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Angular position measurement of pulsars based on X-ray intensity correlation

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ABSTRACT

The positioning accuracy of the X-ray pulsar navigation depends on the angular position accuracy of the pulsars. Currently, long baseline interferometer technology provides 0.1 mas pulsar positioning accuracy, which is far away from the requirement of X-ray pulsar navigation. Using classical statistical optics, we studied the relationship between direct observation of the pulsar angular position and second-order correlation. As an application, we give a proposal to realize angular position measurement of pulsars based on X-ray intensity correlation.

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1. Introduction

Pulsar is a neutron star with high rotational speed and extremely stable rotation frequency. The long term stability of self rotation frequency of partial millisecond pulsar is better than the most stable cesium atomic clock [1]. As a result, the position coordinates of the pulsars constitute a high precision inertial reference frame. XNAV (X-ray Pulsar Navigation) is a technology demonstration will use photons from X-ray pulsars for navigation and spacecraft attitude determination [2]. However, the pulsar angular position error will lead to poor positioning accuracy of navigation. To reach a 10-meter navigation accuracy, the resolution of pulsar angular position should better than 10 μas [3]. At present, the angular positioning accuracy of pulsars is around 0.1 mas by VLBI (Very Long Baseline Interferometry), and it is far away from the requirement of XNAV.

\[
D \cdot \cos \theta = c \cdot \tau
\]

(1)

The basic principle of VLBI is using multiple radio telescopes to observe the same radio source. Then the angular position of the source can be calculated by the time difference between the arrivals of the radio signal at different telescopes [4]. Actually, this time difference is inversely proportional to the degree of the first-order correlation of the wave field.

The resolution of angular position measurement can be indicated by the Rayleigh Criterion [5],

\[
\Delta \theta = 1.22 \frac{\lambda}{D}
\]

(2)

where D is the telescope diameter and \( \lambda \) is the observed wavelength. There are two ways to improve the resolution of telescopes, increasing the telescope caliber or observing shorter wavelengths. Long baseline interference technology makes...
the equivalent caliber of the detection system greatly improved, and its highest resolution is equivalent to a single caliber telescope which diameter is as large as the baseline length.

The X-ray radiated from pulsars ranges from 0.1 nm to 20 nm [6]. Theoretically, if we use the VLBI-like principle to observe the X-ray photons, the resolution of measurement will experience a dramatic increase comparing with the use of S-band radio signal in VLBI.

2. Pulsars angular position and X-ray intensity correlation

However, because of the restriction in detecting technology and optical fabrication in X-ray range, long baseline interferometer technology cannot be directly applied to measure the angular position of X-ray pulsars. Basically, most detectors can only output intensity and the arrival time of X-ray photons [6].

As we known, the first-order correlation of the wave field can be described as the interference,

\[ E(x_1) = A e^{-i \omega t_1} = A \cos \omega t_1 \]
\[ E(x_2) = A e^{-i \omega t_2} = A \cos \omega t_2 \]
\[ E(x) = E(x_1) + E(x_2) \]
\[ I(x) = |E(x)|^2 = \langle E(x) E^*(x) \rangle \]
\[ = \langle |E(x_1) + E(x_2)|^2 \rangle \]
\[ = G^{(1,1)}(x_1, x_1) + G^{(1,1)}(x_1, x_2) + G^{(1,1)}(x_2, x_1) + G^{(1,1)}(x_2, x_2) \]
\[ = I_0 + 2I_0 \cos \omega t + I_0 = 2I_0(1 + \cos \omega t) \]

where \( \langle . . \rangle \) means the ensemble average, and the intensity distribution \( I(x) \) is called the fringe pattern. The OPD (Optical Path Difference) \( x \) is proportional to the time difference \( \tau \) between the light from two telescopes. As a result, we can obtain the time difference \( \tau \) from the amplitude of \( I(x) \). Because the first-order serial correlation \( (G^{(1,1)}(x_1, x_1) \) or \( G^{(1,1)}(x_2, x_2) \)) is equal to its initial intensity [7], the first-order cross correlation in charges of the fringe pattern \( I(x) \). Therefore, measuring the angular position of pulsars is equivalent to measure the time difference, while measuring the time difference is equivalent to measure the amplitude of the first-order cross-correlation function.

However, the first-order correlation needs frequency and phase information which is easy to be obtained in RF (Radio Frequency) while difficult in X-ray regime [8]. Moreover, it is very difficult to fabricate optical components that function in the X-ray regime, obtain the first-order correlation of pulsar X-ray photons by carrying out a “Young’s interference” type experiment is nearly impossible currently because of the huge focal distance.

The information of photons is erased in the intensity distribution, but can be reproduced in the second-order correlation, which also known as intensity correlation [9,10].

\[ G^{(2,2)}(x_1, x_2) = \langle E^*(x_1) E^*(x_2) E(x_1) E(x_2) \rangle \]
\[ = \langle E^*(x_1) E(x_1) \rangle \langle E^*(x_2) E(x_2) \rangle + \langle E^*(x_1) E(x_2) \rangle \langle E^*(x_2) E(x_1) \rangle \]
\[ = G^{(1,1)}(x_1, x_1) \cdot G^{(1,1)}(x_2, x_2) + G^{(1,1)}(x_1, x_2) \cdot G^{(1,1)}(x_2, x_1) \]
\[ = |I_0|^2 + |G^{(1,1)}(x_1, x_2)|^2 = |I_0|^2(1 + \cos^2 \omega t) \]

where \( x_1 \) and \( x_2 \) represent two spatial points. As a result, the second order correlation function of the wave field consists of the first order serial correlation function and the square of the first order cross correlation function. Therefore, we can obtain the time difference \( \tau \) from the intensity correlation of the wave field. And the intensity correlation degree indicates the value of the time differenter. For example, the maximum output of the intensity correlation function corresponds to the minimum \( \tau \), vice versa.

3. Measurement error constraint

Van Cittert-Zernicke theory gives the general result of the quasi-monochromatic incoherent extension source illuminating the complex coherence between two spatial points. The complex coherence between these two points is equal to the absolute value of the normalized Fourier transform of the intensity distribution function of the source [11,12]. It can be represented in formula (5), where \( \Gamma \) is the cross-correlation between light at P1 and P2. Z, \( r_1, r_2 \) represent the distance from Os to O1, P1, P2 respectively, \( \lambda \) is the average wavelength, and \( I(x_1, y_1) \) is the intensity distribution of the star surface.

\[ \Gamma = \exp\left[i \frac{2\pi}{\lambda} (r_1 - r_2) \right] \times \frac{\iint p(x_1, y_1) \exp\{-i \frac{2\pi}{\lambda} [(x_1 - x_2)x_1 + (y_1 - y_2)y_1] \} dx_1 dy_1}{\iint p(x_1, y_1) dx_1 dy_1} \]
Assuming that the pulsars are centrally stacked in a circularly distributed noncoherent extended light source, the above complex coherence can be expressed by a first order Bessel function [13].

\[ I'(D) = \frac{2J_1(\pi \Delta \theta D / \lambda)}{\pi \Delta \theta D / \lambda} \]  

\[ \Delta \theta = \frac{[(x_{12} - x_{11})^2 + (y_{12} - y_{11})^2]^{\frac{1}{2}}}{z} \]  

Where \( \Delta \theta \) is the angle of the source to the object, it is also the smallest change of the angular position that can be resolved. Therefore, \( \Delta \theta \) represents the theoretical accuracy of the angular position measurement. \( (x_{11}, y_{11}) \) and \( (x_{12}, y_{12}) \) represent the minimum distinguishable two points on the source plane.

\( J(x) \) is the first order Bessel function, when \( x = \pi \Delta \theta / \lambda = 0, J(x) \neq 0 \), \( I'(D) \) is maximum. On the contrary, when \( x = \pi \Delta \theta / \lambda = 3.83, J(x) = 0 \), and \( I'(D) \) is 0. As a result, the resolution of measuring is \( \Delta \theta = 1.22 \frac{\lambda}{D} \).

4. Implementation

The X-ray photons emitted by pulsars are intensity correlated. Theoretically, we can measure the pulsar angular position by the intensity correlation of X-ray photons from the pulsar. However, the pulsar can be regarded as a thermal light source with very weak flow, and its coherence time is far less than 1 ps, which is much shorter than the randomly scattered light from a He-Ne laser in common experiments. In other words, we need measure the intensity and the arrival time of X-ray photons simultaneously.

We provide a proposal which can be used to realize the angular position measurement of pulsars based on X-ray intensity correlation (Fig. 1).

At time t, the signals received by the two detectors can be expressed as \( I_1(t) \) and \( I_2(t) \). However, the time at which the detector 1 receives the signal is t, while the time at which the detector 2 receives the same signal is \( t + \tau \). Obviously, \( I_1(t) \) and \( I_2(t) \) do not correspond to the same wave front. In order to correlate the signals received by the two detectors, \( I_2(t) \) should be delayed by a estimated time compensation \( \tau_m \). Let us introduce the intensity fluctuations in the two detectors \( \Delta I_1(t) \) and \( \Delta I_2(t + \tau_m) \). The correlation between the intensity fluctuations at two detectors is

\[ < \Delta I_1(t) \Delta I_2(t + \tau_m) > = < I_1(t) I_2(t + \tau_m) > - < I_1(t) > < I_2(t) > = < I_1(t) > < I_2(t) > |\gamma_{12}(\tau_m)|^2 \]  

where \( \gamma_{12}(\tau_m) \) is the normalized first order cross-correlation function of the wave field. And this correlation function is experimentally measurable. Therefore, if the estimated time compensation \( \tau_m \) is exactly equal to the time difference \( \tau \), the correlator will output its maximum value.

5. Conclusions

High precision pulsar navigation requires high precision pulsar position. Currently, the VLBI method which monitors the pulsar’s radio frequency signals can only provides 1 mas pulsar positioning accuracy. Theoretically, X-ray interferometry measurement can improve the positioning accuracy of pulsars dramatically. However, because of the limitation of current detecting and optical component manufacturing, it is difficult to measure 1st correlation of X-rays. This paper presents an angular position measurement method of X-ray pulsars. Two separate detectors with a baseline distance D observe the same X-ray pulsar, each detector record the X-ray photon intensity and time of arrival. The output of the correlator reflects the time difference of the photon arrival between the two detectors, as well as the angular position of the pulsar.
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References
