LASER GYROS FREQUENCY BIASING AND FIBER OPTIC GYROS PHASE BIASING: SIMILARITIES AND DIFFERENCES

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The report presents a brief overview and comparative analysis of laser gyroscopes frequency biasing and fiber optic gyroscopes phase biasing. Similarities and differences are commented. For the first time, it has been proposed to use practical experience for optimization of the laser gyroscopes frequency biasing for further optimization of fiber optic gyroscopes phase biasing in order to further improve the accuracy of the fiber optic gyroscopes and strapdown inertial orientation systems based on them.

Keywords: laser gyroscope, frequency biasing, fiber optic gyroscope, phase biasing.

I. INTRODUCTION

Laser gyroscope (LG) and fiber optic gyroscope (FOG) are generalized terms. There are about a million different optical physical schemes (OPS) of LG (only neon-helium, without solid-state LG) and about a thousand FOG OPS (interferometric, without taking into account resonator ones).

One of the co-authors of this report was able to see simultaneously the diversity of various LG, and the first models of FOG on October 17, 1975 (the exact date was saved by the passport) at the excursion to the Scientific Research Institute for Applied Physics (SRIAP). At that time, Nikolay Krobka was a third-year student of the group 355 of Physical and Quantum Electronics Faculty of the Moscow Institute of Physics and Technology (MIPT), which was attached to the Physical Electronics Chair in SRIAP [1]. In accordance with the “Fiztech’s System”, the third year students of MIPT at the beginning of the V semester were distributed to the laboratories of “bases” (group 355 – to the laboratories of its “base” – SRIAP). In September-October 1975, for the group 355 the study excursions that preceded the distribution of students to the laboratories of SRIAP were organized. During the excursion in the department no. 6, the students were shown various variants of LG, including already mastered in series production and accepted for operation, and a couple of FOG models, explained the principle of the action of LG and FOG – “based on the Sagnac effect” (SE) [2, 3], and the features of the nonlinearities of the transfer functions of LG and FOG. From the reviews [4, 5] one can be sure that in 1975 the developers of LG and FOG in SRIAP understood correctly the manifestation of SE in LG and FOG and intelligibly explained to students using the terms “geometric area” and “optical length”. The general for LG and FOG is that their principle of action is based on the SE [2, 3]. The difference between LG and FOG is that their sensitivity to absolute rotation occurs in different ways. In LG, the frequency difference \( \Phi_{LG} \) of the counterpropagating waves (CW) is proportional to the projection \( \Omega \) of the absolute angular velocity on the sensitivity axis, and in FOG the phase difference \( \Phi_{FOG} \) of CW is proportional to \( \Omega \)

\[
\Phi_{LG} = K_{LG} \Omega; \quad K_{LG} = 8\pi S_{LG} / \lambda L;
\]

\[
\Phi_{FOG} = K_{FOG} \Omega; \quad K_{FOG} = 8\pi S_{FOG} N / \lambda L;
\]

\[
S_{FOG} = SN
\]

\( K_{LG} \) and \( K_{FOG} \) are the scale factors (SF) of LG and FOG. In the case of LG \( S_{LG} \) is the geometric area of the resonator, \( L \) is the optical length of the perimeter of the resonator. In the case of FOG \( S \) is the geometric area of the one turn of the optical fiber, \( N \) is the number of turns of the FOG’s fiber, \( \lambda \) is the wavelength of light in vacuum. The LG and FOG information outputs are also fundamentally different: LG uses 2-area photodetectors, which allows us to determine the sign (“+” or “−”) of \( \Omega \) and count the number of strips of the traveling interference pattern (LG is the integrating gyroscope), proportional to the integral from \( \Omega \) for the cycle of information retrieval, and in FOG (FOG is an angular velocity sensor) the photodetectors with a single photosensitive area are used. In reality the transfer functions of LG and FOG (depending \( \Phi_{LG} \) and \( \Phi_{FOG} \) from \( \Omega \)) are nonlinear and essentially differ from ideal linear ones (1).

The transfer function of the LG (with accuracy sufficient for students) has the form

\[
\Phi_{LG} = K_{LG} \Omega + \Omega_0 = \Phi_{LG} + \rho_{LG},
\]

(2)
where $\Omega_0$ is the half-width of the static “lock-in zone”, $\nu_{\text{LG}}$ is the frequency biasing (FB) of LG.

In FOG the measured value $\Omega$ is extracted from the photodetector signal which is proportional to the intensity of the interference pattern $J$, which is invariant with respect to the shift of the phase difference $\Phi_{\text{FOG}}$ by $2\pi N$ ($N$ is the natural number), and with respect to the sign change $\Phi_{\text{FOG}}$

$$J \sim [1 + \cos(\Phi_{\text{FOG}})] = [1 + \cos(\Phi_{\text{FOG}} + 2\pi N)];$$

$$J \sim [1 + \cos(\Phi_{\text{FOG}})] = [1 + \cos(-\Phi_{\text{FOG}})].$$

(3)

In FOG the value of the measured angular velocity $\Omega$ corresponds usually to the value of the phase difference CW in the range $[-\pi; +\pi]$, and the maximum value corresponds to the value $\pi$

$$\Phi_{\text{max}} \leftrightarrow \Phi_{\text{max}} = \pi.$$

(4)

To uniquely determine the sign of $\Omega$ and provide the displacement of the “operating point” of the FOG’s transfer function to the linear region, one or another modulation of the phase difference of the CW is required

$$J \sim [1 + \cos(\Phi_{\text{FOG}} + \nu_{\text{FOG}})]$$

(5)

and the following demodulation of the signal of the photodetector, $\nu_{\text{FOG}}$ is the phase bias (PB) of the FOG.

In the explanations of the developers of LG and FOG on October 17, 1975, in part of LG, almost everything was clear, but as for the way of information retrieval from FOG – “not quite” (FOG had to be developed in future, it was more than six months till the first publications about FOG [6, 7]). Nikolay Krobka could not stand it and asked the FOG developers (S.A. Gordon’s group): And why so difficult (modulation and demodulation)? You can make it easier. But students already have been persistently asked to withstand the excursion’s schedule and go to the next laboratory, where a three-axis monoblock LG with single vibrator was demonstrated. A week later, the distribution of students of group 355 on the laboratory of SRIAP was held. Three of the sixteen students of the group were assigned to the department No. 6: Vladimir Zhuk, Valery Logozinsky (FOG subject [8-12]) and Nikolai Krobka (optical amplifiers subject [13-15]). Many years later, in connection with the development of gyroscopes based on the waves of de Broglie, N.I. Krobka recalled his forgotten idea about FOG without PB that spontaneously arose on October 17, 1975, and, possibly, will report it many years after, at Jubilee XXV St. Petersburg International Conference on Integrated Navigation Systems.

II. WORKS ON LASER AND FIBER OPTIC GYROSCOPY AT THE SCIENTIFIC RESEARCH INSTITUTE FOR APPLIED PHYSICS

After the development of continuous neon-helium lasers [16], the interest to ring lasers (RL) and LG was arised [17, 18]. In SRIAP, the development of the LG was deployed on the initiative of L.N. Kurbatov in 1962.

The world’s first laboratory model LG (not transportable) was demonstrated by Sperry Gyroscope (USA) in the beginning of 1963 [19, 20] (Fig. 1). After that, other companies-developers of gyroscopic and navigation equipment started developing LG in the USA (Fig. 2, 3).

In the USSR the first (but immediately transportable) LG model was created in SRIAP in the middle of 1963 (V.N. Kuryatov and his group: E. Nasedkin, G. Koshkin) [21]. The LG model was suspended to the ceiling of the laboratory, like Foucault’s pendulum, to create the FB of LG by torsional vibrations [21]. In this LG model, the three original technical solutions were realized: the prism of the total internal reflection for the resonator, the RF-pumping of the active medium and the pneumatic adjustment of the resonator perimeter. This first LG prototype was the prototype of a series of LG-type “KM”: a four-prism resonator, a sital monoblock, a high-frequency electrodeless discharge, pneumatic adjustment of the perimeter, which provides maximum resistance to mechanical influences, mechanical vibrational FB and built-in support electronics [21].

Developments of LG were deployed almost simultaneously at SRIAP and SRI “Polyus” (Moscow).

In 1964, the first LG “T-130” (B.V. Rybakov) was created at SRI “Polyus” [22, 23].

In 1967, the group of V.N. Kuryatov moved from the SRIAP to SRI “Polyus”, having formed the direction of high-precision LG for marine and aviation applications (Figures 4-6) [21].

![Fig. 1. LG first model [16, 17] (Sperry Gyroscope, USA, 1962)](image1)

![Fig. 2. “Miniature” LG (Honeywell, USA, 1965)](image2)

![Fig. 3. Modern LG (Honeywell)](image3)

![Fig. 4. LG KM-20 [18] (http://www.honeywell.com.)](image4)
Applied developments of LG in the SRIAP in 1960s-1970s are the laser gyrocompasses and direction keepers for ground applications. By the beginning of the 1970s in the SRIAP the LG technology for the solution of tasks of direction storage and gyrocompassing was developed and mastered in serial production (“Arsenal Plant”) in the first half of the 1970s (Fig. 7).

In SRIAP, the Bershtein’s experiment [24] on the verification of SE for radio waves (multi-turn interferometer was a prototype of FOG), methods for measuring small phase differences [25-28], and patents [29, 30] were well known. Therefore, experiments with FOG in SRIAP were started in the first half of the 1970s as soon as the Institute of Radio Electronics of the USSR AS developed the technology of optical fibers [31].

In 1985, works on LG and FOG in SRIAP were stopped. Department No. 6 of SRIAP with topics on LG and FOG was transferred to the Scientific Research Institute for Applied Mechanics (SRIAM) of the Research and Production Association "Rotor" (Chief Designer Academician V. I. Kuznetsov) [32].

Since 1989, the developments of FOG, begun at SRIAP [8-12, 33-35], continues to develop successfully in the Scientific and Technical company "Fizoptika" [36-39].

III. WORKS ON LASER AND FIBER-OPTIC GYROSCOPY AT THE SCIENTIFIC RESEARCH INSTITUTE FOR APPLIED MECHANICS

In SRIAM (since 1993 – SRIAM named after Academician V. I. Kuznetsov) works on LG and FOG were started in 1985 in connection with the tendency of LG and FOG application in rocket technology [32]. The technical requirements for the development of LG in the SRIAM are presented in Table 1 (notations: CS control system; LV – launch vehicle; OSS – orbital space stations; SINS – strapdown inertial navigation system).

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Technical requirements for accuracy of LG (1σ)</th>
<th>Stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS of maneuverable objects (1985 – 1989)</td>
<td>0.01-0.1</td>
<td>0.001-0.01</td>
</tr>
<tr>
<td>CS of the LV and landing spacecraft (launch, post-launch, correction, landing) (1980–1995)</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>CS of OSS (1990–1995)</td>
<td>0.001-0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>SINS for the USSR and RF aircrafts (1989-1996)</td>
<td>0.02</td>
<td>0.001</td>
</tr>
</tbody>
</table>

In the development of the LG of SRIAM was used (tested LG OPS in the SRIAP): the single-mode four-mirror neon-helium RL with a linear polarization of the radiation, a wavelength 0.63 μm, the excitation of a gas discharge with a direct current, a symmetrical discharge scheme: one cathode and two anodes, mechanical devices for FB to eliminate the “lock-in”

Table 2 shows the options of the LG OPS (more than one million OPS), 24 hatched LG variants were developed in SRIAM.

<table>
<thead>
<tr>
<th>DC discharge</th>
<th>High-frequency discharge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ne</td>
<td>He</td>
</tr>
<tr>
<td>Monoblock resonator design</td>
<td>The modular resonator design</td>
</tr>
<tr>
<td>Mirrors of the resonator</td>
<td>Prisms of total reflection of the resonator</td>
</tr>
<tr>
<td>Triangular configuration of the resonator</td>
<td>Quadrilateral resonator configuration</td>
</tr>
<tr>
<td>The planar optical contour of the LG resonator</td>
<td>The nonplanar optical contour of the LG resonator</td>
</tr>
<tr>
<td>1 spherical mirror of the LG resonator</td>
<td>2 spherical mirrors of the LG resonator</td>
</tr>
<tr>
<td>The wavelength of the LG radiation λ = 0.63 μm</td>
<td>The wavelength of the LG radiation λ = 1.19 μm</td>
</tr>
<tr>
<td>Single-mode (2-frequency) LG</td>
<td>Two-mode (4-frequency) LG</td>
</tr>
<tr>
<td>Linear polarization of the LG radiation</td>
<td>Circular polarization of the LG radiation</td>
</tr>
<tr>
<td>Movable mirrors with piezodrive of LG resonator</td>
<td>Pneumatic adjustment of the LG resonator</td>
</tr>
<tr>
<td>Mechanical rotation</td>
<td>PNE based on the Faraday effect</td>
</tr>
<tr>
<td>Permanent FB</td>
<td>Periodic FB</td>
</tr>
<tr>
<td>Unidirectional rotation</td>
<td>Vibrator</td>
</tr>
</tbody>
</table>
To complete the SINS of its own design, the SRIAM (with a planned accuracy of 1 mile/hour (2σ), intended for civil aviation aircrafts for conversion topics), since 1992, at the “Zvezda” company a complex of technologies for the production of LG was mastered, including: drilling gas-discharge channels of the complex configurations in the sitall resonators; polishing the edges of resonators for optical contact with accuracy (1-2) angular second; deep polishing of quartz substrates of mirrors with roughness of (1-2) Å (flat) and (2-5) Å (spherical); spraying the mirrors (SiO₂/TiO₂) with integral scattering less than 0.01%; the production of anodes from titanium and cathodes from beryllium (resource – more than 60 000 hours); chemical polishing of the sitall; chemical purification of LG elements; vacuum manufacturing and sealing of LG; methods of testing the accuracy characteristics of LG.

In the development of the SINS-90 of SRIAM, a reversibly rotating base was used for the block of three LG (without vibrators) and three accelerometers, similar to Litton’s LN-94R system [23] (with improved reverse rotation parameters due to the use of shock springs), so the requirements for the accuracy of LG to ensure the accuracy of the SINS-90 system were low: bias stability – 0.02 deg/h (1σ); random drift – 0.001 deg/h²/2 (1σ); the stability of SF – 15 ppm (1σ). Table 2 shows the characteristics of LG experimental batches manufactured in 1992-1994 for completing and testing the SINS-90 system.


<table>
<thead>
<tr>
<th>No.</th>
<th>Date</th>
<th>Factory number</th>
<th>Application</th>
<th>Technical Requirements</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SF stability (1σ) 1.5 E-05</td>
<td></td>
</tr>
<tr>
<td>1.94</td>
<td>36</td>
<td>60 deg/s 80 deg/s 60 deg/s</td>
<td>Established The results of the acceptance tests SINS-90 testing</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.92</td>
<td>11.06.96</td>
<td>1.20 E-06 1.30 E-06 0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>13056016</td>
<td>2.00 E-06 3.00 E-06 0.019</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>13016123</td>
<td>Resource testing</td>
<td>1.00 E-07 1.90 E-06 0.011</td>
<td>Working hours 12,159 h 11.06.96</td>
<td></td>
</tr>
</tbody>
</table>
Sewing together the real branches of FC, one can obtain:

\[ \Gamma = \left[ K\Omega^2 - (\Omega_0/K)^2 \right]^{1/2} \leftrightarrow [\Omega] \approx \Omega_0/K \]  

At angular velocities \( |\Omega| \leq \Omega_0/K \) and an arbitrary constant value \( \Omega \), FC \( \dot{\Phi} = 0 \) in the range \( |\Omega| \leq \Omega_0/K \). This effect is called the lock-in effect – the synchronization ("capture") of the frequencies of the RL CW. Lock-in effect was observed by the developers of LG in the early stages of LG development [21, 146-149]. To reduce the influence of this effect on the accuracy of LG the FB are used

\[ \dot{\Phi} = K\Omega + \Omega_0 \sin \Phi + p(t). \]  

In practice, six types of FB are used: 1) permanent: \( p(t) = \text{const} = p \); 2) periodic: \( p(t) = p(t) \); 3) multifrequency: \( p(t) = p(v, v, ..., v, v(t)) \); 4) noise (of various forms and statistics): \( p(t) = \xi(t) \); 5) combined: \( p(t) = p(t, \xi(t), u(t)) \); 6) adaptive: \( p(t) = p(t, \xi(t), u(t)) \).

The FC can be realized both mechanically (controlled rotation of the resonator) and optoelectronic using intracavity nonreciprocal elements (based on the nonreciprocal effects of Faraday, Kerr, Zeeman, etc.).

The meaning of constant FB is obvious: the constant FB shifts the range of measured angular velocities from the region of small angular velocities to the region of large angular velocities, where the effect of backscattering of counterpropagating waves is weakly manifested

\[ \dot{\Phi} = (K\Omega + p)(1 - (\Omega_0/(K\Omega + p))^2)^{1/2}. \]  

In the case of periodic FB, the effects of parametric synchronization remain, but the FC is smoothed – the zone \( \Omega_0 \) transforms into an infinite series of local zones of parametric synchronization with half-widths \( \Omega_{0m} \) at frequencies \( \dot{\Phi} = mv \) that are multiples of the frequency \( v \) of the periodic FB, with shifts \( \beta_m \) of the centers of the zones of local parametric synchronization from an ideal FC \( \dot{\Phi} = \Omega \)  [150, 151]

\[ \Omega_{0m} = \Omega_0 f_m; \]  

\[ f_m = \frac{1}{2\pi} \int_0^{2\pi} dt \exp \left[ \frac{i}{v} \int_0^t p(t) d\tau - imt \right]; \]  

\[ \beta_m = -\frac{1}{2\pi} \sum_{m+n=0}^{\infty} \Omega_{0m}. \]  

The widths of the zones of local parametric synchronization essentially depend on the shape \( p_0(t) \) of FB \( (p(t) = cp_0(t)) \), where \( c \) – FB amplitude, \( \max \left| p_0(t) \right| = 1 \). For example [150, 151]:

- for harmonic FB: \( p(t) = p(vt) = c\sin vt \)
  \[ \Omega_{0m} = I_m(\gamma), \quad \gamma = c/v, \]  
  where \( I_m(\gamma) \) – Bessel function of the first kind of the order \( m \) [152].
- for FB in the form of a meander
  \[ p(t) = M(vt) = cm(vt); \]  
  \[ m(vt) = \begin{cases} +1 & 0 \leq vt \leq \pi \\ -1 & -\pi \leq vt \leq 2\pi \end{cases} \]
  \[ \Omega_{0m} = \frac{2\gamma}{\pi} \frac{\sin \frac{\pi}{2}(\gamma + m)}{\gamma - m}; \]
  \[ \max \Omega_{0m} = \frac{\Omega_0}{2}. \]

In the case of multifrequency FB the structure of FC is analogous to the case of single-frequency periodic FBs, while parametric synchronization zones \( \Omega_{0m} \) under the following conditions: \( m v = m_1 v_1 + ... + m_N v_N \) (\( m, m_1, ..., m_N \) – integers).

The use of noise stationary FB (symbol \( \langle \ldots \rangle \) is averaging over the ensemble)

\[ p(t) = \xi(t); \]
\[ \langle \xi(t) \rangle = 0; \]
\[ \langle \xi(t) \xi(t_2) \rangle = \langle \xi(t) \xi(t + \tau) \rangle = k(\tau) \]

more significantly smooths the FC. In the case of a delta-correlated FB: \( k(\tau) = k_0 \delta(\tau) \) (\( \delta(\tau) \) – Dirac delta function) there is no synchronization \( \Omega_{0m} = 0 \), and the averaged FC has the form [153]

\[ \langle \dot{\Phi} \rangle = \Omega_0 [1 - (\Omega_0^2/2)\Omega^2 + k_0^2/4]^{-1}, \]  

but there is phase diffusion, where the diffusion coefficient decreasing with increasing of noise FB intensity \( (k_0^{-1}) \) and the magnitude of the measured angular velocity \( \Omega^2 \):

\[ \langle (\dot{\Phi}(t) - \langle \dot{\Phi}(t) \rangle)^2 \rangle = D t; \quad D = \Omega_0^2 k_0 (\Omega^2 + k_0^2/4)^{-1}. \]  

In practice, the most widely used are combined FB \( p(t) = p(vt, \xi(t)) \) of various types. In the case of an additive combination of the periodic and noise components of the FB: \( p(t) = p(vt + \xi(t)) \) the local zones of parametric synchronization remain, similarly to (3.8.1.2.15), but the FC is additionally smoothed out analogously to (15) in regions of
zones of local parametric synchronization: \( K\Omega \approx mv \), \( \Omega \rightarrow \Omega - mv \), \( \Omega_0 \rightarrow \Omega_{0m} \) [151, 153].

It is technically difficult to compensate noise FB in the form of stationary random processes in the difference frequency (in the measured angular velocity). It is more simple to compensate the noise FB in the form of stationary random processes in the difference phase. For combined FB of the type: \( p(t) = p(vt) + \xi(t) \), where \( \xi(t) \) is the time derivative of white noise

\[
\langle \xi(t) \rangle = 0; \quad \langle \xi(t_1)\xi(t_2) \rangle = \langle \xi(t)\xi(t+\tau) \rangle = k(\tau); \quad (17)
\]

the local zones \( \Omega_{0m} \) of parametric synchronization decrease exponentially with increasing intensity of the noise component of the FB [153]:

\[
\Omega_{0m} \rightarrow \Omega_{0m} \exp(-\sigma^2/2). \quad (18)
\]

When electromechanical vibrators are used to create the FB LG, the most simple to realize are the multiplicative combinations of the periodic and noise components of the FB: \( p(t) = \xi(t) \cdot p(vt) \). The use of opto-electronic devices for the creation of FB allows to use many other forms of combined FB [154]. For example [155]:

- combined FB in the form of a meander with a random phase reduces the zones of parametric synchronization in twice: \( \Omega_{0m} \rightarrow \Omega_{0m}/2 \).
- combined FB in the form of a meander with a random period \( T_{min} \leq T \leq T_{max} \) completely eliminates the zones of parametric synchronization \( \Omega_{0m} = 0 \) under condition \( |\nabla \Phi_{\max} - \nabla \Phi_{\min}| \geq 2\pi \).

The idea of adaptive FB (AFB) is that the error in measuring the angle of the apparent rotation (the integral of the measured angular velocity) due to the “lock-in” effect was bounded from above (did not increase in time) over the whole interval of LG functioning.

In accordance with (9), the nonlinear LG error \( \varepsilon(t) \) due to the “lock-in” effect has the form:

\[
\varepsilon(t) = \Phi(t) - K\Theta(t) - P(t); \quad \Theta(t) = \int_0^t \Omega(t)d\tau; \quad P(t) = \int_0^t p(t)d\tau \quad (19)
\]

and satisfies a nonlinear differential equation:

\[
\dot{\varepsilon} = \Omega_0 \sin[\varepsilon(t) + K\Theta(t) + P(t)] = \Omega_0 \sin \Phi(t); \quad \varepsilon(t)|_{t=0} = 0. \quad (20)
\]

Taking into account an arbitrary initial phase, (omitted in (2), (9) without loss of generality for the analysis of FC), and the phases of the backscattering coefficients, we have an equation of the general form:

\[
\dot{\varepsilon} = \Omega_0 \sin[\varepsilon(t) + K\Theta(t) + P(t) + \beta] = \Omega_0 \sin[\Phi(t) + \beta]; \quad \varepsilon(t)|_{t=0} = 0. \quad (21)
\]

An approximate solution of equation (21) can be represented as a sum of increments on intervals in the neighborhood of the moments of time \( t_i \) in which the derivative of the difference frequency vanishes \( \Phi(t_i) = 0 \) (around these moments the error \( \varepsilon(t) \) acquires increments):

\[
\varepsilon(t) \rightarrow \varepsilon(t_N) \approx \sum_{i=1}^{N} \nabla \varepsilon_i; \quad \nabla \varepsilon_i = \left\{ \begin{array}{l} \frac{(t_{i+1}+t_i)/2}{(t_{i+1}+t_{i-1}/2)2} \int \dot{\varepsilon}(\tau)d\tau; \quad (22) \\ \max \nabla \varepsilon_i \leq \Phi(t_i) \equiv 0 \Leftrightarrow \Phi(t_i) \equiv -\Phi(t_i)(t_i-t_i)^2/2. \end{array} \right.
\]

With an error of no more than 10%, integration over finite segments can be replaced by integration over infinite limits [155]:

\[
\nabla \varepsilon_i = \int_{(t_{i+1}+t_{i-1})/2}^{\infty} \dot{\varepsilon}(\tau)d\tau = \int_{-\infty}^{(t_{i+1}+t_{i-1})/2} \dot{\varepsilon}(\tau)d\tau. \quad (23)
\]

The expression for the increment of the nonlinear error takes the form:

\[
\nabla \varepsilon_i = \Omega_0 (\pi/\Phi_i)^{1/2} \cdot \{ \sin(\beta(\cos \Phi_i - s_i \sin \Phi_i) + \cos(\sin \Phi_i + s_i \cos \Phi_i)) \}
\]

\[
s_i = \text{sgn} \Phi_i \quad (24)
\]

and, accordingly, for the accumulated error:

\[
\varepsilon(t_N) = \sum_{i=1}^{N} \nabla \varepsilon_i = \Omega_0 (\pi/\Phi_i)^{1/2} (\sin(\beta(\sum_{i=1}^{N} S_i + \cos\beta \sum_{i=1}^{N} C_i)) ; \quad (25)
\]

\[
S_i = |\Phi_i|^{-1/2} (\cos \Phi_i - s_i \sin \Phi_i); \quad C_i = |\Phi_i|^{-1/2} (\sin \Phi_i + s_i \cos \Phi_i).
\]

It is possible to control the AFB in various ways, for example, by controlling the amplitude of the FB to satisfy the conditions:

\[
\text{sgn}(S_M) = -\text{sgn}(\sum_{i=1}^{M-1} S_i); \quad \text{sgn}(C_M) = -\text{sgn}(\sum_{i=1}^{M-1} C_i). \quad (26)
\]

In the case of the model equation (21) (phase \( \beta \) is an arbitrary constant), the control (26) allows to obtain a limited value of the nonlinear error over a long time interval (confirmed by modeling on the "supercomputer" BESM-6 [155]).
\[ \varepsilon(t_N) \leq \text{Const}. \quad (27) \]

The condition (27) corresponds to an ideal FC

\[ \dot{\Phi} = K\Omega. \quad (28) \]

and the absence of the angle random walk of the LG caused by the partial contribution of the backscattering of the counterpropagating waves of RL. However, for the realization of the regime (27), (28), stability of the static capture zone, amplitudes and phases of the scattering coefficients is necessary.

V. PHASE BIASING OF FIBER OPTIC GYROSCOPES

The purpose of the FOG PB differs from the purpose of the LG FB. FOG PB is used to accurately "extract" the phase \( \phi_{\text{FOG}} \) (1) from the interference pattern (5) detected by the photodetector. In essence, all the technical decisions of FOG PB are one or another technical implementation of a mathematical operation \( X = \arccos Y \), or, in most modern FOG developments, operations \( x = \cos y \) in two versions: 1) FOG with open feedback loop; 2) FOG with closed feedback loop.

In the process of FOG development, the evolution of the FOG PB forms \( P_{\text{FOG}}(t) \) for the past 40 years, partially, but not yet completely, has passed the path of development of the LG FB. The constant FOG PB (shift to \( \pi/2 \)) was proposed at the turn of the 1970s-1980s, then single-frequency and multifrequency periodic PBs were used. Noise, combined and adaptive FOG PBs by analogy with the LG FB are still "in reserve".

In 2007, in the SRIAM named after Academician V.I. Kuznetsov, works on the FOG were resumed (with the involvement of the FOG of "RPE "Gyroptika" Ltd.) for the purpose of developing in SRIAM named after Academician V. I. Kuznetsov the FOG with a closed feedback loop (KIND11-221, KIND11-222 and KIND11-240) and devices based on them for space applications [156, 157] (Fig. 8, 9).

![Fig. 8. Experimental model of a triaxial FOG KIND34-062 [156]](image1)

![Fig. 9. Experimental model of strapdown inertial measurement unit KIND34-059 [157]](image2)

At the initial stage of the development, it was noted [158] that in the FOG noise of "RPE "Gyroptika" Ltd. there are powerful noise sources with a spectral noise power density at zero frequency equal to zero. The most probable source of such noise are not ideally compensated FOG PBs during the information retrieval in FOG. It is known from the experience of laser gyroscopy [154, 159, 160] that noises of this type, which are usually neglected by the developers of the sensitive elements, lead to large errors of strapdown inertial orientation systems (SIOS) that accumulate in time. The FOG noise research cycle, the identification of the FOG noise structure and the effect of FOG noise on the accuracy of SIOS are presented in [161-186]. In works [187-196] the results of the research of the FOG and further FOG development in SRIAM named after Academician V.I. Kuznetsov are presented.

From the experience of LG development one can offer another analogy for use in FOG. If it is necessary to measure the large angular velocities in order to expand the range (4) of FOG, one can use the analogy with the technical solution worked out for LG in the 1960s — “shifting the perimeter of LG to the next longitudinal mode”, which means the following. For LG functioning in a wide range of temperatures, the stabilization system of the perimeter of LG resonator is used. If the control range is not sufficient, the perimeter \( L \) varies in an abrupt manner by an amount of \( \pm \Delta L \) corresponding to the generation of LG CW in the next longitudinal mode (similarly \( \pm \Delta N \Delta L \) for an arbitrary \( N \)). But because of the non-ideal alignment of the resonator of LG, different longitudinal modes can correspond to different values of the systematic bias and SF, which are the subject of calibration. In the case of FOG (for most OPS), there is no such problem — the change of FOG’s PB by \( \pm 2\pi N \) ("the transfer of the FOG’s PB to the next interference fringe") does not change the geometry of the FOG. As a gift from laser gyroscopists, the FOG developers are encouraged to try and use the simply realizable operation to expand the dynamic range of FOG — "transfer of the FOG PB to the neighboring interference fringe"

\[ P_{\text{FOG}}(t) \rightarrow P_{\text{FOG}}(t) \pm 2\pi N. \quad (29) \]

VI. CONCLUSIONS

In the 1970s, LG accuracy was achieved in the world practice at a level sufficient for most applications: bias stability (0.001-0.01) deg/h (1\( \sigma \)); angle random walk (0.0005-0.005) deg/h1/2 (1\( \sigma \)), the stability of the scale factor is better than for all mechanical gyroscopes (0.001-1.0) ppm (10-9-10-6) [197, 198]. The increase in the accuracy of FOG to the level of accuracy of LG (in terms of bias stability) occurred at the beginning of the 21st century as a result of 25 years of FOG development [199, 200]. By the mid-1980s, the leading developers achieved nonlinearity and instability of LG SF (10-9 – 10-8), in the SRIAM named after Academician V.I. Kuznetsov by the mid-1990s (10-7 – 10-6) [197, 198]. The current level of instability and non-linearity of the FOG SF is inferior to LG by three orders of magnitude. For an equivalent replacement of the LG by FOG, it is required to equalize not only the values of the random drift, but also the instability and non-linearity of the LG and FOG SF. In this report, it was suggested in future developments of FOG (based on the experience of LG development) investigate and use new types of FOG PB: noise, combined and adaptive types to increase the FOG accuracy, and also provide strict compensation of FOG PB at information extraction to improve FOG based SIOS accuracy.


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