = 176.58 \) s, \( t_3 = 114.89 \) s

Fig. 12 \( t_1 = 229.97 \) s, \( t_2 = 231.02 \) s, \( t_3 = 229.30 \) s

Table 1  Position of the changing eddy

<table>
<thead>
<tr>
<th>Number</th>
<th>Time</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 0 \leq t &lt; 70 )</td>
<td>(200, 0)</td>
</tr>
<tr>
<td>2</td>
<td>( 70 \leq t &lt; 100 )</td>
<td>(160, 30)</td>
</tr>
<tr>
<td>3</td>
<td>( 100 \leq t &lt; 130 )</td>
<td>(120, 60)</td>
</tr>
<tr>
<td>4</td>
<td>( 130 \leq t &lt; 160 )</td>
<td>(80, 90)</td>
</tr>
<tr>
<td>5</td>
<td>( 160 \leq t )</td>
<td>(40, 120)</td>
</tr>
</tbody>
</table>

Table 2  Path planning time of single AUV (without ocean current environment) (1)

<table>
<thead>
<tr>
<th>Number</th>
<th>Voyage time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>162.37</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>210.67</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>93.71</td>
</tr>
</tbody>
</table>

Table 3  Path planning time of single AUV (without ocean current environment) (2)

<table>
<thead>
<tr>
<th>Number</th>
<th>Voyage time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>193.29</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>231.02</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>237.68</td>
</tr>
</tbody>
</table>

\( D_{time} \)%: 11.97
4.1 Non-cooperative simulation of multi-AUV path planning

In this section of the experiments, as shown in Fig. 3–Fig. 7, each AUV’s coordination coefficient $k$ is set to 0, which means that the search process is carried out without coordination, besides, from the figures we can see that the paths of AUVs are overlapping partly. The non-cooperative simulation of path planning consists of two parts: firstly, Fig. 3 and Fig. 4 show the paths found by three AUVs without consideration of oceans currents. The arrows in those two figures are used to maintain consistency with other figures. Table 2 lists the corresponding time spend by three AUVs reaching their common goal. Secondly, Fig. 5–Fig. 7 are experiment results which are conducted under the circumstances of taking the ocean currents into consideration comparing to the first part. Table 3 shows the corresponding time spend reaching their common goal by three AUVs and their total time difference ratio. The second part of the experiment is also the first path planning defined in Section 2.1. Finally, the difference between paths in Fig. 4 and Fig. 7 and the data gap between Table 2 and Table 3 demonstrate ocean currents have a great influence on AUVs, thus we must take the impact of ocean currents into consideration for the path planning.

4.2 Cooperative simulation of multi-AUV path planning

Fig. 8–Fig. 12 are the sequence diagrams of ocean current distribution and motion trajectory of AUVs, and they show the position of three AUVs at the specified point-in-time. Table 4 shows the corresponding time spend by the three AUVs and their total time difference ratio.

After the experiment of cooperative path planning for three AUVs is conducted, we could evaluate the cooperating effect by the comparison of $D_{time}$ in the first path planning and the second path planning under the environment of time-varying ocean flows. The value of $D_{time}$ in the second path planning is 0.32%. It is much smaller than that of the first path planning, which is 11.97%, showing that the cooperating model of multi-AUV and the dynamic programming algorithm used in this paper realized the goal of cooperative path planning basically. Furthermore, the experiments in Section 4.1 not only are the basis of the experiment in Section 4.2, but also make a sharp contrast with each other in paths, total time and time-difference, and the contrast proves that the cooperating algorithm has taken effect for the path planning.

5. Conclusions

This paper conducts multiple collaborative paths planning in time-varying environment by a dynamic programming algorithm. Firstly, we propose a mathematical model for single and multiple AUV's path planning and calculate the time complexity of the algorithm, which shows that the proposed algorithm has good feasibility and practicality. Secondly, we introduce the time calculation method used in this paper, then the time spend between two search points in time-varying ocean flows could be calculated with some approximation by the method. Finally, a three AUVs cooperating path planning in the time-varying environment is conducted. From simulation results, we could get the longest time-consuming AUV by letting each AUV do their own path planning, for each one among three AUVs is not disturbed by the other two AUVs. After that, we pick up the longest one from them and make it the target AUV that the others should cooperate with in the second path planning. The comparison of the $D_{time}$ in the two times path planning shows that the three AUVs could reach their common goal almost at the same time, thus the algorithm has good applicability.

References


Biographies

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