Cycle Slip Detection and Repair with Triple Frequency Combination Method

Lin Zhao  
College of Automation  
Harbin Engineering University  
Harbin, China  
zhaolin@hrbeu.edu.cn

Yue Liu  
College of Automation  
Harbin Engineering University  
Harbin, China  
lysyyxq@163.com

Liang Li  
College of Automation  
Harbin Engineering University  
Harbin, China

Na Li  
College of Automation  
Harbin Engineering University  
Harbin, China

Abstract—As the application field of GNSS (Global Navigation Satellite System) is expanding, users have increasing demand of real-time positioning accuracy. The modernization of the global positioning system and the development of BDS (BeiDou Navigation Satellite System) will lead to a more high-accuracy on-the-fly positioning system. Unlike GPS, BDS is consists of MEO, GEO and IGSO satellites. The presence of the S band provided by RDSS from GEO satellites has introduced more degrees of freedom for observation data combination. We establish geometric-free and ionosphere-free linear combinations of BDS observations on the three of B1-B2-B3-S band with the aim to enhance the sensibility of detecting and repair cycle slips in real time. The small cycle slips can be magnified by the linear combinations. Therefore, the three combination cycle slips are detected. Once the combination cycle slips have been detected, the real cycle slips on each frequency can be estimated using the combination coefficients and the jumps can be fixed. These combinations are defined with the aims of long wavelength and weakening geometric and ionosphere errors. In this paper, some principles are established to appraise the performance of these combinations in order to obtain the optimal combinations. A combination method to detect and repair the small cycle slips for triple frequency of BDS is proposed. Some simulations based on real collection data have been carried out and the results have showed the proposed method is able to detect and repair both big and small cycle slips (even smaller than 1 cycle) on each carriers.

Keywords—BDS; triple frequency; linear combination; cycle slip detection

I. INTRODUCTION

BDS is a late-model, all-weather satellite navigation system that developed independently by China. It can provide rapid positioning, brief message communication and time service for users. The development of BDS makes it possible to achieve high-precision relative positioning autonomously. Comparing with other navigation system, BDS can provide active positioning and message service additionally. Although active positioning needs requirement from users and transmits through ground stations, the implementation time of first positioning is much shorter than passive positioning. Also, message transmission service is an innovation of BDS, which is the first system in the world that equipped with this function. In the environment without communication network (like ocean, desert or wild field), BDS users can locate themselves and report their position to the outside world through text message. Unlike other GNSS, BDS is composed of mixed constellations, which are MEO, IGSO and GEO satellites. It is a model innovation of BDS, which maximize the reduction of the impact to orbital measurement accuracy that caused by the lack of global monitor stations and high-precision atomic clocks. This kind of construction makes it possible to ensure the orbital monitoring and control rate. The construction of mixed constellations guarantees the regional service performance by few numbers of satellites in more economical way. Also, the payload of GEO satellite consists of RDSS, RNSS and C/C can provide with active, passive positioning, navigation, timing, brief message communication and position report function. RDSS (Radio Determination Satellite System) is one of the satellite radio services that used for users location. The distance measurement between users and satellites and position calculation cannot complete independently by users, it must be done by external system through response of users. While the positioning is completed, the user location report is transmitted to external system, also the integration of positioning and communication can be achieved. ITU (International Telecommunication Union) has assigned S frequency band 2483.5MHz~2500MHz frequencies to RDSS that can also be used in satellite navigation. BeiDou regional navigation system is the first to use the S band. The centre carrier frequency is 2491.75MHz. The presence of S band can provide more degrees of freedom for multi frequency linear combination. Taken the S band as the fourth frequency, we can make more triple frequency combinations with B1, B2 and B3. It can provide more choices in different situations and may promote the performance of RNSS.
Precision real-time dynamic positioning system requires not only the moving receivers complete calculating integer ambiguity without static initialization, but also the receivers can accept the sporadic loss of lock during tracking satellite signal. In dynamic navigation and positioning, the environment of users is often changed dramatically. Short-term loss of lock of signal, radio interference, unstable troposphere and multipath effect may cause cycle slip. Whether can cycle slip be detected and repaired will directly impact on the accuracy of positioning result. There are four cycle slip detection methods widely used, which are high-order difference method, ionosphere residual method, polynomial fitting and pseudorange/carrier phase combination method.

High-order difference method detects the presence of cycle-slip based on the regularity of the carrier phase measurement without cycle-slip changed over time. When cycle-slip presents in the measurement, the first difference to the fifth difference about this measurement will be abnormal, and the diversity will become larger with the increasing order. According to 5–6 correct observations and their high-order differences before cycle-slip presents, the first corrected observation after tracking recovered can be obtained. However, this method is more suitable for data with high sampling rate, and it is hard to detect small cycle-slip.

Goad presented ionosphere residual method. Without considering measurement noise and multipath effect, in conditions of stable ionosphere and short sampling interval, if there is a mutational ionosphere residual between two adjacent epochs, it means that cycle-slip may exist in one of the carrier phase observations on two frequencies. However, when cycle-slips presented simultaneously in both two frequencies carrier phase observations meet a specific relationship, the ionosphere residual changes insignificantly, or ionosphere has major impact on epochs, it will be difficult to detect cycle-slip.

Polynomial fitting uses n correct carrier phase observations without cycle-slip to fit a kth-order polynomial. Error equation is set up according to the polynomial model, then cycle-slip is detected and repaired through the least squares principle. Polynomial fitting not only suits for static measurement but also dynamic observation. However, when the dynamic change of target presents dramatically, there is large model error that makes it only have the ability to detect more than 5 cycle-slips. Also polynomial fitting is only suitable for the receivers that can provide the rate of carrier phase change.

In addition to ionosphere delay, multipath effect and measurement error, other errors have the same affect both on carrier phase and pseudorange. Therefore, when the pseudorange reaches certain accuracy, cycle-slip can be detected and repaired by combining pseudorange and carrier phase observations. The estimation accuracy of cycle-slip depends on the change of ionosphere delay, multipath effect between epochs, the observation error of pseudorange and carrier phase and the wavelength of carrier phase. The accuracy of conventional pseudorange/carrier phase combination method is too low to meet the requirement of dynamic precision positioning.

Although the previous four methods can well detect large cycle-slip occurring in static environment and stable dynamic environment, there are still some difficulties in small cycle-slip detection for high dynamic environment. Therefore the combined characteristics of triple frequency observation based on BDS are considered to apply to the detection and repair of cycle slip in high dynamic environment, meanwhile based on the extension of conventional pseudorange/carrier phase cycle slip detection fundamentals, the detection and repair of small cycle slip in complex dynamic environment can be completed. Under the premise in the integer feature and validity of integer ambiguity, using the carrier phase observations on each bands, geometry-free and/or ionosphere-free linear combination models can be structured. Different coefficient schemes with long combined wavelength that meet requirements of geometric-free or ionosphere-free are chosen through numerical simulation. Based on the extension of conventional pseudorange/carrier phase cycle slip detection equation, the combined cycle slip is computed using the triple frequency combination observations. The equation set consists of three cycle slips on each frequency and combination coefficients can be set up, then introducing the computed combination cycle slips back into the equation set, the solution of cycle slips on each frequency measurements is obtained. Thereby the purpose of cycle slip detection and repair is achieved. The proposed method is not strictly bound by data sampling rate and still has the ability to detect small cycle slips when the dynamic change of target is large. After the combination of triple frequency observations, the reduced effect of geometric error on observations and the increased wavelength can enhance the estimation accuracy of cycle slip.

II. PRINCIPLE OF CYCLE SLIP DETECTION WITH TRIPLE FREQUENCY COMBINATION

If combined B1, B2 and B3 frequency bands with S band signal of RDSS, the performance of RNSS may be promoted. We assume the observation on S band as the fourth frequency observation, choose three of these four bands as triple frequency observations to combine.

A. Choosing combination coefficient

Double differential carrier phase observation is commonly used in precision positioning as raw observation data. It can decreasing even eliminating space, satellite, station related system error, reduce the number of parameter that need to estimate. Double differential carrier phase observation on different frequency bands as below

\[
\begin{align*}
\lambda_1 \Delta \nabla \phi_1^i(k) &= \Delta \nabla \rho_1^i(k) - \Delta \nabla \lambda_1^i(k) + \Delta \nabla \epsilon_1(k) \\
\lambda_2 \Delta \nabla \phi_2^i(k) &= \Delta \nabla \rho_2^i(k) - \beta_1 \Delta \nabla \lambda_2^i(k) + \Delta \nabla \epsilon_2(k) \\
\lambda_3 \Delta \nabla \phi_3^i(k) &= \Delta \nabla \rho_3^i(k) - \beta_2 \Delta \nabla \lambda_3^i(k) + \Delta \nabla \epsilon_3(k) \\
\lambda_4 \Delta \nabla \phi_4^i(k) &= \Delta \nabla \rho_4^i(k) - \beta_3 \Delta \nabla \lambda_4^i(k) + \Delta \nabla \epsilon_4(k)
\end{align*}
\]
where $\Delta \mathbf{V}$ is the double difference operator, $\lambda$ is wavelength, $\lambda_1 = 0.1920 m$, $\lambda_2 = 0.2483m$, $\lambda_3 = 0.2363m$, $\lambda_4 = 0.1203m$. $\phi^i$ is the carrier phase observation for satellites $i$ and $j$, $\rho^i$ is the geometry error for satellites $i$ and $j$, $I^i$ is the ionosphere propagation delay for satellites $i$ and $j$, $N^i$ is the integer for satellites $i$ and $j$, $\epsilon^i$ terms are carrier measurement errors due to receiver noise and multipath.

In (1) ~ (4), satellite and receiver clock error are already eliminated in double differential carrier phase observation, remnant error mainly contains satellite orbit error, ionosphere delay, troposphere delay, multipath effect and receiver noise.

Structuring geometry-free and ionosphere-free linear combination models via proper coefficient distribution by using modulated carrier phase observations on B1-B2-B3-S band can get optimum performance both in co-observations wavelength and noise, improve the sensibility of cycle slip detection and repair.

The general triple frequency carrier phase observations linear combination expression as below

$$\phi_{LC} = a\phi_1 + b\phi_2 + c\phi_3 + d\phi_4$$

(5)

where $\phi_{LC}$ is the combined carrier phase observation, $\phi$ is the carrier phase observation on corresponding frequency band, $a$, $b$, $c$, $d$ are the linear combination coefficients.

$N_{iC}$ that corresponding to $\phi_{iC}$ can express as below

$$N_{iC} = aN_1 + bN_2 + cN_3 + dN_4$$

(6)

where $N$ is the integer ambiguity corresponded to carrier signal. In order to ensure the integer nature of integer ambiguity, $a$, $b$, $c$ should be integer in actual computation.

So the combination carrier phase observation equation expresses as below

$$\Delta \mathbf{V}\phi_{LC} = \Delta \mathbf{V}\rho^i(k) \left( \frac{a}{\lambda_1} + \frac{b}{\lambda_2} + \frac{c}{\lambda_3} + \frac{d}{\lambda_4} \right)$$

$$- \Delta \mathbf{V}I(k) \left( \frac{a}{\lambda_1} + \beta_1 \frac{b}{\lambda_2} + \beta_2 \frac{c}{\lambda_3} + \beta_3 \frac{d}{\lambda_4} \right)$$

$$+ \Delta \mathbf{V}N_{LC}$$

$$\left[ \frac{\Delta \mathbf{V}\epsilon^i_{\delta\phi}(\delta)}{\lambda_1} + \frac{\Delta \mathbf{V}\epsilon^j_{\delta\phi}(\delta)}{\lambda_2} + \frac{\Delta \mathbf{V}\epsilon^i_{\delta\phi}(\delta)}{\lambda_3} + \frac{\Delta \mathbf{V}\epsilon^j_{\delta\phi}(\delta)}{\lambda_4} \right]$$

(7)

 Combined wavelength is

$$\lambda_{LC} = \left( \frac{a}{\lambda_1} + \frac{b}{\lambda_2} + \frac{c}{\lambda_3} + \frac{d}{\lambda_4} \right)^{-1}$$

(8)

In (8), it should be satisfied with $\frac{a}{\lambda_1} + \frac{b}{\lambda_2} + \frac{c}{\lambda_3} + \frac{d}{\lambda_4} > 0$.

Assuming carrier phase measurement noise mean-square error is $\alpha$ times corresponding wavelength, which means $\sigma = \alpha \cdot \lambda$, difference frequency carrier phase observation errors are mutual independence.

According to error propagation law, double difference measurement noise on each bands are

$$\Delta \mathbf{V}\sigma_1 = 2 \cdot \alpha \cdot \lambda_1$$

$$\Delta \mathbf{V}\sigma_2 = 2 \cdot \alpha \cdot \lambda_2$$

$$\Delta \mathbf{V}\sigma_3 = 2 \cdot \alpha \cdot \lambda_3$$

$$\Delta \mathbf{V}\sigma_4 = 2 \cdot \alpha \cdot \lambda_4$$

(9) (10) (11) (12)

Linear combination noise can be expressed as

$$\Delta \mathbf{V}\sigma_{LC} = 2 \cdot \sqrt{\sigma_1^2 \lambda_1^4 + \sigma_2^2 \lambda_2^4 + \sigma_3^2 \lambda_3^4 + \sigma_4^2 \lambda_4^4}$$

(13)

Too much noise will reduce the efficiency and success rate of calculating ambiguity, therefore, the requirement of calculation accuracy should be considered to choose the proper coefficient. In order to ensure the validity of integer ambiguity, the combination measurement noise should be satisfied with

$$\Delta \mathbf{V}\sigma_{LC} < \lambda_{LC}$$

(14)

According to (7), the geometric error term should be as below

$$\Delta \mathbf{V}G_{LC} = \Delta \mathbf{V}\rho^i(k) \left( \frac{a}{\lambda_1} + \frac{b}{\lambda_2} + \frac{c}{\lambda_3} + \frac{d}{\lambda_4} \right)$$

(15)

where $\frac{a}{\lambda_1} + \frac{b}{\lambda_2} + \frac{c}{\lambda_3} + \frac{d}{\lambda_4}$ means the magnify factor of geometric error, recorded as $a_G$.

Likewise according to (7), the ionosphere delay term should be as below

$$\Delta \mathbf{V}I_{LC} = \Delta \mathbf{V}\rho^i(k) \left( \alpha_1 \frac{a}{\lambda_1} + \alpha_2 \frac{b}{\lambda_2} + \alpha_3 \frac{c}{\lambda_3} + \alpha_4 \frac{d}{\lambda_4} \right)$$

(16)

where $\frac{a}{\lambda_1} + \frac{b}{\lambda_2} + \frac{c}{\lambda_3} + \frac{d}{\lambda_4}$ means the magnify factor of ionosphere delay, recorded as $a_i$.

The table below shows the possible combination coefficients and the corresponding magnify factor of geometric, ionosphere and measurement noise and wavelength respectively. Here we assume $\alpha_1 = \alpha_2 = \alpha_3 = 1%$ , $\alpha_4 = 2.5%$.
As can be seen from Table I, we can choose the triple frequency combinations that have low geometric error and long wavelength.

1) B2-B3-S band combination
   a) The combination wavelength of (-50, 26, 11) combination is not long enough to ignore the affect caused by the combination carrier measurement noise, therefore, this combination is not considered.

   b) (-48, 28, 9), (-6, -10, 8) and (-4, -6, 5) combinations have long wavelengths and small measurement noises, but the ionosphere magnify factor are extraordinary big that make it difficulf to calculate integer ambiguity. So this kind of combination do not meet the standards.

   c) The optimal combinations can be (-3, -5, 4), (-2, 0, 1) and (-1, -1, 1). These combination coefficients have the character of long wavelength, low geometric error, and acceptable ionosphere delay simultaneously.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>αc</th>
<th>αs</th>
<th>σLC(m)</th>
<th>λLC(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-50</td>
<td>26</td>
<td>11</td>
<td>0.1140</td>
<td>-80.2461</td>
<td>0.6959</td>
<td>8.8775</td>
</tr>
<tr>
<td>0</td>
<td>-48</td>
<td>28</td>
<td>9</td>
<td>0.0053</td>
<td>-68.4308</td>
<td>0.6839</td>
<td>188.5487</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
<td>-10</td>
<td>8</td>
<td>0.0199</td>
<td>-46.8714</td>
<td>0.1478</td>
<td>50.3007</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
<td>-6</td>
<td>5</td>
<td>0.0636</td>
<td>-29.3434</td>
<td>0.0916</td>
<td>15.7206</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
<td>-5</td>
<td>4</td>
<td>0.0099</td>
<td>-23.4357</td>
<td>0.0739</td>
<td>100.6015</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0.2584</td>
<td>-6.1025</td>
<td>0.0256</td>
<td>3.8698</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0.0537</td>
<td>-5.9076</td>
<td>0.0118</td>
<td>18.6322</td>
</tr>
<tr>
<td>-50</td>
<td>0</td>
<td>36</td>
<td>13</td>
<td>0.0166</td>
<td>12.7449</td>
<td>0.6462</td>
<td>60.3204</td>
</tr>
<tr>
<td>-48</td>
<td>0</td>
<td>8</td>
<td>26</td>
<td>0.0045</td>
<td>-113.8604</td>
<td>0.4958</td>
<td>223.7257</td>
</tr>
<tr>
<td>-24</td>
<td>0</td>
<td>4</td>
<td>13</td>
<td>0.0022</td>
<td>-56.9302</td>
<td>0.2479</td>
<td>447.4514</td>
</tr>
<tr>
<td>-20</td>
<td>0</td>
<td>7</td>
<td>9</td>
<td>0.2788</td>
<td>-29.260</td>
<td>0.0996</td>
<td>3.5865</td>
</tr>
<tr>
<td>-13</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>0.0072</td>
<td>34.5376</td>
<td>0.0906</td>
<td>139.4384</td>
</tr>
<tr>
<td>-8</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>0.0511</td>
<td>-15.7920</td>
<td>0.0839</td>
<td>19.5687</td>
</tr>
<tr>
<td>-4</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0.0256</td>
<td>-7.8960</td>
<td>0.0419</td>
<td>39.1374</td>
</tr>
<tr>
<td>-50</td>
<td>-24</td>
<td>0</td>
<td>43</td>
<td>0.3982</td>
<td>-281.6983</td>
<td>0.0025</td>
<td>2.5110</td>
</tr>
<tr>
<td>-41</td>
<td>20</td>
<td>0</td>
<td>16</td>
<td>0.0204</td>
<td>-26.6177</td>
<td>0.0019</td>
<td>49.0656</td>
</tr>
<tr>
<td>-31</td>
<td>5</td>
<td>0</td>
<td>17</td>
<td>0.0055</td>
<td>-72.2940</td>
<td>0.0013</td>
<td>180.5979</td>
</tr>
<tr>
<td>-19</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>0.0219</td>
<td>-29.1719</td>
<td>0.0082</td>
<td>38.0447</td>
</tr>
<tr>
<td>-11</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>0.3847</td>
<td>-14.0316</td>
<td>0.0482</td>
<td>2.5997</td>
</tr>
<tr>
<td>-7</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>0.0470</td>
<td>13.9502</td>
<td>0.0440</td>
<td>21.2619</td>
</tr>
<tr>
<td>-4</td>
<td>-1</td>
<td>0</td>
<td>3</td>
<td>0.0792</td>
<td>-17.7760</td>
<td>0.0442</td>
<td>12.6228</td>
</tr>
<tr>
<td>-50</td>
<td>29</td>
<td>34</td>
<td>0</td>
<td>0.2743</td>
<td>152.8059</td>
<td>0.0289</td>
<td>3.6453</td>
</tr>
<tr>
<td>-46</td>
<td>8</td>
<td>49</td>
<td>0</td>
<td>0.0149</td>
<td>128.3428</td>
<td>0.0294</td>
<td>67.2180</td>
</tr>
<tr>
<td>-13</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>0.0072</td>
<td>34.8376</td>
<td>0.0906</td>
<td>139.4384</td>
</tr>
<tr>
<td>-4</td>
<td>-1</td>
<td>6</td>
<td>0</td>
<td>0.5324</td>
<td>10.8863</td>
<td>0.0816</td>
<td>1.8782</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
<td>-2</td>
<td>0</td>
<td>0.0752</td>
<td>11.9663</td>
<td>0.0833</td>
<td>13.3064</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0.2047</td>
<td>-0.3378</td>
<td>0.0171</td>
<td>4.8842</td>
</tr>
<tr>
<td>-1</td>
<td>-5</td>
<td>6</td>
<td>0</td>
<td>0.0478</td>
<td>-0.4282</td>
<td>0.0947</td>
<td>20.9206</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
<td>1</td>
<td>0</td>
<td>0.0068</td>
<td>-12.3630</td>
<td>0.0694</td>
<td>146.95</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>-5</td>
<td>0</td>
<td>0.2525</td>
<td>0.0088</td>
<td>0.0778</td>
<td>8.7736</td>
</tr>
</tbody>
</table>
Therefore, when the change of ionosphere delay is negligible, subtract (18) of adjacent epochs, the estimation of cycle slip can be obtained

\[ \Delta N = N(t_1) - N(t_2) \]
\[ = \phi(t_1) - \phi(t_2) - \frac{R(t_1) - R(t_2)}{\lambda} \]  

(18)

The estimation accuracy of (19) depends on the change of ionosphere delay between epochs, the measurement error and the wavelength of carrier phase. Therefore, the performance of cycle slip detection with triple frequency depends on the combined wavelength, the reduction of geometric error and ionosphere delay. The expression of triple frequency carrier phase/pseudorange cycle slip detection can be written as

\[ \Delta N_{lc} = N_{lc}(t_2) - N_{lc}(t_1) \]
\[ = \phi_{lc}(t_2) - \phi_{lc}(t_1) - \frac{R(t_2) - R(t_1)}{\lambda_{lc}} \]  

(19)

where \( \Delta N_{lc} \) is the estimation of combined cycle slip.

In practical process of detection and repair, different cycle slips can be detected by (20). Substitute the combined cycle slips in the equation set below

\[ \begin{cases} 
 a_1 N_1 + b_1 N_2 + c_1 N_3 = \Delta N_1 \\
 a_2 N_1 + b_2 N_2 + c_2 N_3 = \Delta N_2 \\
 a_3 N_1 + b_3 N_2 + c_3 N_3 = \Delta N_3 
\end{cases} \]  

(20)

where \( \Delta N_1, \Delta N_2, \Delta N_3 \) present the detected cycle slips by different combinations, \( N_1, N_2, N_3 \) are the cycle slips on each single bands. Therefore, the cycle slips on each bands can be calculated through the above equation set.

According to (21), the combination coefficients will magnify the small cycle slip by integer times even dozens of times. Therefore, the combination cycle slips will be easy to detect.

Based on the above analysis of choosing optimal combination coefficients and cycle slip detection method, the experiments can be done using these combinations.

C. Integer ambiguity resolution

The fundamental problem of carrier phase precision relative positioning is integer ambiguity calculation. LAMBDA algorithm that widely used nowadays is a rapid double difference integer ambiguity solution. Firstly compute the float solution of ambiguity, then search the optimal integer solution after spatial transformation and finally the baseline vector is obtained. Since LAMBDA algorithm is geometry related solution that the calculation accuracy affected by geometric error, and for motive receivers, it may need to conjecture their location coordinates, while geometric unrelated algorithm is not affected by the movement of receiver. Also the later algorithm is insensitive to effect of troposphere delay and the calculating is aimed at the measurement of one satellite, which make real-time dynamic system tend to adopt geometric unrelated algorithm to calculate ambiguity. TCAR combines the observations on
each three frequency bands, and then computes the combined ambiguity stepwise from the widest to the narrowest alley by rounding method. The whole algorithm is geometric unrelated and can be used for real-time dynamic measurement. In this case, we can replace the original exceeded wide-lane, wide-lane observations with the combination observations used to detect cycle slip to obtain the longer wavelength.

Assuming the frequencies of three bands are \( f_1, f_2 \) and \( f_3 \) respectively. The exceeded wide-lane frequency is 
\[
f_{EWL} = a_1 f_1 + b_1 f_2 + c_1 f_3, \tag{28}
\]
where \( a_1, b_1, c_1 \) are the combination that has the longest wavelength of the three combinations we used to detect cycle slips, the exceeded wide-lane wavelength is 
\[
\lambda_{EWL} = c / f_{EWL}, \tag{29}
\]
\( c \) is velocity of light, exceeded wide-lane integer ambiguity floating point value is expressed as
\[
\hat{\rho}_{EWL} = \rho - \phi_{EWL} + \Delta N_{LC1} \tag{21}
\]
where \( \phi_{EWL} = a_1 \phi_1 + b_1 \phi_2 + c_1 \phi_3 \) is exceeded wide-lane observation, \( \rho \) is the highest accuracy pseudorange in base band, \( \Delta N_{LC1} \) is the detected cycle slips. \( \hat{\rho}_{EWL} \) is gained by flooring the integer ambiguity floating point value and Substitute \( \hat{\rho}_{EWL} \) in (22) can get the exceeded wide-lane pseudorange
\[
\rho_i = \left( \hat{\rho}_{EWL} + \phi_{EWL} \right) \times \lambda_{EWL} \tag{22}
\]

The rest can be done in the same manner, wide-lane integer ambiguity float point value \( \hat{\rho}_{WL} \) and pseudorange \( \rho_2 \) can be expressed as
\[
\hat{\rho}_{WL} = \rho - \phi_{WL} + \Delta N_{LC2} \tag{23}
\]
\[
\rho_2 = \left( \hat{\rho}_{WL} + \phi_{WL} \right) \times \lambda_{WL} \tag{24}
\]
where \( \phi_{WL} = a_2 \phi_1 + b_2 \phi_2 + c_2 \phi_3 \), \( \Delta N_{LC2} \) is the detected cycle slips. Here \( a_2, b_2, c_2 \) are the combination coefficient that has the second long wavelength of the three combinations we used to detect cycle slips, the exceeded wide-lane wavelength is 
\[
\lambda_{WL} = c / f_{WL} = \lambda_{LC2}. \tag{30}
\]

The base band integer ambiguity float point value \( \hat{\rho}_{BASE} \) is 
\[
\hat{\rho}_{BASE} = \rho - \phi_{BASE} \tag{25}
\]

After flooring \( \hat{\rho}_{BASE} \), the final integer ambiguity \( \rho_{BASE} \) can be acquired.

Assuming pseudorange measurement noise is \( \sigma_{\rho} \), combination carrier phase observation noise is recorded as \( \sigma_{\phi} \), the carrier phase observation noise of basic frequency band is recorded as \( \sigma_{\phi} \).

The success rate of each ambiguity-approaching steps is estimated by Gauss normal distribution function as below
\[
P_{success} = \int_{0.5}^{0.5} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} dx \tag{26}
\]
where \( \sigma \) is the noise mean-square of the estimation.

\[
\sigma_{EWL} = \sqrt{\frac{1}{2} \sigma_{\rho}^2 + \sigma_{\phi}^2} \tag{27}
\]
\[
\sigma_{WL} = \sqrt{\frac{2}{3} \sigma_{\phi}^2 + \sigma_{\phi}^2} \tag{28}
\]
\[
\sigma_{BASE} = \sqrt{\frac{2}{3} \sigma_{\phi}^2 + \sigma_{\phi}^2} \tag{29}
\]

III. SIMULATION OF CYCLE SLIP DETECTION AND REPAIR

The triple frequency observation data of BDS is unavailable for secondary users. Therefore, we choose the triple frequency observation data of GPS to verify the effectiveness of combination method. Different artificial cycle slips have been added to 2-seconds triple frequency GPS data taken from http://storage.javad.com/downloads/almanacs/. Only satellite PRN1 has the third frequency data therefore only this satellite is considered. According to the above analysis of coefficient selection based on BDS, the coefficient based on GPS also can be selected. Here, we choose \((4, -8, 3), (1, -7, 6), (-3, 1, 3)\) as the combination coefficients.

We designed different scenarios to test the proposed approach. The original carrier phase data are confirmed as cycle slip free, hence some cycle slip groups should be added to the original carrier phase data to test the algorithm. After the cycle-slips are fixed, they will be removed directly from the raw carrier phase data and hence will not affect the following epochs any more.

A. Small cycle slip on single frequency band

Add 0.5 cycle slip at 30th epoch on L2 artificially, therefore the theoretical detective cycle slip should be \(-4, -3.5, 0.5\) respectively. The results of simulation are showed as below.
Fig. 1. Detected cycle slip on (4,-8,3)

Fig. 2. Detected cycle slip on (1,-7,6)

Fig. 3. Detected cycle slip on (-3,1,3)

**B. Small cycle slip on double frequencies**

Add 0.5 cycle slip at the 30th epoch both on L1 and L2 artificially, therefore the theoretical detective cycle slip should be -2, -3, and -1 respectively. The results of simulation are showed as below.

**C. Small and big cycle slips on two different frequencies**

Add 100 cycle slips on L1 and -1 cycle slip on L2 at the 30th epoch artificially, so the theoretical detective cycle slip should be 408, 107 and -301 respectively. The results of simulation are showed as below.
IV. CONCLUSION AND FUTURE WORK

Optimal linear combinations of BDS are obtained to achieve the cycle slip detection and repair in carrier phase observations. The proposed method can be applied in real-time system because it only requires comparisons between observations of consecutive epochs. As we can see from the above test results, the proposed method can correctly detect and estimated all combinations of artificial cycle slips on the three carriers under the considered hypothesis.

As to future work, the method will be checked with BDS data if possible. Also, the practical success rate of using the proposed method to deal with the problem of integer ambiguity resolution, which can enhance the accuracy and speed rate of calculation theoretically using similar linear combinations.

ACKNOWLEDGMENT

This work was supported by National Science Foundation of China (NSFC, Grant Ref. 61273081, 61304235), and the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

REFERENCES


