Multi-sensor optimal fusion fixed-interval Kalman smoothers

Shu-Li Sun *

Department of Automation, Electronic Engineering College, Heilongjiang University, Road Xuefu, No. 74, Harbin, Heilongjiang 150080, China

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Abstract

Based on the optimal weighted fusion algorithms in the linear minimum variance sense, the optimal fusion fixed-interval Kalman smoothers are given for discrete time-varying linear stochastic control systems with multiple sensors and correlated noises, which have a three-layer fusion structure. The first and the second fusion layers both have netted parallel structures to determine the cross-covariance matrices of prediction and smoothing errors between any two-sensor subsystems, respectively. The third fusion layer is the fusion centre to determine the optimal weights and obtain the optimal fusion fixed-interval smoothers. Smoothing error cross-covariance matrix between any two-sensor subsystems is derived. Applying it to a tracking system with three-sensors shows the effectiveness.

Keywords: Multi-sensor; Optimal information fusion; Fixed-interval Kalman smoother; Cross-covariance; Tracking system

1. Introduction

Fixed-interval smoothers can be applied to the track reconstruction of the maneuver target such as airplane, missile, robot, etc. [1]. In the distributed multi-sensor environment, it is very important to know how to fuse the information from multiple sensors together. Generally, we combine all measurement vectors from different sensors into one measurement vector, and then we can obtain the centralized filter by using the standard Kalman filtering methods. But the centralized filter will bring a large computational burden in the fusion center due to the high-dimension computation and large data memory. Recently, the information fusion distributed Kalman filter has been further studied and widely applied in communication and control fields since the parallel structures can increase the input data rates and have reliability. Carlson [2] presents the famous federated square root filter. But, to some extent, it is conservative because of using the upper bound of the variance matrix of the process noise instead of the variance matrix itself. Bar-shalom [3] deals with the fusion filter for two-sensor systems and gives the computation formula of cross-covariance matrix between two-sensors, where the process noise is independent of the measurement noises. Under the assumption of the normal distribution, Kim [4] and Chen et al. [5] give the optimal fusion filter for systems with multiple sensors based on the maximum likelihood estimation, respectively, and assume the process noise to be independent of the measurement noises. Vorobyov et al. [6] give the fusion estimator for a scalar signal as a special case of [4,5]. Li et al. [7] give the unified fusion rules based on a unified linear model for centralized, distributed and hybrid fusion architectures in weighted least square sense and best linear unbiased estimation sense. Sun [8] gives three optimal weighted fusion algorithms weighted by matrices, diagonal matrices and scalars in the linear minimum variance sense, where the optimal fusion algorithm weighted by matrices is same as [4,5,7], but the derivation is not based on any model and the assumption of the normal distribution is avoided. Furthermore, the weighted fusion algorithms are applied to the multi-channel ARMA signal filtering [9], the optimal fusion filter for discrete time-varying linear systems with correlated noises [10] and the white noise fusion estimation [11,12]. The data fusion problem in sensor networks is addressed in [14] where the probabilistic
models and Bayesian data fusion methods are emphasized. The distributed Kalman filter with embedded consensus filter is investigated in [15]. The consensus problems are addressed under a variety of assumptions on the network topology being fixed or switching, presence or lack of communication time-delays, and directed or undirected network information flow in [16]. So far, the fusion estimation for the system state is mainly focused on the filtering fusion. However, the smoothing fusion is seldom reported, particularly, for the smoothing fusion in a fixed finite interval.

In this paper, three distributed optimal weighted fusion fixed-interval Kalman smoothers with a three-layer fusion structure are given for discrete time-varying linear stochastic control systems with multiple sensors and correlated noises based on three optimal weighted fusion algorithms weighted by matrices, diagonal matrices and scalars in the linear minimum variance sense. They have better accuracy than any local smoother does. The fixed-interval smoothing error cross-covariance matrix is derived between any two-sensor subsystems.

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The rest is organized as follows: the problem formulation is given in Section 2. In Section 3, the smoothing error cross-covariance matrix between any two-sensor subsystems is derived. In Section 4, the distributed optimal fusion fixed-interval Kalman smoothers are given. A three-layer fusion structure is presented. The simulation example in a tracking system with three-sensors is shown in Section 5 where local, distributed fusion and centralized smoothers are compared. Finally, the conclusion is drawn.

2. Problem formulation and lemmas

Consider the discrete time-varying linear stochastic control system with $L$ sensors

$$
x(t + 1) = \Phi(t)x(t) + B(t)w(t) + \Gamma(t)w(t)
$$

$$
y_i(t) = H_i(t)x(t) + v_i(t), \quad i = 1, 2, \ldots, L
$$

where $x(t) \in \mathbb{R}^n$ is the state, $y_i(t) \in \mathbb{R}^m_i$ is the measurement, $w(t) \in \mathbb{R}^n$ is a known control input, $w(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n_i$ are white noises, and $\Phi(t), B(t), \Gamma(t), H_i(t)$ are time-varying matrices with suitable dimensions. The subscript $i$ denotes the $i$th sensor and $L$ is the number of sensors.

In the following, $I_n$ denotes the $n \times n$ identity matrix, and $0$ denotes the zero matrix with suitable dimension.

**Assumption 1.** $w(t)$ and $v_i(t)$, $i = 1, 2, \ldots, L$ are correlated white noises with zero mean and

$$
E \begin{bmatrix}
w(t) \\
v_i(t)
\end{bmatrix} = Q_i(t)\delta_{i0},
$$

$$
Q_i(t) = \begin{bmatrix}
Q_{ii}(t) & S_i(t) \\
S_i^T(t) & R_i(t)
\end{bmatrix}
$$

where $R_i(t)$ is the variance of $v_i(t)$, i.e., $R_i(t)$, the symbol $E$ denotes the mathematical expectation, the superscript $T$ denotes the transpose, and $\delta_{i0}$ is the Kronecker delta function.

**Assumption 2.** The initial state $x(0)$ is independent of $w(t)$ and $v_i(t)$, $i = 1, 2, \ldots, L$, and

$$
E[x(0)] = \mu_0, \quad E[(x(0) - \mu_0)(x(0) - \mu_0)^T] = P_0.
$$

Our aim is to find the optimal fusion fixed-interval Kalman smoothers $\hat{x}_i(N)$ for the state $x(t)$ from the weighted fusion of local smoothers $\bar{\hat{x}}_i(N)$ based on measurements

$$
y_j(N), y_j(N - 1), \ldots, y_j(1), \quad i = 1, 2, \ldots, L;
$$

$$
t = 0, 1, \ldots, N - 1, \quad \text{the fixed integer} N > 0, \quad \text{based on the optimal (i.e., linear minimum variance) fusion algorithms}
$$

weighted by matrices, diagonal matrices and scalars [8], which will satisfy that

(a) $\hat{x}_i(t|N) = A_i^{(1)}(t)\hat{x}_i(t|N) + A_i^{(2)}(t)\hat{x}_i(t|N) + \cdots + A_i^{(L)}(t)\hat{x}_i(t|N)$,

(b) Unbiasedness, namely, $E[\hat{x}_i(t|N)] = E[x(t)]$.

(c) Optimality, namely, to find the optimal weights $A_i^{(l)}(t), i = 1, 2, \ldots, L$ to minimize the traces of variance matrices of the weighted fusion smoothers $\hat{x}_i(t|N)$, respectively.

One has the following preliminary lemmas before stating the main results.

**Lemma 1.** Under Assumptions 1 and 2, the $i$th local sensor subsystem of system (1) and (2) with multiple sensors has the optimal Kalman first-step predictor [17]:

$$
\hat{x}_i(t + 1|t) = \Phi(t)\hat{x}_i(t|t - 1) + B(t)u(t) + K_P(t)e_i(t),
$$

$$
e_i(t) = y_i(t) - H_i(t)\hat{x}_i(t|t - 1),
$$

$$
K_P(t) = \left[\Phi(t)P_i(t - 1) + \Gamma(t)S_i(t)\right]Q_i^{-1}(t),
$$

$$
Q_i(t) = H_i(t)P_i(t - 1) + H_i(t)^T(t) + R_i(t),
$$

with the initial value $\hat{x}_i(0|0) = \mu_0, P_P(t)$ is the prediction gain matrix, $e_i(t)$ is the innovation sequence, and the first-step prediction error variance matrix $P_i(t|t - 1)$ satisfies the following recursive equation:

$$
P_i(t + 1|t) = \left[\Phi(t) - K_P(t)H_i(t)\right]P_i(t|t - 1)\left[\Phi(t) - K_P(t)H_i(t)\right]^T + \left[\Gamma(t) K_P(t)\right]Q_i(t)\left[\Gamma(t) K_P(t)\right]^T,
$$

with the initial value $P_i(0|0) = P_0$.

**Lemma 2.** Under Assumptions 1 and 2, the $i$th local sensor subsystem of system (1) and (2) with multiple sensors has the optimal fixed-interval Kalman smoother [17]:

$$
\hat{x}_i(t|N) = \hat{x}_i(t|t - 1) + \sum_{k = 0}^{N - t} M_i(t|t + k)\hat{e}_i(t + k),
$$

where the gain matrix $M_i(t|t + k)$ is computed by

$$
M_i(t|t + k) = P_i(t|t - 1)\Phi_i^T(t + k, t)H_i^T(t + k)Q_i^{-1}(t + k).
$$
One defines that

\[ \Phi^T_i(t + k, l) = \prod_{s=l}^{t+k-1} \left[ \Phi(s) - K_{p_i}(s)H_i(s) \right]^T, \]

(12)

\[ \Phi_i(s, s) = I_n. \]

The fixed-interval smoothing error variance matrix is given by

\[ P_i(t|N) = P_i(t|t - 1) - \sum_{k=0}^{N-t} M_i(t|t+k)Q_n(t+k)M_i^T(t|t+k) \]

(13)

**Proof**. From the projection theory [17], one has that

\[ \hat{x}_i(t|N) = \hat{x}_i(t|N - 1) + M_i(t|N)\varepsilon_i(N) \]

(14)

(14) by iteration yields (10). From the definition

\[ M_i(t|t+k) = E[x(t)\xi_i(t+k)|Q_n^{-1}(t+k) \text{ of the smoothing} \]

\[ + \sum_{k=0}^{N-t} E[\hat{x}_i(t-1)\xi_i^T(t+k)]M_i^T(t|t-1) \]

(15)

From (11) one has the smoothing error equation

\[ \hat{x}_i(t|N) = \hat{x}_i(t|t-1) - \sum_{k=0}^{N-t} M_i(t|t+k)e_i(t+k). \]

(16)

From (11) and the definition of the smoothing gain matrix

\[ M_i(t|t+k), \] and noting \( x(t) = \hat{x}_i(t|t-1) + \hat{x}_i(t|t-1) \) and

\[ \hat{x}_i(t|t-1) \perp e_i(t+k), k \geq 0 \text{ where the symbol } \perp \text{ denotes} \]

(17)

Substituting (17) into (16) one has (13). \( \square \)

**Lemma 3.** Under Assumptions 1 and 2, the first-step prediction error covariance between the \( i \)th and the \( j \)th sensor subsystems has the following recursive form [12]:

\[ P_{ij}(t+1|t) = \left[ \Phi(t) - K_{p_i}(t)H_i(t) \right]P_{ij}(t|t-1)\left[ \Phi(t) - K_{p_i}(t)H_i(t) \right]^T + \left[ \Gamma(t) - K_{p_i}(t) \right]Q_{ij}(t)\left[ \Gamma(t) - K_{p_i}(t) \right]^T \]

(18)

with the initial value \( P_{ij}(0|0) = P_0. \)

**Proof.** From (1), (2), (5) and (6), one has the first-step prediction error equation for the \( i \)th local sensor subsystem:

\[ \hat{x}_i(t+1|t) = \Phi(t) - K_{p_i}(t)H_i(t)\hat{x}_i(t|t-1) + \Gamma(t)w(t) - K_{p_i}(t)\varepsilon_i(t). \]

(19)

Since \( \hat{x}_i(t|t-1) \) is uncorrelated with \( w(t) \) and \( \varepsilon_i(t) \), one can obtain (18) from (19) by using the projection theory and (3). \( \square \)

3. Smoothing error cross-covariance

**Theorem 1.** Under Assumptions 1 and 2, the fixed-interval smoothing error cross-covariance matrix between the \( i \)th and the \( j \)th sensor subsystems has the following form:

\[ P_{ij}(t|N) = \Psi_i(t|N)P_{ij}(t|t-1)\Psi_j^T(t|N) + \sum_{k=0}^{N-t} G_i(t+k)Q_j(t+k)G_j^T(t+k), \]

(20)

where \( Q_0(t) \) is defined by (3). \( \Psi_i(t|N) \) and \( G_i(t+k) = [G_{ii}^n(t+k), G_{ij}^l(t+k)] \) are defined by

\[ \Psi_i(t|N) = I_n - \sum_{k=0}^{N-t} M_i(t|t+k)H_i(t+k)\Phi_i(t+k,t), \]

(21)

\[ G_i^l(t+k) = - \sum_{l=k+1}^{N-t+1} M_i(t+l)H_i(t+l)\Phi_i(t+l,t+k+1) \times \Gamma(t+k), k = 0, 1, \ldots, N - t - 1, G_i^l(t|N) = 0, \]

(22)

\[ G_i^n(t+k) = \sum_{l=k+1}^{N-t+1} M_i(t+l)H_i(t+l)\Phi_i(t+l,t+k+1) \times K_{pi}(t+k) - M_i(t+k), \]

(23)

\[ k = 0, 1, \ldots, N - t - 1, G_i^n(t|N) = -M_i(t|N). \]

**Proof.** From (10) one has the fixed-interval smoothing error equation:

\[ \hat{x}_i(t|N) = \hat{x}_i(t|t-1) - \sum_{k=0}^{N-t} M_i(t|t+k)e_i(t+k). \]

(24)

From (2) and (6) one has

\[ e_i(t+k) = H_i(t+k)\hat{x}_i(t+k|t+k-1) + \varepsilon_i(t+k). \]

(25)

From (19) one has the first-step prediction error equation:

\[ \hat{x}_i(t+k|t+k-1) = \left[ \Phi(t+k-1) - K_{p_i}(t+k-1) \right] \times H_i(t+k-1)\hat{x}_i(t+k-1|t+k-2) + \Gamma(t+k-1)\varepsilon_i(t+k-1) - K_{p_i}(t+k-1)\varepsilon_i(t+k-1) \times \Gamma(t+k-1)w(t+k-1) \]

(26)

(26) by iteration yields

\[ \hat{x}_i(t+k|t+k-1) = \Phi_i(t+k,t)\hat{x}_i(t|t-1) + \sum_{l=0}^{k-1} \Phi_i(t+k,t+l+1) \times [\Gamma(t+l)w(t+l) - K_{pi}(t+l)\varepsilon_i(t+l)], \]

(27)
where $\Phi(t+k,l)$ is defined by (12). Substituting (27) into (25) yields
\[
e_i(t+k) = H_i(t+k)\Phi_i(t+k, t)\tilde{x}_i(t-1) + H_i(t+k) \\
\times \sum_{l=0}^{k-1} \Phi_i(t+k, t+l+1)\Gamma(t+l)w(t+l) \\
- K_{\Phi_i(t+k)}v_i(t+k).
\] (28)

Substituting (28) into (24) yields the fixed-interval smoothing error equation for the state $x(t)$:
\[
\tilde{x}_i(t|N) = \left[ I_n - \sum_{k=0}^{N-1} M_i(t+k|t)H_i(t+k)\Phi_i(t+k, t) \right] \tilde{x}_i(t-1) \\
- \sum_{k=0}^{N-1} \sum_{l=0}^{k-1} M_i(t+k|t)H_i(t+l)\Phi_i(t+l, t+1)\Gamma(t+k)w(t+k) \\
+ \sum_{k=0}^{N-1} \sum_{l=0}^{k-1} M_i(t+k|t)H_i(t+l)\Phi_i(t+l, t+1)K_{\Phi_i(t+k)}v_i(t+k) \\
- \sum_{k=0}^{N-1} M_i(t+k|t)v_i(t+k).
\] (29)

Using (21), (22) and (23), (29) is rewritten as
\[
\tilde{x}_i(t|N) = \Psi_i(t|N)\tilde{x}_i(t-1) + \sum_{k=0}^{N-1} G_i^k(t+k|t)w(t+k) \\
+ \sum_{k=0}^{N-1} G_i^k(t+k|t)v_i(t+k),
\] (30)
where $\Psi_i(t|N)$, $G_i^k(t+k|t)$ and $G_i^k(t+k|t)$ are defined by (21)–(23).

From the projection property [17], since $\tilde{x}_i(t-1)$ consists of linear combination of $(w(t-1), \ldots, w(0), x_0(t), v_0(t-1), \ldots, v_0(1))$, so one has that $\tilde{x}_i(t-1) \perp w(t+k)$, $\tilde{x}_i(t-1) \perp v_i(t+k)$, $(k \geq 0)$, where the symbol $\perp$ denotes the orthogonality, $i, j = 1, 2, \ldots, L$. Using (30) and (3), one has (20). \hfill \Box

4. Optimal fusion fixed-interval Kalman smoothers with a three-layer fusion structure

Based on the local smoothers and cross-covariance matrices in Lemmas 1–3, one has the following distributed fusion fixed-interval smoothers by applying three weighted fusion algorithms in the linear minimum variance sense.

**Theorem 2.** Under Assumptions 1 and 2, system (1) and (2) with multiple sensors has the optimal weighted fusion fixed-interval Kalman smoothers:
\[
\tilde{x}_o(t|N) = A_{11}^O(t)\tilde{x}_1(t|N) + A_{12}^O(t)\tilde{x}_2(t|N) + \cdots + A_{1L}^O(t)\tilde{x}_L(t|N) + \cdots + A_{L1}^O(t)\tilde{x}_1(t|N) + A_{L2}^O(t)\tilde{x}_2(t|N) + \cdots + A_{LL}^O(t)\tilde{x}_L(t|N), \quad N > 0,
\] (31)
where local smoothers $\tilde{x}_o(t|N)$, $i = 1, 2, \ldots, L$ are computed by (10).

(a) For matrix weighting case, optimal matrix weights $A_i^O(t)$, $i = 1, 2, \ldots, L$ are computed by
\[
A_i^O(t) = \Sigma^{-1}(t|N)e^T\Sigma^{-1}(t|N)e^{-1}
\] (32)
where $\Sigma(t|N) = (P_i(t|N))_{l=L+1}^N$ is an $nL \times nL$ matrix where covariance matrices $P_i(t|N)$, $i = j, j = 1, 2, \ldots, L$ are computed by (13) and (20). $A_i^O(t) = [A_i^O(t), A_i^O(t), \ldots, A_i^O(t)]^T$ and $e = [E_{a1}, \ldots, E_{al}]^T$ are $nL \times n$ matrices.

The variance of the matrix weighting optimal fusion smoother is computed by
\[
P_o(t|N) = (e^T\Sigma^{-1}(t|N)e)^{-1}.
\] (33)

Furthermore, one has $P_o(t|N) \leq P_i(t|N)$, $i = 1, 2, \ldots, L$.

(b) For diagonal matrix weighting case, optimal diagonal matrix weights $A_i^O(t)$, $i = 1, 2, \ldots, L$ are computed by $a_i^O(t) = \left( P_i^O(t|N) \right)^{-1}e $, $k = 2, \ldots, n$.

where $A^O_i(t) = [a_i^O(t), \ldots, a_i^O(t)]^T$ and $e = [1, 1, \ldots, 1]^T$ are both L-dimension vectors. $a_i^O(t)$ is the element in $(k, k)$ place of $A_i^O(t)$, $i = 1, 2, \ldots, L$. The $L \times L$ matrix $P^O(t|N)$ is defined by
\[
P_i^O(t|N) = \begin{bmatrix} P_{11}^O(t|N) & \cdots & P_{1L}^O(t|N) \\
\vdots & \ddots & \vdots \\
P_{L1}^O(t|N) & \cdots & P_{LL}^O(t|N) \end{bmatrix},
\]
\[
k = 1, 2, \ldots, n,
\] (35)
where $P_{k1}^O(t|N)$ is the element in $(k, k)$ place of covariance matrix $P_i(t|N)$, $i = j, j = 1, 2, \ldots, L$. The variances of components of the diagonal matrix weighting optimal fusion smoother are computed by
\[
P_o^O(t|N) = \frac{1}{e^T\left( P^O(t|N) \right)^{-1}e} , \quad k = 1, 2, \ldots, n.
\] (36)

Furthermore, one has the result $P_o^O(t|N) \leq P_i^O(t|N)$, $i = 1, 2, \ldots, L; k = 2, \ldots, n$.

(c) For scalar weighting case, optimal scalar weights $A_i^O(t)$, $i = 1, 2, \ldots, L$ are computed by
\[
A_i^O(t) = \frac{\left( \Sigma(t|N) \right)^{-1}e}{e^T\left( \Sigma(t|N) \right)^{-1}e},
\] (37)
where $\Sigma(t|N) = [\text{tr} P^O(t|N)]_{l=L+1}^N$ is an $L \times L$ matrix where the symbol tr denotes the trace of a matrix. $e = [1, 1, \ldots, 1]^T$ and $A_i^O(t) = [A_i^O(t), A_i^O(t), \ldots, A_i^O(t)]^T$ are both L-dimension column vectors. The variance of scalar weighting optimal fusion smoother is given by
\[
P_o^O(t|N) = \sum_{i=1}^L A_i^O(t)A_i^O(t)P_i(t|N).
\] (38)

Furthermore, one has $\text{tr} P_o^O(t|N) \leq \text{tr} P_i(t|N)$, $i = 1, 2, \ldots, L$.

**Proof.** Applying three optimal weighted fusion estimation algorithms in the linear minimum variance sense [8], which include matrix weighting fusion, diagonal matrix weighting fusion and scalar weighting fusion, to the distributed fixed-interval smoothing problem, one can obtain Theorem 2 directly. \hfill \Box
Remark 1. In three distributed weighted fusion algorithms in the linear minimum variance sense, the precision of matrix weighting fusion (MWF) is higher than that of diagonal matrix weighting fusion (DMWF), and the precision of diagonal matrix weighting fixed-interval Kalman smoothers (DMWFKS) is higher than that of scalar weighting fusion (SWF). However, their computational burden is reverse since MWF requires the inverse of one $nL \times nL$ high-dimension matrix to compute matrix weights, DMWF requires in parallel the inverses of $n$ groups of $L \times L$ matrices to compute diagonal matrix weights, and SWF requires the inverse of one $L \times L$ matrix to compute scalar weights [8,13].

Remark 2. In three weighted fusion algorithms, to compute weights all require the inverses of some matrix. Generally, their inverses exist since they are positive semidefinite [8]. If their inverses do not exist at some time of fusion process, one takes the best local estimator to be the fusion estimator at this time [12].

The optimal fusion fixed-interval Kalman smoothers (31) have a three-layer fusion structure as Fig. 1. In Fig. 1, the first and second fusion layers have netted parallel structures where the first-step prediction and smoothing errors between any two-sensor subsystems are fused to determine the prediction and smoothing error cross-covariance matrices at each time step, respectively. The third fusion layer is the fusion center where the smoothers and smoothing error variances of all local subsystems, and the smoothing error cross-covariance between local subsystems coming from the second fusion layer are fused to determine the optimal weights and yield the optimal weighted fusion fixed-interval smoothers.

5. Simulation example

Consider the tracking system with three-sensors:

Fig. 1. Three-layer fusion structure of the optimal weighted fusion fixed-interval Kalman smoothers.

\[ x(t + 1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} w(t) \]  
\[ y_i(t) = H_i x(t) + v_i(t), \quad v_i(t) = c_i w(t) + \xi_i(t), \quad i = 1, 2, 3, \]  
where $T$ is the sampling period, the state $x(t) = [s(t), \dot{s}(t)]^T$ where $s(t)$ and $\dot{s}(t)$ are the position and velocity of the target at time $t$, respectively, $y_i(t)$, $i = 1, 2, 3$ are the measurement signals of three-sensors with the measurement matrices $H_1 = [1, 0]^T$, $H_2 = [1, 0.5]^T$ and $H_3 = [1, 1]^T$, respectively, $v_i(t)$, $i = 1, 2, 3$ are measurement noises correlated with the Gaussian white noise $w(t)$ with zero mean and variance $\sigma_i^2$, with the constant correlated coefficients $c_i$, $i = 1, 2, 3$, and Gaussian white noises $\xi_i(t)$, $i = 1, 2, 3$ with zero mean and variances $\sigma_i^2$ are independent of $w(t)$. Our aim is to find the optimal weighted fusion fixed-interval Kalman smoothers $\hat{s}_0(t|N)$ and $\hat{s}_i(t|N)$.

In the simulation, we take $T = 0.2$, $\sigma_w^2 = 1$, $\sigma_1^2 = 1.5$, $\sigma_2^2 = 1$, $\sigma_3^2 = 2$, $c_1 = 0.5$, $c_2 = 0.6$, $c_3 = 0.8$, $x(0) = [0, 0]^T$, $P_0 = 0.1 I_2$, $N = 60$. For every single-sensor subsystem of system (39) and (40) with three-sensors, applying Lemma 2, one can obtain the local optimal fixed-interval Kalman smoothers (LKS). Applying Theorem 2, one has three optimal weighted fusion fixed-interval Kalman smoothers, i.e., matrix weighting fusion Kalman smoother (MWFKS), diagonal matrix weighting fusion Kalman smoother (DMWFKS) and scalar weighting fusion Kalman smoother (SWFS). Comparison of variances of LKS, MWFKS, DMWFKS and SWFS is shown in Fig. 2. In Fig. 2(a) and (b) we see that the steady-state smoothers are gradually reached after about 20 steps. Since the distributed fusion smoothers have approximate precision, their variances at the initial 5 steps are re-shown in Fig. 2(c) and (d) to be able to distinguish their precision clearly. Fig. 2 verifies that distributed fusion smoothers have better accuracy than any LKS does. MWFKS has better accuracy.
than DMWFKS does. DMWFKS has better accuracy than SWFKS. However, MWFKS requires the inverse of one $6 \times 6$ matrix to compute matrix weights at each time step, DMWFKS requires in parallel the inverses of two $3 \times 3$ matrices to compute diagonal matrix weights at each time step, and SWFKS requires the inverse of one $3 \times 3$ matrix to compute scalar weights at each time step. Hence their computational burdens are reverse with their accuracy. To compare with the centralized Kalman smoother (CKS) where all measurement vectors are merged into one augmented measurement vector, the variances of CKS are also shown in Fig. 2. From Fig. 2 we see that CKS has best accuracy. But it requires the high-dimension computation by merging measurements. Though the distributed fusion smoothers have lower precision than CKS does, they have better reliability since they have the parallel structure.

6. Conclusion

Based on three optimal weighted fusion algorithms weighted by matrices, diagonal matrices and scalars in the linear minimum variance sense, three optimal weighted fusion fixed-interval Kalman smoothers are given for discrete time-varying linear stochastic control systems with multiple sensors and correlated noises. The fixed-interval smoothing error cross-covariance between any two-sensor subsystems is derived. A three-layer fusion structure is given where the first and the second fusion layers both have netted parallel structures to determine the prediction and smoothing errors cross-covariance matrices, respectively. The third fusion layer is the fusion center to determine the optimal weights and obtain the optimal fusion fixed-interval smoothers.

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