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Is it risky to go green? A volatility analysis of the green bond market

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ABSTRACT

Since its inception in 2007, the green bond market has experienced a compound growth rate of 50% annually. In 2014, green bond issuance totaled USD 36.6 billion, more than threefold its previous year’s level of USD 11 billion. This new market is a response to the growing demand of investors for financial investments that are beneficial both environmentally and economically. As the green bond market continues to grow, it is important to obtain a better understanding of the risk and return behavior of the market. This paper is the first to analyze the volatility behavior of the green bond market using data on daily closing prices of the S&P green bond indices between April 2010 and April 2015. Building on a multivariate GARCH framework, my empirical results show that the ‘labeled’ segment of the green bond market experiences large volatility clustering while the pattern of volatility clustering is weaker in the ‘unlabeled’ segment of the market. I also found that a shock in the overall conventional bond market tends to spill over into the green bond market, where this spillover effect is variable over time. These results are meaningful insights into this new, yet very promising market, therefore, have important implications for asset pricing, portfolio management and risk management.

1. Introduction

Gandhi once said ‘Earth provides enough to satisfy every man’s needs, but not every man’s greed.’ Indeed, increasing amounts of greenhouse gas emissions from human activities are not only a major contributor to climate change but also bring about various negative consequences on the human society. The urging question of our time is how to prosper economically without impacting the ecological systems beyond irrevocable changes. In fact, in recent years, countries and multinational agencies across the world have made tremendous efforts to promote environmentally friendly investments. Yet, fundraising efforts to date have been inadequate to meet the immense amount of funding required to address climate change. According to the World Bank, climate change mitigation efforts in developing countries could cost about USD 275 billion annually over the next 20 years, which is more than double the current level development assistance of USD 100 billion per year.
(World Bank 2015b). Therefore, new sources of financing for climate change must be considered. In the financial world, new financial instruments have been created to facilitate the increasing demand for green investing. One very promising financial instrument of that kind is green bonds, which are debt instruments with a bonus environmental feature. Since its first appearance in 2007, the green bond market has expanded at more than 50% compounded annual growth rate, providing funding for environmental projects all over the world (Kochetygova and Jauhari 2014; World Bank 2015a). Given the potential environmental and economic benefits of green bonds, it is crucial to understand the volatility behavior of this green financial instruments in comparison with other conventional investments as the market continues to expand. In this paper, my goal is to provide an insight into the volatility behavior of the green bond market and study the relationship between the green bond market volatility and the volatility of the conventional bond market.

The World Bank defines a green bond as ‘a debt security that is issued to raise capital specifically to support climate-related environmental projects’ (World Bank 2015b). A green bond could either be 'labeled' or 'unlabeled'. Labeled green bonds are formally marketed as ‘green’ by the issuers, where the issuers define the types of environmental projects they plan to support with the bond proceeds and report back to investors on a regular basis. On the other hand, unlabeled green bonds do not have a formal green tag, but are issued by firms whose businesses are naturally aligned with environmental causes, for example, bonds issued by wind or solar energy companies. The range of projects supported by green bonds is very diverse, with low-carbon transport and clean energy projects being the two largest beneficiaries (Shankleman 2016). From a market pioneered by large development banks, in 2014, two-thirds of all new green bonds came from issuers that are not multilateral development banks, attracting a broad group of investors such as asset managers, pension funds, companies, foundations and religious organizations (Kochetygova and Jauhari 2014; Damutz 2016).

The fast growth of the green bond market gives rise to the need to address its risk and return characteristics so that to equip investors with informative insights into the market. Within this context, this paper is the first to answer the following research questions: How does the volatility of the green bond market behave in comparison with the conventional bond market? Are there any spillover effects between the green bond market and the conventional bond market? How much does a shock in the green bond market contribute to the volatility of the conventional bond market and vice versa? To answer this question, I first build a framework to model the volatility of a financial asset based on the multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) framework, a family of statistical models originally proposed by Bollerslev (1986) and Engle (2002) and have been widely used in the literature studying the relationship between different financial time series’ volatilities (Bauwens, Laurent, and Rombouts 2006). With this model, I am able to test not only the pattern of volatility in the green bond market but also how the volatility in the green bond market transmits to the broader conventional bond market.

Using time series data on daily closing prices of the S&P green bond indices between April 2010 and April 2015, the results from my analysis suggest that there is significant volatility clustering in the green bond market, where periods of high volatility are often followed by further periods of high volatility and periods of low volatility are followed
by periods of low volatility. This volatility clustering effect is particularly stronger for the labeled green bond sector, as compared to its unlabeled counterpart and the broader conventional bond market, since most labeled green bonds are of similar credit ratings while the markets for unlabeled green bonds and conventional bonds consist of a more diverse sets of bonds. Moreover, the green bond market is also interdependent with the conventional bond market. My empirical results show that a shock in the green bond market tends to spill over to the conventional bond market and this spillover effect variable over time. Furthermore, the data also suggest that there has been an upward trend in the correlation of volatility in the labeled green bond segment with the conventional bond market. One explanation is that there exists convergence of returns between the green bond market and the broader bond market as the green bond market continues to attract a broader group of investors.\textsuperscript{4} The results are robust after accounting for the presence of extreme observations in the data.

This paper is related to three main strands of the literature. First, this paper relates to the literature modeling volatility of the financial market. This paper follows the work of Bollerslev (1986) and Engle (2002), who propose an econometric framework to study volatility clustering and volatility spillover among different time series. While this framework has been applied to multiple markets in the financial work, this paper is the first to apply the GARCH modeling techniques to studying the volatility of the green bond market, a new investment option that can potentially meet the growing demands for sustainable and socially responsible investing.

Second, this paper is also in line with the literature studying the fixed-income financial market (e.g. Das 1998; Beber, Brandt, and Kavajecz 2006; Harford and Uysal 2014). While this line of literature spans over a long period of time and covers multiple areas of the bond market, none of them has addressed the role of the fixed-income market in promoting environmentally sound investment. This paper contributes to the literature by focusing on the green fixed-income market and providing the first empirical evidence for the performance of this new, yet promising market segment.

This paper also relates to the literature studying the characteristics of environmentally friendly financial instruments. Ortas and Moneva (2013) study the risk and return performance of 21 Clean Techs equity indexes using a state-space approach and found that the Clean Techs equity indexes outperform the market portfolio in terms of returns and that their returns are highly volatile even in uprising markets. Many other studies document the volatility spillover between the green equity market and other sectors of the market such as the conventional equity market, the oil and carbon market and find that there are significant interdependence between the green equity market and the broader financial market (e.g. Kumar, Managi, and Matsuda 2012; Sadorsky 2012). Climent and Soriano (2011) and Chang, Nelson, and Witte (2012) study the performance of green mutual funds and find that green mutual funds have underperformed on a risk-adjusted basis compared to conventional funds. On the other hand, Gil-bazo, Ruiz-verdu, and Santos (2010) find that company management plays an important role in the performance of socially responsible mutual funds. While the literature studying green financial instruments has been well-developed, most of the emphasis so far has been on analyzing the performance of the equity sector of the market. To the best of my knowledge, this paper is among the first attempts in characterizing the behavior of the green fixed-income market. Understanding the performance of the green bond
market is important because together with the broader USD 100 trillion bond market, the green bond market can serve as a low-cost financing tool toward a green economy (Caldecott 2010; Mathews et al. 2010; Mathews and Kidney 2012). Moreover, for investors interested in environmentally beneficial investing, the fixed-income market is a good starting point as it is often considered as a lower risk market than other green investment options.

The rest of the paper is organized as follows. Section 2 provides an overview of the green bond market while 3 specifies the framework for modeling volatility in the green bond market. Section 4 describes the data and Section 5 presents the empirical results and analysis. Finally, Section 6 provides a concluding remark.

2. Overview of the green bond market

According to a survey of high net worth investors in 2016 by Morgan Stanley, sustainable investing is becoming more popular among investors, where 55% of investors reported that they are interested in sustainable investing and 32% view sustainable investing as a good investment approach for the future (Morgan Stanley 2016). Yet, regardless of the growing interest among investors about sustainable investing, there still exists a large funding gap for low-carbon projects, which cannot be supported by public sources alone (World Bank 2015b). This emphasizes the role of private funding in the transition toward a low-carbon economy.

While sustainable investing has been popular in the equity market, the green bond concept is a relatively new development. The most fundamental distinction between green bonds and conventional bonds is that all the proceeds from green bonds are used to finance environmentally friendly projects. The identification and labeling of green bonds typically follow the Green Bond Principles (GBPs), a set of voluntary standards established by industry participants including major banks and non-profit organizations (International Capital Market Association 2015). The GBPs consist of four elements. First, in order to be labeled ‘green’, a bond’s proceeds must be used for environmentally beneficial capital expenditures, such as investments in alternative energy, energy efficiency, pollution prevention and control, sustainable water and green buildings. Second, the green bonds’ documentation must include specific criteria and process for determining eligible projects or investment. Third, a formal process that regulates the use of net proceeds must be disclosed in the bond prospectus or supporting document. And fourth, issuers of green bonds should report at least annually on the specific investments made from the green bond proceeds and document the environmental impacts of the specific investments. Total ‘labeled’ green bonds outstanding were USD 65.9 billion in June 2015. In 2014, the issuance of the ‘labeled’ green bond totaled USD 36.6 billion, which was more than three times the previous year’s amount of USD 11 billion (Kochetygova and Jauhari 2014; Damutz 2016).

Besides bonds that follow the GBPs and are formally labeled as ‘green’, many bonds in the market have been issued without a green label but having clear environmental benefits such as financing wind farms or solar installations. According to Standard and Poor's, this ‘unlabeled’ segment is potentially several times larger in size than the ‘labeled’ green bond market segment. In 2015, ‘unlabeled’ green bond outstanding totaled USD 531.8 billion, which is significantly larger than the total outstanding amount of USD 65.9 billion in
the ‘labeled’ green bond market. From a small market pioneered by large development banks and institutional investors, the green bond market has attracted many different types of issuers and investors such as corporations, mutual funds, asset managers, insurance companies, subnational and municipal government entities. In 2014, about two-thirds of all new green bonds came from issuers that are not multilateral development banks in more than 20 different currencies (Kochetygova and Jauhari 2014).

The growth of the green bond market reflects the increasing interests of investors in low-carbon projects. In fact, since its inception, the green bond market has attracted a diverse group of investors, such as asset managers, pension funds, companies, foundations and religious organizations (Kochetygova and Jauhari 2014; Damutz 2016).

3. Modeling the green bond market volatility

As the green bond market continues to grow, it is important to understand the volatility dynamics of this market segment in relation with other sectors in the financial market. A widely used technique to in the literature studying the volatility of financial time series is GARCH, which uses an autoregressive structure to model the conditional variance of a time series, thereby allowing volatility shocks to persist over time. Under this framework, the volatility of an asset’s returns is given in the following set of equations:

\[ R_t = E_{t-1}[R_t] + \varepsilon_t, \quad \varepsilon_t|I_{t-1} \text{iid}(0, \sigma^2_t), \]

where \( E_{t-1}[R_t] \) denotes the conditional mean of the asset returns at time \( t \) given the information set \( I_{t-1} \); \( \varepsilon_t \) is the error term and \( \sigma^2_t \) is the conditional variance of asset returns at time \( t \).

The specifications of \( E_{t-1}[R_t] \) and \( \sigma^2_t \) are of the following form:

\[ E_{t-1}[R_t] - \mu = \sum_{h=1}^{r} \phi_h(R_{t-h} - \mu) + \sum_{k=1}^{s} \psi_k \varepsilon_{t-k}, \]

\[ \sigma^2_t = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma^2_{t-j}, \]

where \( \mu = E[R_t] \) denotes the unconditional mean of the asset returns. Altogether, parameters \( a_i \) \((i = 1, \ldots, p)\) and \( b_j \) \((j = 1, \ldots, q)\) determine the extent of volatility clustering in asset returns. A high and significant \( a_i \) and \( b_j \) \((i = 1, \ldots, p; j = 1, \ldots, q)\) indicate the existence of volatility clustering, where periods of high volatility are followed by periods of high volatility and vice versa. Finally, the lag lengths \( p, q, r, s \) are determined using the Schwartz information criteria.

To compare the volatility of the green bond market with that of the conventional bond market, one approach is to estimate Equations (1)–(3) for each market separately. While this approach allows us to study the pattern of volatility of individual markets, it ignores the interactions between the green bond market and the broader conventional bond market. To capture the possible volatility spillovers between the green bond market and the broader conventional bond market, I also extend the above univariate model in
Equations (1)–(3) to a multivariate case where the volatility of an asset’s returns not only depends on its past values but also on the volatility of other assets in the financial market. One feature of the multivariate GARCH model is that it allows time-varying conditional variances of asset returns as well as covariances between the returns of different assets. This allows the analysis of the volatility structure of individual assets as well as the interaction between various assets. In this paper, the specification of this multivariate model consists of two components. First, returns are modeled using a vector autoregression framework. Then, a multivariate GARCH model is used to model the time-varying variances and covariances. Specifically, let $R_{G_t}$ be the returns on the green bond market at time $t$ and $R_{M_t}$ be the returns on a benchmark conventional bond market at time $t$. Let $\mu_G$ and $\mu_M$ be the unconditional means of the returns on the green bond market and the benchmark market. The specification for $R_{G_t}$ and $R_{M_t}$ is of the following form:

$$
R_{G_t} - \mu_G = \phi_{11}(R_{G_{t-1}} - \mu_G) + \phi_{12}(R_{M_{t-1}} - \mu_M) + \phi_{13}(R_{G_{t-r_G}} - \mu_G) + \phi_{14}(R_{M_{t-r_G}} - \mu_M) + \epsilon_{G_t},
$$

$$
R_{M_t} - \mu_M = \phi_{21}(R_{G_{t-1}} - \mu_G) + \phi_{22}(R_{M_{t-1}} - \mu_M) + \phi_{23}(R_{G_{t-r_M}} - \mu_G) + \phi_{24}(R_{M_{t-r_M}} - \mu_M) + \epsilon_{M_t},
$$

with $\epsilon_t | I_{t-1} \sim WN(0, \Sigma_t)$, where $\Sigma_t = \begin{bmatrix} \sigma_{Gt}^2 & \sigma_{GtMt} \\ \sigma_{GtMt} & \sigma_{Mt}^2 \end{bmatrix}$ denotes the conditional variance–covariance matrix at time $t$. The lag lengths $r_G$ and $r_M$ are jointly determined using the Schwartz information criteria.

To model the time-varying volatility of the return series $\{R_{G_t}\}$ and $\{R_{M_t}\}$, I apply maximum likelihood estimation techniques to Engle’s (2002) dynamic conditional correlation (DCC) model. Under this approach, the conditional covariance matrix $\Sigma_t$ is modeled based on the univariate GARCH modeling of the individual series $\{R_{G_t}\}$ and $\{R_{M_t}\}$. Compared to other approaches which model the conditional variance–covariance matrix $\Sigma_t$ directly, the DCC approach has clear computational advantages because its flexibility allows for the estimation of very large correlation matrices. The estimation of the DCC model relies on the decomposition of the conditional covariance matrix $\Sigma_t$ into

$$
\Sigma_t = D_t Q_t D_t,
$$

where $Q_t = \begin{bmatrix} 1 & \rho_{GtMt} \\ \rho_{Mg} & 1 \end{bmatrix}$ is the correlation matrix and $D_t = \begin{bmatrix} \sigma_{Gt} & 0 \\ 0 & \sigma_{Mt} \end{bmatrix}$ is a diagonal matrix with the standard deviations of the two series on the diagonal.

The estimation of the DCC model’s parameters involves two steps. In the first step, the volatility measures of each individual series are estimated under the following univariate GARCH model:

$$
\sigma_{Gt}^2 = a_0 + \sum_{i=1}^{p_G} a_{Gi} \epsilon_{Gt-i}^2 + \sum_{j=1}^{q_G} b_{Gj} \sigma_{Gt-j},
$$

where $L. PHAM$
\begin{equation}
\sigma^2_{M_t} = a_{0M} + \sum_{i=1}^{p_M} a_{iM} \varepsilon^2_{M_{t-i}} + \sum_{j=1}^{q_M} b_{jM} \sigma^2_{M_{t-j}}. \tag{8}
\end{equation}

Then an estimate for the standardized residuals is calculated from the above GARCH models: \( \hat{z}_{it} = (\hat{e}_{it}/\hat{\sigma}_{it}) \) (\( i = G, M \)). In the second step, the conditional correlation between the standardized residuals is modeled using the following GARCH (1,1) framework:

\begin{equation}
\hat{q}_{ijt} = \text{Cov}(\hat{z}_{it}, \hat{z}_{jt} | I_{t-1}) \\
= \hat{E}[\hat{z}_{it} \hat{z}_{jt}](1 - \alpha - \beta) + \alpha \hat{z}_{it-1} \hat{z}_{jt-1} + \beta \hat{q}_{ijt-1}, \quad i, j = G, M. \tag{9}
\end{equation}

Thus, the correlation estimator is \( \hat{\rho}_{ijt} = \hat{q}_{ijt}/\sqrt{\hat{q}_{ii}} \hat{\bar{q}}_{jjt} \).

Finally, the univariate volatility estimates in the first step and the bivariate conditional correlation estimates in the second step are combined to estimate \( \Sigma_t = D_t R_t D_t \). Altogether, the parameters \( a_i \) and \( b_j \) in Equations (6) and (7) show the magnitude of the volatility clustering within the returns series while the parameters \( \alpha \) and \( \beta \) in Equation (8) show the magnitude of volatility spillover from one time series to the other. Moreover, the value of each element in the estimated \( \Sigma_t \) will reveal about the magnitude of volatility clustering in asset returns and volatility spillovers from one asset’s returns to another’s.

### 4. Data

To analyze the volatility behavior of the green bond market in relation to the overall conventional bond market, this study requires time series data on the market performances of green bonds and conventional bonds. Specifically, I use the daily closing prices of the S&P Green Bond (GB) Index and the S&P Green Project Bond (GPB) Index as indicators of the green bond market performance. The performance of the conventional bond market is approximated using the S&P US Aggregate Bond (AB) Index. The sample period for the data spans between 30 April 2010 and 29 April 2015.

The S&P GB Index and the S&P GPB Index are complementary indices that serve as a tool to track the global green bond market. The GB Index is constructed using bonds that have been independently verified to comply with the GBPs. The goal of the index is to track the performance of the ‘labeled’ green bond market. The majority of the bonds included in the GB Index are of investment-grade, with 54% of the bonds are AAA rated by Standard and Poor as of August 2014 (Kochetygova and Jauhari 2014). Figure 1 shows the daily returns on the GB Index in the sampling period. It can be seen from the figure that the index is more volatile at the beginning of the sampling period. Moreover, the index seems to exhibit mean reverting behavior in returns, which suggests the data are stationary. In fact, results from a unit root test shows that returns on the S&P GB Index is stationary, as can be seen in Table 1.

While the green-labeled bond market is growing, a significant part of the market consists of bond issues without a green label but having obvious environmental benefits such as financing wind farms or solar installations. The S&P GPB Index is constructed to track the broader ‘unlabeled’ green bond market. While bonds need not have a green label to be included in the GPB Index, they must fall into one of the following categories: bonds...
issued by special purpose entities to finance or refinanced a green project, asset-backed securities to securitize cash flows from pools of green assets and corporate bonds issued by companies whose revenues originate only from green activities. In contrast to the S&P GB Index, which is dominated by bonds with high credit rating, the S&P GPB Index include both investment- and subinvestment-grade bonds. As of September 2014, 51% of the bonds in the GPB index are investment-graded while 42% of the bonds in the index are subinvestment graded, with B- being the lowest credit rating (Kochetygova, Arora, and Jauhari 2014). Figure 2 shows the daily returns on the GPB Index between April 2010 and April 2015. A unit root test of the index’s daily returns shows that the series is stationary (Table 1).

To compare the performance of the green bond indices with the broader conventional bond market, I use the S&P U.S. AB Index as a benchmark index for the analysis. The index provides an overview of the market for publicly-issued U.S. dollar denominated investment-grade bonds, where all bonds have a minimum credit rating of BBB- or equivalent. Similar indices that track the performance of publicly-issued US dollar denominated

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**Table 1. Unit root tests.**

<table>
<thead>
<tr>
<th></th>
<th>ADF test statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns on GB Index</td>
<td>−37.197***</td>
</tr>
<tr>
<td>Returns on GPB index</td>
<td>−37.559***</td>
</tr>
<tr>
<td>Returns on US AB index</td>
<td>−37.998***</td>
</tr>
</tbody>
</table>

*p < 10%.

**p < 5%.

***p < 1%.

---

**Figure 1.** GB Index daily returns (Source: S&P). (a) Returns, (b) Squared returns and (c) Absolute value of returns. Sampling period: Daily 4/30/2010–4/29/2015.
investment-grade bonds have been used in previous studies as a benchmark for the overall bond market (e.g. Daskalaki and Skiadopoulos 2011; Case, Yang, and Yildirim 2012). Figure 3 shows the daily returns on the US AB Index between April 2010 and April 2015. Again, a unit root test shows that the index’s daily returns follow a stationary process (Table 1).

Table 2 summarizes the descriptive statistics of the indices’ returns and Figure 4 plots the autocorrelation functions of the returns. The autocorrelation functions show that present returns have little correlation with their lagged values, which is consistent with the stylized fact that financial market returns are more or less unpredictable. Among the three indices, the GPB Index has the highest average returns while the GB Index has the highest standard deviation. The standard deviations in all three series are larger than their mean. The GB Index has the highest Sharpe ratio. In other words, among

**Table 2.** Descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>GB Returns</th>
<th>GPB Returns</th>
<th>AB Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0001</td>
</tr>
<tr>
<td>Min</td>
<td>-0.0308</td>
<td>-0.0192</td>
<td>-0.0089</td>
</tr>
<tr>
<td>Max</td>
<td>0.0257</td>
<td>0.0131</td>
<td>0.0082</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.0049</td>
<td>0.0027</td>
<td>0.0021</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1846</td>
<td>-0.5710</td>
<td>-0.1640</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.5840</td>
<td>5.3900</td>
<td>1.1780</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.0260</td>
<td>0.0050</td>
<td>0.0054</td>
</tr>
<tr>
<td>Weighted average maturity</td>
<td>6.83 years</td>
<td>13.94 years</td>
<td>6.60 years</td>
</tr>
<tr>
<td>Par weighted price</td>
<td>103.53</td>
<td>104.1</td>
<td>106.21</td>
</tr>
</tbody>
</table>

Note: GB = Green Bond Index, GPB = Green Project Bond Index and AB = US Aggregate Bond Index.
Source: S & P.
the three indices, the GB Index provides investors with the best returns given the same amount of risk or equivalently, the lowest risk given the same amount of returns. Specifically, the GB Index offers a 2.60% excess returns per unit of risk, which is much higher than that of the GPB Index and the US AB Index (0.50% and 0.54%, respectively).11 A closer look at the squared returns for each index (middle panels of Figures 1–3) indicates that all three series exhibit volatility clustering, where periods of high volatility tends to be followed by periods of high volatility and vice versa.12

Figures 5 and 6 show the 20-day rolling covariances and correlations between the GB Index, the GPB Index and the benchmark US AB Index. There are considerable variations in the rolling covariances and correlations between the indices. The rolling covariances and correlations between the GB Index and the US AB Index (Figure 5) are mostly lower than their unconditional covariance and correlation for the first half of the sampling period and begin to rise above the unconditional measures for the second half of the sampling period. On the other hand, the rolling covariances and correlations between the GPB Index and the US AB Index tends to stay above their unconditional counterparts during the sampling period.

5. Results and discussion

5.1. Preliminary tests

To test the validity of the GARCH models discussed in Section 3 given the available data, I first run a Box–Ljung test on the squared index returns. The Box–Ljung test is often used to test the independence of a given time series, where the null hypothesis is that there is no serial correlation within the series. Results from this test on squared index returns
(Table 3) reject the null hypothesis of no serial correlation, thereby validating the existence of volatility clustering effect in the data where current volatility depends on the magnitudes of volatility in past periods. Moreover, I also found that positive and negative shocks have similar impacts on the volatility of the data, therefore, it is not necessary to incorporate a measure for the asymmetric leverage effects into the GARCH framework in Section 3. Finally, the Schwartz information criteria suggest that the return Equations (1), (4), (5) are best described by excluding all the lagged values (i.e. \( r = r_M = r_G = s = 0 \)) and the volatility Equations (3), (7), (8) are best described by a GARCH(1,1) process (i.e. \( \rho = \rho_M = \rho_G = q = q_M = q_G = 1 \)).
5.2. Univariate GARCH empirical results

The preliminary test results suggest that we can reduce the univariate GARCH model in Section 3 to the following set of equations:

\[ R_t = E_{t-1}[R_t] + \epsilon_t, \quad \epsilon_t | I_{t-1} \sim iid(0, \sigma_t^2), \tag{10} \]

Table 3. Box-Ljung test of squared returns.

<table>
<thead>
<tr>
<th></th>
<th>GB Returns</th>
<th>GPB Returns</th>
<th>AB Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-statistic</td>
<td>380***</td>
<td>17*</td>
<td>350***</td>
</tr>
</tbody>
</table>

Note: GB = Green Bond Index, GPB = Green Project Bond Index and AB = US Aggregate Bond Index.

*\ p < 10%.
**\ p < 5%.
***\ p < 1%.

Figure 5. Rolling covariance and rolling correlation between GB and AB indices (Source: S&P). (a) Rolling covariance and (b) Rolling correlation. GB = Green Bond Index; GPB = Green Project Bond Index; AB = US Aggregate Bond Index. The straight line is the unconditional covariance/correlation between the two series. Sampling period: Daily 4/30/2010–4/29/2015.

Figure 6. Rolling covariance and rolling correlation between GPB and AB indices (Source: S&P). (a) Rolling covariance and (b) Rolling correlation. GB = Green Bond Index; GPB = Green Project Bond Index; AB = US Aggregate Bond Index. The straight line is the unconditional covariance/correlation between the two series. Sampling period: Daily 4/30/2010–4/29/2015.

5.2. Univariate GARCH empirical results

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\[ R_t = E_{t-1}[R_t] + \epsilon_t, \quad \epsilon_t | I_{t-1} \sim iid(0, \sigma_t^2), \tag{10} \]
\[ E_{t-1}[R_t] = \mu, \quad (11) \]
\[ \sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2, \quad a_0 > 0; \ a_1 > 0; \ b_1 > 0, \quad (12) \]

Table 4 shows the estimated parameters of the univariate GARCH model for the two green bond return series (GB Index and GPB Index) as well as their market benchmark (US AB Index). Figure 7 plots the conditional standard deviation of the three time series based on the univariate GARCH estimates.

Compared to the US AB Index, the ‘labeled’ segment of the green bond market, which is characterized by the GB Index, tends to exhibit higher level of volatility clustering. The estimates for the ARCH/GARCH parameters \( a_1 \) and \( b_1 \) are higher for the GB Index returns than for the US AB Index. In fact, it takes 258.6 days for a shock to the GB Index to reduce its impact by 50%. On the other hand, the half-life of a shock to the US AB Index is only 68.3 days. Moreover, the conditional standard deviation for the GB Index is higher and more volatile than the conditional standard deviation for the US AB Index. As can be seen in Figure 7, the conditional standard deviation of the GB Index ranges between 0.00200 and 0.01120 over the sampling period, while the conditional standard deviation of the US AB Index ranges between 0.00130 and 0.00390 over the same period.

The pattern of volatility clustering seems to lower significantly once we incorporate the ‘unlabeled’ green bond market segment into the model, as illustrated by the empirical results for the GPB Index (Column (2) of Table 4). In fact, it only takes 34.41 days for a shock to the GPB Index to reduce its impacts by 50%, which is a significant decline compared to a half-life of 258.6 days for the GB Index. Moreover, the conditional standard deviation of the GPB Index returns ranges between 0.00201 and 0.00457 (Figure 7), which is much lower than that for the GB Index. A possible explanation for the lower volatility clustering in the GPB Index return relies on the fact that there is more diversity in the bond portfolio used to calculate the Green Project Bond Index, which include both investment-grade- and subinvestment-grade- bonds (Kochetygova and Jauhari 2014). As a
result, there are lower correlations among the bonds within the Green Project Bond Index, thus reducing the magnitude of volatility clustering in the GPB Index compared to the GB Index. Under this univariate GARCH framework, the unconditional average index return ($\mu$) is also the highest for the GPB Index, which is consistent with the descriptive statistics presented in Section 4.

**Figure 7.** Conditional standard deviation ($\sigma_t$). Model: Univariate GARCH (1,1). Variables: GB, GPB and AB. Sampling period: Daily 4/30/2010–4/29/2015. (a) GB Index, (b) GPB Index and (c) US AB Index.
In addition to the analysis using raw data on the bond indices’ returns, I also exclude all outliers in the estimates to account for the potentially high sensitivity of volatility measures to extreme observations or outliers. This will provide a more robust and stable estimation of the model parameters. Table 5 and Figure 9, which summarize the results of the estimation with the outlier-free data, show that the main results still hold even after we exclude outliers from the analysis.

5.3. Multivariate GARCH empirical results

While the univariate GARCH framework above can capture the pattern of volatility clustering for a single time series, it fails to capture the potential volatility spillover from one
time series to another. In fact, Figure 7 shows that there are comovements between the green bond market volatility (as characterized by the GB and GPB Indices) and the conventional bond market volatility (as characterized by the US AB Index). Therefore, it is appropriate to analyze the volatility movement of the green bond market in relation with that of the conventional bond market in a multivariate setting. In this section, I present the empirical results of the bivariate GARCH model with two variables: a Green Bond Market Index and a Conventional Bond Market Index. In this setting, the index for the green bond market is either the GPB Index or the GPB Index while the benchmark market index is the US AB Index. The conditional standard deviations for each individual series and the conditional correlations among the series are estimated using the DCC model proposed by Engle (2002). Compared to other models, this model’s flexibility in modeling time-varying conditional correlations has clear computational advantages as it allows for the estimation of very large correlation matrices.

According to the preliminary test results, we can reduce the bivariate GARCH model in Section 3 to the following set of equations:

\[ R_{Gt} = \mu_G + \epsilon_{Gt}, \]  
\[ R_{Mt} = \mu_M + \epsilon_{Mt}, \]  
\[ \epsilon_t | I_{t-1} = \begin{bmatrix} \epsilon_{Gt} \\ \epsilon_{Mt} \end{bmatrix} I_{t-1} \sim WN(0, \Sigma_t), \]  
\[ \Sigma_t = D_t R_t D_t; \quad R_t = \begin{bmatrix} 1 & \rho_{GMT} \\ \rho_{MGt} & 1 \end{bmatrix}; \quad D_t = \begin{bmatrix} \sigma_{Gt} & 0 \\ 0 & \sigma_{Mt} \end{bmatrix}, \]  
\[ \sigma^2_{Gt} = a_0G + a_1G \epsilon^2_{Gt-1} + b_1G \sigma^2_{Gt-1}, \]  
\[ \sigma^2_{Mt} = a_0M + a_1M \epsilon^2_{Mt-1} + b_1M \sigma^2_{Mt-1}. \]

Table 5. Univariate volatility modelling – GARCH(1,1) with outlier-free data.

<table>
<thead>
<tr>
<th></th>
<th>(1) GB Returns</th>
<th>(2) GPB Returns</th>
<th>(3) AB Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu)</td>
<td>0.00003</td>
<td>0.00037***</td>
<td>0.00013***</td>
</tr>
<tr>
<td></td>
<td>(0.00009)</td>
<td>(0.00007)</td>
<td>(0.00005)</td>
</tr>
<tr>
<td>(a_0)</td>
<td>0.00000006**</td>
<td>0.00000020***</td>
<td>0.00000004**</td>
</tr>
<tr>
<td></td>
<td>(0.0000002)</td>
<td>(0.00000008)</td>
<td>(0.00000002))</td>
</tr>
<tr>
<td>(a_1)</td>
<td>0.0423***</td>
<td>0.0463***</td>
<td>0.0453***</td>
</tr>
<tr>
<td></td>
<td>(0.0075)</td>
<td>(0.0129)</td>
<td>(0.0109)</td>
</tr>
<tr>
<td>(b_1)</td>
<td>0.9547***</td>
<td>0.9227***</td>
<td>0.9452***</td>
</tr>
<tr>
<td></td>
<td>(0.0074)</td>
<td>(0.0218)</td>
<td>(0.0137)</td>
</tr>
<tr>
<td>Unconditional mean in mean equation ((\mu))</td>
<td>0.00003</td>
<td>0.00037</td>
<td>0.00013</td>
</tr>
<tr>
<td>Persistence ((a_1 + b_1))</td>
<td>0.9971</td>
<td>0.9691</td>
<td>0.9905</td>
</tr>
<tr>
<td>Unconditional variance ((a_0/(a_1 + b_1)))</td>
<td>0.00000006</td>
<td>0.00000021</td>
<td>0.00000004</td>
</tr>
<tr>
<td>Half-life (days)</td>
<td>((\ln(0.5)/\ln(a_1 + b_1)))</td>
<td>239.4</td>
<td>22.09</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses are standard errors
GB = Green Bond Index, GPB = Green Project Bond Index and AB = US Aggregate Bond Index.
\*\(p < 10\%\).
\**\(p < 5\%\).
\***\(p < 1\%\).
\( \hat{q}_{ijt} = \text{Cov}(\hat{z}_{it}, \hat{z}_{jt} | I_{t-1}) = \hat{E}[\hat{z}_{it}\hat{z}_{jt}](1 - \alpha - \beta) + \alpha\hat{z}_{it-1}\hat{z}_{jt-1} + \beta\hat{q}_{ijt-1}, \)

\( i, j = G, M, \)
where \( \hat{z}_{it} = (\hat{e}_{it}/\hat{\sigma}_{it}) \) \( (i = G, M) \) is the standardized residuals is calculated from the GARCH models in Equations (16) and (17).\(^{15}\)

Table 7 summarizes the results of the bivariate GARCH modeling for the green bond market. Specifically, Column (1) of the table shows the results of a bivariate GARCH model for the returns of the labeled green bond market (as captured by the GB Index) and the conventional bond market (as captured by the GPB Index). On the other hand, Column (2) of the table shows the results of the same model for the returns of the unlabeled green bond market (as captured by the GPB Index) and the conventional bond market. Overall, the labeled sector of the green bond market exhibit large volatility clustering compared to the unlabeled green bond sector and the conventional bond market. Figure 10(a) and 10(b) shows the estimates of the conditional standard deviation of the GB Index relative to the US AB Index while Figure 11(a) and 11(b) shows the estimates of the conditional standard deviation of the Green Project Bond Index relative to the US AB Index. It can be seen from the figures that the conditional standard deviation of the GPB Index is lower than that of the conventional bond market while the conditional standard deviation of the GB Index tends to be higher than that of the conventional bond market. This is consistent with the results obtained from the previous univariate GARCH model.

Moreover, the bivariate GARCH model also suggests that there exists volatility spillover between the green bond market and the overall fixed-income market since the parameters \( \alpha \) and \( \beta \) are both positive and statistically significant. This provides evidence for the existence

<table>
<thead>
<tr>
<th>Parameter estimates for the Green Bond Market Index :</th>
<th>(1) Green Bond Index = GB</th>
<th>(2) Green Bond Index = GPB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_G )</td>
<td>0.000035</td>
<td>0.000367***</td>
</tr>
<tr>
<td></td>
<td>(0.000101)</td>
<td>(0.000073)</td>
</tr>
<tr>
<td>( a_{0G} )</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>(0.000003)</td>
<td>(0.000000)</td>
</tr>
<tr>
<td>( a_{1G} )</td>
<td>0.051400</td>
<td>0.028907***</td>
</tr>
<tr>
<td></td>
<td>(0.068685)</td>
<td>(0.008305)</td>
</tr>
<tr>
<td>( b_{1G} )</td>
<td>0.946498***</td>
<td>0.955050***</td>
</tr>
<tr>
<td></td>
<td>(0.064320)</td>
<td>(0.008399)</td>
</tr>
</tbody>
</table>

Parameter estimates for the benchmark conventional bond market (AB)

<table>
<thead>
<tr>
<th>Parameter estimates for the benchmark conventional bond market (AB)</th>
<th>(1) Green Bond Index = GB</th>
<th>(2) Green Bond Index = GPB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_M )</td>
<td>0.000139***</td>
<td>0.000139***</td>
</tr>
<tr>
<td></td>
<td>(0.000050)</td>
<td>(0.000051)</td>
</tr>
<tr>
<td>( a_M )</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>(0.000002)</td>
<td>(0.000002)</td>
</tr>
<tr>
<td>( a_{1M} )</td>
<td>0.044203</td>
<td>0.044203</td>
</tr>
<tr>
<td></td>
<td>(0.080317)</td>
<td>(0.080279)</td>
</tr>
<tr>
<td>( b_{1M} )</td>
<td>0.949094***</td>
<td>0.949094***</td>
</tr>
<tr>
<td></td>
<td>(0.077196)</td>
<td>(0.077158)</td>
</tr>
</tbody>
</table>

Estimates for the conditional correlation parameters

<table>
<thead>
<tr>
<th>Parameter estimates for the conditional correlation parameters</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.022922***</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>(0.005019)</td>
<td>(0.000006)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.975424***</td>
<td>0.908743***</td>
</tr>
<tr>
<td></td>
<td>(0.005770)</td>
<td>(0.115694)</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses are standard errors.

GB = Green Bond Index, GPB = Green Project Bond Index and AB = US Aggregate Bond Index.

\(^*\)\( p < 10\% \).

\(^{**}\)\( p < 5\% \).

\(^{***}\)\( p < 1\% \).
of a non-constant interaction between the green bond indices and the market benchmark index with respect to conditional correlation. Figure 10(c) shows the conditional correlation between the GB Index and the market benchmark index while Figure 11(c) shows the conditional correlation between the GPB Index and the market benchmark index. The figures show that there are increasing correlations between the GB Index and the market benchmark over time while there is no clear pattern of correlations between the GPB Index and the market benchmark. However, on average, both the GB Index and the Green Project Bond Index are positively correlated with the market benchmark (i.e. the US AB Index). This is explained by the fact that besides the use of proceeds towards environmentally friendly projects, green bonds are no different from conventional bonds and are often priced very close to regular bonds (World Bank 2015c).

To account for the potential impacts of extreme observations on the robustness of the results, I also repeat the above empirical analysis after cleaning the data from all outliers. Table 7 shows the estimation result of the multivariate GARCH model after accounting for outliers in the data and Figures 12 and 13 show the conditional standard deviation and conditional correlation of the two green bond indices in comparison with the benchmark US AB Index. Overall, the main empirical results above still hold even after controlling for the impacts of outliers.

The above estimates of the multivariate GARCH model can be used to construct hedge ratios. Following Kroner and Sultan (1993), the hedge ratio between the GB Index $G$ and
the Bond Market Benchmark Index $M$ at time $t$ is

$$\beta_{GMt} = \frac{\sigma_{GMt}}{\sigma_{Mt}}$$

(20)
where $\sigma_{GMt}$ denotes the covariance between the GB Index and the benchmark Index at time $t$ and $\sigma^2_{Mt}$ denotes the variance of the benchmark index at time $t$. A positive hedge ratio $\beta_{GMt}$ shows the extent to which a long position in the green bond market can be hedged by a short...
position in the overall conventional bond market. On the other hand, a negative hedge ratio shows the extent to which a short position in the green bond market can hedge by a long position in the overall conventional bond market. Figures 14 and 15 show the time-
varying hedge ratio between the two green bond indices (the GB Index and the GPB Index) and the US AB Index while Table 8 provides a summary of the hedge ratios. As can be seen in the figures and table, the estimated hedge ratios exhibit a lot of variability, which reflects the

**Figure 13.** Conditional standard deviation and conditional correlation. Model: Bivariate DCC M-GARCH (1,1). Variables: GPB Index and AB Index without outliers. Sampling period: Daily 4/30/2010–4/29/2015. (a) GPB’s conditional standard deviation, (b) AB’s conditional standard deviation and (c) GPB-AB conditional correlation.
non-constant interaction of volatility between the green bond market and the overall broader bond market.

6. Conclusion

With the growing awareness of environmental issues among investors and the general public, the urge to mobilize the global debt market as a low-cost financing instrument...
for a green economy has been stronger than ever. The development of green bonds and other environmentally friendly investments offer a mechanism to promote investments that can both economically and environmentally sound. This paper is the first to study the volatility behavior of the green bond market in relation with the broader conventional bond market in the hope to provide investors with extra insight into this new, yet promising market. The paper utilizes data on the daily closing prices of the S&P GB Index, GPB Index and U.S. AB Index between 4/30/2010 and 4/29/2015 and analyzes their volatility under both a univariate and multivariate GARCH framework. Overall, both the univariate and multivariate models suggest that there exists volatility clustering within each individual index and the multivariate model suggests that there are also evidence for time-varying volatility spillover between the green bond market and the conventional bond market, where both the ‘labeled’ and ‘unlabeled’ segments of the green bond market are positively correlated with the conventional bond market. These results are robust even after accounting for the impacts of extreme values in the data.

The paper has several implications for investors as well as policymakers. First, the estimation results can be used to construct the optimal risk-minimizing portfolio mix between green bonds and conventional bonds. However, the variability in the estimated hedge ratios between the green bond market and the conventional bond market indicates that the optimal portfolio mix requires frequent updating. Second, the results also show that there was an increase in the correlation between the ‘labeled’ green bond market and the conventional bond market, indicating a convergence of returns between the ‘labeled’ green bond market and the conventional bond market. Therefore, as the green bond market continues to grow, it is important to introduce stronger differentiation strategies between green bonds and conventional bonds in order to attract a broader pool of investors. Finally, policies aimed at standardizing the certification process of green bonds and increasing investors’ awareness can allow the green bond market to reach a broader group of investors.

The analysis in this paper provides several suggestions for future research. First, the analysis could benefit from using a longer time series in the future to reflect the behavior of green bonds during a full business cycle. Second, it would be interesting to investigate the role of green bonds in mitigating climate or environmental risk and the relationship of green bonds with other financial markets, such as energy markets and equity markets. Third, depending on data availability, future research could study how the behavior of green bonds changes with changes in types and locations of issuers. Finally, a more complete understanding of the environmental impacts of the projects funded by green bonds would play an important role in fostering the growth of this new market.

Notes
1. In 2012, greenhouse gas emissions from human activities were 47.6 metric tons of carbon dioxide equivalent, which was a 40% increase from 1990 (World Resource Institute 2016).
2. The adverse impacts of human activities on the natural environment have been documented by many environmental scientists (Stenseth et al. 2002; Oreskes 2004). However, concerns over climate change are broader than just a negative environmental impact. For example, Stern et al. (2006) claim that climate change can dampen economic growth, while Portier et al. (2010) anticipates that climate change can lead to increasing risks of cancer, cardiovascular diseases, heat-related illness and many other health disorders.
3. For example, green bonds are funding a wide variety of projects that improve agricultural productivity, energy efficiency, forest management, and transportation. See Mathews and Kidney (2012) and World Bank (2015a) for case studies and examples of projects funded by green bonds in various countries.

4. The first participants in the green bond market were large institutions like multinational development banks. Nowadays the green bond market has attracted a broad group of investors such as asset managers, pension funds, companies, foundations and religious organizations. Issuers and investors use green bonds as a way to communicate their commitment to sustainability and social responsibility to their stakeholders. Kochetygova and Jauhari (2014), OECD (2015) and Damutz (2016)

5. According to the Organization of Economic Co-operation and Development, between 2012 and 2014, sustainable investments increased by 61%, where half of the investments are allocated to bonds (OECD 2015).


7. $\sigma_i$ denotes the standard deviation of series $i$ while $\sigma_{ij}$ denotes the covariance between series $i$ and series $j$, $i,j=G,M$; $i \neq j$.


9. $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}; i,j=G,M$.

10. The lag lengths $p_G$, $p_M$, $q_G$, $q_M$ are determined using the Schwartz information criteria

11. Sharpe ratio of an asset $= (R_a - R_f)/\sigma_a$, where $R_a$ and $\sigma_a$ denote the returns and standard deviation of the asset in consideration, and $R_f$ denotes the returns on a risk-free asset.

12. I use 3-month yields on US Treasury bill as a proxy for the risk-free asset returns. Data for US Treasury bill yields are obtained from St. Louis FRED.

13. Appendix discusses the methodology used to test for asymmetric leverage effects of a time series.

14. I used the methodology specified in Khan, Van Aelst, and Zamar (2007) and Boudt, Peterson, and Croux (2008) to clean the dataset from outliers. Under this approach, all observations are first ranked by their extremeness, which is measured by their squared Mahalanobis distance from the mean and variance. Then any observation with an estimated squared Mahalanobis distance greater than the 99.9% quantile will be identified as an outlier. Under this approach, the number of extreme values is 14 for both the GB Index and the GPB Index, while the US AB Index contains nine extreme values. Figure 8 shows the comparison between the raw data with outliers and the cleaned data that are outlier free.

15. As one reviewer pointed out, one concern with using the bivariate GARCH model is the amount of overlap in assets between the green bond indices and the US AB Index. In this case, the overlap in assets between the GB Index and the GPB Index will not affect the results, because the two indices are used in two separate estimates of the above bivariate GARCH model. The overlap of assets between the green bond indices and the US AB Index does not affect the analysis of the bivariate GARCH model since the empirical results are consistent between the univariate and the bivariate GARCH models, as shown in the subsequent discussion of the results. Moreover, the amount of overlap in assets between the green bond indices and other bond indices that are not sustainability-themed is small, since the green bond indices are used to track a very specialized and small portion of the bond market. In 2015, total ‘labeled’ and ‘unlabeled’ green bond outstanding was USD 65.9 billion and USD 531.8 billion respectively, which represents less than 1% of the total value outstanding of the overall bond market (World Bank 2015c).

Disclosure statement

No potential conflict of interest was reported by the author.
Notes on contributor

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References


Appendix. Testing for asymmetric leverage effects

One stylized fact in the financial world is that volatility tends to be higher in response to negative shocks than to positive shocks. This section presents a simple framework to uncover this possible asymmetric leverage effects in a given time series, according to Zivot (2008).
Let \( \{R_t\} \) be a time series of asset returns. To test for the presence of asymmetric leverage effects in \( \{R_t\} \), we first obtain the residuals from the following conditional mean regression:

\[
E_{t-1}[R_t] = c + \sum_{h=1}^{r} \phi_h(y_{t-h}) + \sum_{k=1}^{s} \psi_k e_{t-k} + \epsilon_t,
\]  

(A1)

where the lag lengths \( r \) and \( s \) are determined using the Schwartz information criteria.

Next, we estimate the following regression:

\[
\hat{e}_t^2 = \gamma_0 + \gamma_1 \hat{\omega}_{t-1} + \xi_t,
\]  

(A2)

where \( \hat{e}_t \) is the estimated residuals from Equation (20), \( \hat{\omega}_t \) is a dummy variable that equals 1 when \( \hat{e}_t < 0 \) and 0 otherwise. A significant \( \gamma_1 \) provides evidence for asymmetric leverage effects in the ARCH/GARCH model.