Autonomous Lateral Following Considerations for Vehicle Platoons

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Abstract
In this paper the problem of lateral control of vehicles operating in a platoon without the use of road infrastructure is analyzed. It is shown that the use of sensors that monitor the preceding vehicle's relative lateral position is enough to achieve lateral control for a pair of vehicles, provided that certain limitations are taken into account. For a platoon of multiple vehicles, the lateral error propagation is a serious issue that can be solved if performance is compromised. The use of inter-vehicle communication is proposed in order to recover platoon stability and satisfactory performance. Experimental data and simulations for both passenger and tractor-semi-trailer vehicles illustrate the analytical results.

INTRODUCTION
One of the major factors that lead to the improvement of transportation is the development of Intelligent Vehicle Highway Systems (IVHS), since a well-designed automated system increases safety and throughput on highways. However, the complexity of the problem necessitates its division into sub-problems. The automated platoon, that is, a group of vehicles operating in close proximity to each other, is one of these sub-problems. There are two main issues that have to be solved when dealing with automated platoons: (i) control strategies that ensure platoon stability, and (ii) infrastructure needed to implement these strategies. As far as platoon stability is concerned, the term itself implies uniform boundedness of the system errors. In the longitudinal (or lateral) direction, this means that any spacing (or lateral) error at some point in the platoon does not propagate along the rest of the platoon. Swaroop and Hedrick defined string stability for interconnected nonlinear systems and proposed a parameter adaptation law to ensure longitudinal platoon stability, [13]. The control strategy that they suggested assumed that the distance from the preceding vehicle would be measured and that the longitudinal velocity and acceleration of the lead vehicle would be communicated to the rest of the platoon. Similar system configurations have been used by other researchers on passenger vehicles and trucks, [3,8]. Chien and Ioannou avoided the use of inter-vehicle communication by introducing a speed-dependent spacing policy, but achieved platoon stability for large spacing among vehicles only, [1].

While extensive research on longitudinal platoon stability has been conducted, the literature concerning platoon stability in the lateral direction is limited. Lateral control of platoons is generally achieved by the combined use of road and vehicle infrastructure such as magnet-magnetometer and lane marker-camera schemes. These techniques constitute what is called "lane-keeping" and basically the system is not interconnected, since each vehicle independently tries to follow the reference line, [7, 12]. Hence, the platoon stability analysis is not needed, but the required road infrastructure remains an issue for economic reasons. On the other hand, autonomous following schemes, that is, techniques that create an "electronic tow-bar" and do not require road infrastructure have been suggested. White and Tomizuka proposed an autonomous following system for trucks by use of a laser scanning radar that monitors the lateral position of the preceding vehicle relative to that of the following vehicle, [14]. Lu and Tomizuka implemented a similar scheme on passenger vehicles, [10]. In Europe, a vision-based system and a trajectory-based approach were suggested in the scope of the CHAUFFEUR Project at DaimlerChrysler, [4]. Also, within the PRAXITELE Project, INRIA implemented a similar method to develop an electronic tow-bar for passenger cars, [2]. However, lateral platoon stability remains an issue, since the systems are interconnected. This paper presents the limitations of conventional lateral autonomous following for both passenger and tractor-semi-trailer vehicles and suggests three techniques to overcome these limitations. Next, lateral platoon stability is defined in order to study the lateral error propagation along a vehicle platoon. It is shown that lateral platoon stability is achieved at the expense of performance. Finally, the addition of inter-vehicle communication is suggested in order to combine platoon stability with performance. Experimental data and simulations illustrate the analytical results.

CONVENTIONAL LATERAL FOLLOWING
As discussed earlier, lateral autonomous following is important in IVHS, since it can act either as a backup system in case lane keeping techniques fail, or as a primary system if there is no road infrastructure to support lane keeping. The principle behind lateral autonomous following lies on the fact that by monitoring the relative lateral position of the preceding vehicle, it is possible to control the following vehicle.
For the purposes of analysis and design, roll and pitch motions are neglected, steering and yaw angles are considered small and lateral force on tires is proportional to the slip angle. Under these assumptions, the motion of a tractor semi-trailer is governed by Eq. (1):

\[
\begin{bmatrix}
\dot{q}_r \\
\dot{q}_w
\end{bmatrix}
= \begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
q_r \\
q_w
\end{bmatrix}
+ \begin{bmatrix}
0 \\
M^{-1}F
\end{bmatrix}
\delta_r + \begin{bmatrix}
0 \\
M^{-1}E_1
\end{bmatrix}
\hat{y}_d
\]

(1)

where \(q_r = (y, \phi, \psi)^T\) and \(q_w\) is the lateral error of the tractor's center of mass relative to the road centerline, \(\phi\) is the yaw angle of the tractor relative to the road centerline, \(\psi\) is the trailer's yaw angle relative to the tractor , \(\delta_r\) is the commanded steering angle (input) and \(\hat{y}_d\) is the yaw angle of the road relative to the inertial frame. Matrices \(M, K, C, F, E_1\) and \(E_2\) are explained in [6].

For passenger vehicles the format of the equation remains similar, the only difference being that the state vector is now \((q, \dot{q})^T = (y, \phi, \dot{y}, \dot{\psi})^T\) and the parameter matrices are different from above. Details on the passenger vehicle model are presented in [10].

In order to determine the output equation it is assumed that a sensor is mounted at distance \(d_p\) from the vehicle's center of mass and monitors the relative distance of the rear end of the preceding vehicle, as shown in Figure 1.

\[y_p = x_p \sin \epsilon^{(2)}_p + y_p \cos \epsilon^{(2)}_p\]

(2)

Assuming that the angles \(\epsilon^{(2)}_p\) and \(\epsilon^{(3)}_p\) remain small, Eq. (2) is rewritten:

\[y_p = (y^{(2)}_p - y^{(1)}_p) + (x_p + d_p) \epsilon^{(1)}_p + ((d_p + d_p) \epsilon^{(3)}_p + d_p \epsilon^{(0)}_p)\]

(3)

Similar analysis on passenger vehicles yields:

\[y_p = (y^{(2)}_p - y^{(1)}_p) + (x_p + d_p) \epsilon^{(1)}_p + (d_p \epsilon^{(0)}_p)\]

(4)

where \(d_p\) is the distance of the preceding vehicle's center of mass from its rear end. Both equations clearly show that the sensor measurement is composed of three terms: (i) the relative lateral position, (ii) the look-ahead term, that is, a quantity due to the fact that the preceding vehicle's rear end is at distance \((x_p + d_p)\) from the following vehicle's center of mass, and (iii) the off-tracking term. This last term mathematically describes the intuitive fact that the rear end of a vehicle does not necessarily follow the trajectory of the vehicle's center of mass. For passenger vehicles this term is relatively small, while for tractor-semi-trailer vehicles off-tracking makes accurate control rather challenging. Three methods are presented below that deal with this problem.

**Technique No 1: "The Disturbance Technique"**

The first method groups the undesirable terms together and treats them as disturbances. More specifically, \(y^{(2)}_p\) is considered the actual output of the system, \(y^{(0)}_p\) is the desired output, the terms related to the road curvature are considered input disturbance \(D(s)\) and the off-tracking as well as the look-ahead terms are considered sensor noise \(N(s)\). Under these assumptions, the transfer function from the steering angle \(\delta\) to the lateral acceleration \(\ddot{y}^{(0)}_p\) can be calculated from Eq. (1) and is of the following form:

\[
G_2(s) = \frac{U(s)}{V(s)}
\]

(5)

where \(U(s)\) and \(V(s)\) can be found in [12]. Hence, the control problem to be solved is summarized in Figure 2.

**Technique No 2: "The Look-Ahead Technique"**

One way to eliminate the off-tracking term, is to have the sensor track the preceding vehicle's center of mass instead of its rear end. This requires the installation of a reflective surface on the plane, which the preceding vehicle's center of mass lies on, e.g. on the roof. Clearly, this approach raises practical implementation issues, but if it is...
implemented, then according to Figure 1, the look-ahead distance increases and the sensor output, becomes:

\[ y_r = (x_r^{(3)} - y_r^{(3)}) + (x_r + d_r) e_r^{(3)} + (d_r + d_r) e_r^{(3)} + d_r (e_r^{(3)} - e_r^{(3)}) \]  

where \( x_r^{(3)} \) is now the distance between the sensor and the measured point. Alternatively, it is possible to measure the relative yaw angle \((e_r^{(3)} - e_r^{(3)})\) by scanning two points on the rear end of the preceding vehicle and using simple kinematic equations. Then, by adding in Eq. (3) the term \( d_r (e_r^{(3)} - e_r^{(3)}) \), the output becomes:

\[ y_r = (x_r^{(3)} - y_r^{(3)}) + (x_r + d_r) e_r^{(3)} + ((d_r + d_r) e_r^{(3)} + d_r (e_r^{(3)} - e_r^{(3)})) \]

\[ y_r = (x_r^{(3)} - y_r^{(3)}) + (x_r + d_r) e_r^{(3)} + (d_r (e_r^{(3)} - e_r^{(3)})) \]

Thus, the control points are no longer the centers of mass, but the hitch of the heavy vehicles. Similar analysis for passenger vehicles transforms Eq. (4) to the following:

\[ y_r = (x_r^{(3)} - y_r^{(3)}) + (x_r + d_r) e_r^{(3)} \]  

In this way, the off-tracking term and hence the bias that it introduces is eliminated. The downside of this method is that relative yaw angles are usually very small during highway operation; hence the sensor resolution has to be high enough to yield reliable measurements.

**Technique No 3: "The Look-Down Technique"**

Finally, it is possible to eliminate the look-ahead term by storing the sensor measurements, translating and rotating them according to the motion of the vehicle and using them in the control law when the longitudinal distance between the control points (either centers of mass or hitches) becomes zero. Specifically, for known yaw rate, the sensor measurement is manipulated at each time step as follows:

\[ x_{xy}[k] = (-e_r^{(3)} y_{xy}[k-1] - x) \Delta t + x_{xy}[k-1] \]

\[ y_{xy}[k] = (e_r^{(3)} x_{xy}[k-1] - y) \Delta t + y_{xy}[k-1] \]

Where \( x, y \) is the longitudinal and lateral velocity at the vehicle's reference frame respectively. When the longitudinal distance between the control points becomes zero, then the output of the system becomes:

\[ y_r = y_r^{(3)} - y_r^{(3)} \]

Comparison of the techniques described above is presented in the next Section.

**Experimental Results**

In order to design controllers for trucks and passenger vehicles, the well-known loop shaping technique is used. Due to sensor limitations in the experimental setup, the "disturbance technique" is used. The parameters of tractor semi-trailer trucks are borrowed from [6] and those of passenger cars from [10]. The proposed controllers are described in Eqs. (12) and (13) respectively.

\[ G_P(s) = \frac{(s + 0.1)(s + 1)}{s(s + 10)} \]  

(12)

\[ G_P(s) = \frac{(2s + 1)(18s + 1)}{(0.2s + 1)(56.98s + 1)} \]  

(13)

The proposed control law was experimentally tested on passenger cars. The following vehicle was equipped with a laser scanning radar located on its roof, see Figure 3.

![Figure 3. Laser scanning radar for autonomous following.](image)

The laser scanning radar (LIDAR), manufactured by Mitsubishi Electric Corp., Japan contains three components: the sensor head, the electronic control unit and the interfacing circuits. It sweeps the surround environment. This has proved very useful in terms of facilitating the rejection of clutter and ghost targets. Specifically, the rear end of the preceding vehicle was equipped with a reflective patch and the probabilistic data association filter (PDAF) was expanded to incorporate intensity information. The preceding vehicle was under lane-keeping control using the magnet-magnetometer look-ahead scheme that was successfully implemented at the 1997 NAHSC Demonstration. Schematically, the system setup is depicted in Figure 4.

![Figure 4. System configuration for experimental tests.](image)

For safety reasons, the tests were conducted at low speeds. The results are shown in Figure 5. It is clear that the proposed control law yields acceptable performance, since the relative lateral error remains under 0.13m. Therefore, lateral autonomous following can be successfully achieved by use of a laser scanning radar, provided that the details of the system kinematics and dynamics are taken in account.
PLATOON STABILITY LIMITATIONS

It has to be noted at this point that similar results to the ones demonstrated above would have been obtained if a vision-based system were used, since the only change involves the raw data processing algorithm. The question that arises is what happens if there is a platoon of a large number of vehicles instead of just two. A quick answer can be given by using the simulation parameters borrowed from [6], for a platoon of four identical tractor-semi-trailer trucks. The results are shown in Figure 6.

The figure first justifies the statement that trailer off-tracking can be a major problem for accurate control. It also shows that due to the look-ahead distance, the vehicles cut the corners of the turns that they negotiate. Moreover, it illustrates the fact that the look-down method results in oscillatory tracking, which is a downside in terms of passenger comfort. Finally, it is clear that lateral platoon stability is not achieved since the lateral error propagates along the platoon.

For linear systems, the above condition implies that lateral $L_e$ platoon stability is achieved when (i) the following equation holds:

$$\left| \frac{\gamma^{(i-1)}(\omega)}{\gamma^{(i)}(\omega)} \right| < 1, \quad \forall \omega$$

where $\gamma^{(i-1)}(\omega)$ and $\gamma^{(i)}(\omega)$ is the lateral error of the $(i-1)$th and $i$th vehicle respectively. Since we are concerned about the lateral error propagation, we deal with $L_e$ stability. Hence, the lateral platoon stability condition is:

$$\max_i |\gamma^{(i-1)}(t)| < \max_i |\gamma^{(i)}(t)|$$

For linear systems, the above condition implies that lateral $L_e$ platoon stability is achieved when (i) the following equation holds:

$$\left| \frac{\gamma^{(i-1)}(\omega)}{\gamma^{(i)}(\omega)} \right| < 1, \quad \forall \omega$$

AND (ii) the impulse response of the transfer function from $\gamma^{(i-1)}$ to $\gamma^{(i)}$ does not change sign. Reformulating the problem of the autonomous following scheme and assuming that trailer off-tracking has been compensated for, the block diagram for the look-ahead technique is shown in Figure 7 and the transfer function is described by Eq. (16).

$$\frac{\gamma^{(i)}(s)}{\gamma^{(i-1)}(s)} = \frac{s^2 G_x(s)}{1 + s^2 G_x(s)}$$

where $G_x(s) = \frac{G_x(s) G_x(s)}{1 + s L_a G_x(s) G_x(s)}$ and $L_a$ is the look-ahead distance. For $L_a = 0$, we obtain the look-down technique. Representing the loop gain as a complex number, that is:

$$L(j\omega) = (aj)^2 G_x(j\omega) = r_x(j\omega) e^{j\phi_{xj\omega}}$$

then Eq. (15) becomes:

$$\frac{r_x(j\omega) e^{j\phi_{xj\omega}}}{1 + r_x(j\omega) e^{j\phi_{xj\omega}}} < 1 \Rightarrow$$

$$\frac{r_x(j\omega)}{\sqrt{1 + r_x^2(j\omega) + 2r_x(j\omega) \cos \phi_{xj\omega}}} < 1 \Rightarrow$$

$$r_x(j\omega) \cos \phi > -.5 \Rightarrow \Re\{r_x(j\omega)\} > -.5$$

This means that the first condition for lateral platoon stability in the $L_e$ sense is satisfied if the open-loop Nyquist plot lies on the right of the line $x=-0.5$ for $\omega$. This is a very important conclusion, since it is fast criterion to determine if the platoon is unstable. It should be stressed...
that it does not guarantee platoon stability since it does not give any information about the impulse response of the transfer function. Simulations showed that it is possible to achieve lateral platoon stability by merely measuring the relative distance between the preceding and the following vehicle. In practice, if the look-ahead distance is small the controller gain has to be very small as well to satisfy the stability condition. Conversely, for larger controller gains platoon stability is achieved when the look-ahead distance is increased. In the first case, the result is slow transients resulting in poor performance, while in the second case vehicles cut the corners of the turns that they negotiate. Thus, lateral platoon stability and satisfactory performance is practically impossible to achieve. What can be guaranteed is prevention of error propagation for specific conditions. The example below illustrates such a case.

**Example**

For the tractor-semi-trailer vehicle parameters and the controller used in Section II with zero look-ahead distance, the open-loop Nyquist plot is shown in Figure 8a.

![Figure 8a](image)

**Figure 8.** Open loop Nyquist plot of the tractor-semi-trailer system (a), and simulation of a 4-vehicle platoon, (b).

From this figure, it can be concluded that a platoon consisting of the exact same vehicles is laterally unstable since the Nyquist plot does not lie on the right of the line \( x = -0.5 \). However, for \( \alpha \in (4.76, \infty) \) the lateral error is not expected to propagate along the platoon. Indeed, Figure 8b shows the steady state simulation results for a 4-vehicle platoon whose lead vehicle oscillates at a frequency of \( \text{rad/s} \). It is evident that although this is not a realistic highway curvature profile, the theoretical results are confirmed, that is, there is no error propagation.

**ADDING INTER-VEHICLE COMMUNICATION**

One way to obtain platoon stability without compromising performance is to eliminate the interconnection of the vehicles. In other words, if the reference trajectory for each vehicle is not that of the preceding vehicle but that of the lead vehicle (the very first vehicle of the platoon), then the platoon is no longer a string of vehicles. Hence, platoon stability analysis as conducted in the previous section is no longer valid. The platoon is stable as long as a stabilizing controller is designed for each vehicle.

In order to achieve vehicle “disconnection” without the need of road infrastructure, the addition of inter-vehicle communication is proposed. Ideally, an accurate Global Positioning System (GPS) installed on the lead-vehicle and the transmission of its position to the platoon would be sufficient to achieve a common reference trajectory for all vehicles. However, there are several deployment issues with this solution such as guaranteeing fast real-time information from the GPS. Thus, making use of the existing on-board sensors seems a wise solution. To this end, we propose the transmission of the relative lateral position information between each preceding and following vehicle.

More specifically, for a platoon of \( n \) vehicles, there are \( n-1 \) followers, thus there are \( n-2 \) followers in front of the \( i \)th vehicle of the platoon. Suppose that all vehicles are using the look-down technique as described in Section II. If the \( i \)th vehicle receives the relative lateral errors measured by all the preceding vehicles, and adds them to its own lateral measurement, then the output of the \( i \)th vehicle is:

\[
y_p^i = \left( y_p^0 \right) + \sum_{k=1}^{i-1} \left( y_p^k - y_p^{(k-1)} \right)
\]

or

\[
y_p^0 = y_p^0
\]

(19)

where \( y_p^0 \) is the lateral error of the lead vehicle. It is clear that there is no dependence on the preceding vehicles, thus, if the controller of each vehicle yields a stable system, then the whole platoon is stable. To illustrate this, a simulation of a platoon of four identical trucks is shown in Figure 9.

![Figure 9](image)

**Figure 9.** Platoon response with communication.

It is noted that the lead vehicle is assumed to be under lane keeping look-ahead control. Evidently, the lateral error does not propagate along the platoon, which implies stability. For the look-ahead technique and for equal distance between identical trucks, the output of the \( i \)th vehicle becomes:

\[
y_p^i = \left( y_p^0 \right) + \sum_{k=1}^{i-1} \left( y_p^k - y_p^{(k-1)} \right) + \left( x_p^i + d_x \right) \varepsilon_p^i
\]

\[
y_p^0 = y_p^0 + \left( x_p^0 + d_x \right) \varepsilon_p^0
\]

(20)
Here, there is some dependence on the preceding vehicles, which prevents any clear conclusions about stability. Hence, it is possible to combine the good stability properties of the look-down technique with the smooth control properties of the look-ahead by keeping the look-down technique for communication to the rest of the platoon and implementing the look-ahead scheme for vehicle control.

One important consideration is the communication delay, which is inherent in wireless systems and makes the selection of the means of communication a non-trivial problem. This delay is due to three factors: (i) transmission delay, which is usually negligible, (ii) delay due to packet losses and (iii) communication architectural delay, that is, delay associated with the selection of the architecture of the communication system. In the existing literature, it has been shown that platoon stability in the longitudinal direction is severely compromised if the packet loss rate and the architectural delays are not small enough. In the lateral control case, the trajectory storage method eliminates these concerns as long as each vehicle receives the required data at time intervals smaller than the time to travel one look-ahead distance. In a token-bus protocol, which is the most common protocol when deterministic communication delays are desired, this means that the time to complete a communication cycle has to be smaller than the vehicle spacing divided by their longitudinal speed. If this is not the case, it is suggested that the platoon speed decrease or the vehicle spacing increase so as to satisfy the above criterion.

CONCLUSIONS

This paper focused on autonomous lateral following for platoons of vehicles. The dynamics of both passenger vehicles and tractors semi-trailers were presented, the influence of rear end off-tracking and look-ahead distance was illustrated and three control methods were suggested, that address the limitations of autonomous following without inter-vehicle communication. Next the lateral platoon stability was introduced and it was shown that interconnected platoons are stable under certain conditions. Finally, it was shown that the addition of inter-vehicle communication ensures lateral platoon stability and satisfactory performance since it eliminates the interconnection among the vehicles, provided that the communication delays are short enough. Illustrative examples, simulations and experimental data were used to verify the analytical results.

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