Inertial rotation measurement with atomic spins: From angular momentum conservation to quantum phase theory

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Inertial rotation measurement with atomic spins: From angular momentum conservation to quantum phase theory

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Rotation measurement in an inertial frame is an important technology for modern advanced navigation systems and fundamental physics research. Inertial rotation measurement with atomic spin has demonstrated potential in both high-precision applications and small-volume low-cost devices. After rapid development in the last few decades, atomic spin gyroscopes are considered a promising competitor to current conventional gyroscopes—from rate-grade to strategic-grade applications. Although it has been more than a century since the discovery of the relationship between atomic spin and mechanical rotation by Einstein [Naturwissenschaften, 3(19) (1915)], research on the coupling between spin and rotation is still a focus point. The semi-classical Larmor precession model is usually adopted to describe atomic spin gyroscope measurement principles. More recently, the geometric phase theory has provided a different view of the rotation measurement mechanism via atomic spin. The theory has been used to describe a gyroscope based on the nuclear spin ensembles in diamond. A comprehensive understanding of inertial rotation measurement principles based on atomic spin would be helpful for future applications. This work reviews different atomic spin gyroscopes and their rotation measurement principles with a historical overlook. In addition, the spin-rotation coupling mechanism in the context of the quantum phase theory is presented. The geometric phase is assumed to be the origin of the measurable rotation signal from atomic spins. In conclusion, with a complete understanding of inertial rotation measurements using atomic spin and advances in techniques, wide application of high-performance atomic spin gyroscopes is expected in the near future.

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I. INTRODUCTION

Precision inertial rotation measurement is of great importance in guidance, navigation, and control (GNC) technology as well as fundamental physics research.1,2 Atomic devices are increasingly used to measure various physical quantities such as time and magnetic field strength.3,4 The potential for high precision measurement as well as commercial applications for small-volume low-cost devices has driven scientists and engineers to develop atomic gyroscopes. Pioneering work was done more than half a century ago, and recently atomic gyroscopes are experiencing rapid development.

One of the most important applications for atomic gyroscopes is rotation sensing in an inertial navigation system (INS). An INS is an autonomous navigation system that has many unique features such as anti-interference and high data...
mainly focused on NMRG developments. In 2006, Stoner dates for next generation INSs. As a result, they are thought to be the most promising candidates. In a word, atomic gyroscopes are competitive with optical gyroscopes and conventional gyroscopes. The performance and cost of the gyroscopes grow exponentially with the performance. However, commercial MEMS gyroscopes do not currently meet the demand of navigation-grade applications, and many problems need to be solved to suppress the drift prevalent in MEMS gyroscopes. Therefore, new research on high-precision gyroscopes, as well as low-cost, small-package gyroscopes, has become an intensively studied area. Table I summarizes the performance, size, and cost of both the conventional gyroscopes and the latest atomic gyroscopes. The gyroscopes are categorized into rate/tactical-grade, navigation-grade, and strategic-grade according to the drift level acquired in applications. As shown in Table I, the size and cost of conventional gyroscopes grow exponentially with the performance. However, a nuclear magnetic resonance gyroscope (NMRG) drift is comparable to a ring laser gyroscope (RLG) and a fiber optics gyroscope (FOG), while lower in cost and available in a smaller robust package. Meanwhile, an atomic spin gyroscope in the spin relaxation free (SERF) regime maintains a strategic-grade performance with a lower cost in comparison to a floated gyroscope (FG) and an electrical suspended gyrooscope (ESG). The atomic interferometer gyroscope (AIG) is projected to demonstrate better performance than the ESG, which is the most precise of the conventional gyroscopes. The performance and cost of the SERF gyroscope and the AIG are given in the table according to theoretical evaluation from current laboratory experiments. In a word, atomic gyroscopes are competitive with conventional gyroscopes in performance, cost, and size. As a result, they are thought to be the most promising candidates for next generation INSs.

In the 1980s, atomic gyroscope reviews and reports mainly focused on NMRG developments. In 2006, Stoner and Walsworth reported the SERF gyroscope based on the K-3He comagnetometer researched by Kornack et al. In 2010, Donley reviewed the NMRG techniques and miniaturization trends. In 2011, Kitching et al. reviewed the basic physics and instrumentation issues of several atomic sensors, including the NMRG, the SERF gyroscope, and the AIG. In 2012, Fang and Qin reviewed the AIG and the SERF gyroscope from the viewpoint of inertial navigation applications. Later the same year, Larsen reported their progress on NMRG for micro-packaged positioning, navigation, and timing system. This differs from the AIG—which still has a long way to go before their use as a commercial sensor. Great progress has been made in atomic spin gyroscopes, and they are expected to supersede conventional gyroscopes in next generation INSs, from navigation-grade to strategic-grade implementations. More recently, various atomic spin gyroscopes that utilize different rotation sensing principles have been proposed. However, all measure the rotation based on the atomic spin, so a common thread exists between the different methods. A better understanding of the atomic spin-based rotation-sensing mechanisms will be helpful for the development of atomic spin gyroscopes.

The connection between mechanical rotation and atomic spin was first discussed a century ago. Fig. 1 shows the historical theoretical and technological developments of atomic spin-based rotation measurement. In 1915, Einstein and De Haas found that ferromagnetic material magnetization induces a mechanical rotation. Later, Barnett discovered the inverse phenomenon that mechanical rotation spontaneously magnetizes an uncharged object. Their research indicates the connection between mechanical rotation and atomic spin. The gyroscope was designed and manufactured from the viewpoint of inertial navigation applications. The right column lists developments of atomic spin gyroscope technologies—from early semi-classical theories to the geometric phase theory. The right column lists developments of atomic spin gyroscopes—beginning with the NMRG to the latest nuclear spin gyroscope based on NV centers in diamond.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Types</th>
<th>Drift (deg/h)</th>
<th>Size (mm³)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate/tactical grade</td>
<td>MEMS Gyro.</td>
<td>1000–0.1</td>
<td>&lt;100</td>
<td>&lt;100</td>
</tr>
<tr>
<td></td>
<td>NMRG</td>
<td>0.01</td>
<td>10–10⁻⁴</td>
<td>10⁻⁴–10⁴</td>
</tr>
<tr>
<td></td>
<td>RLG</td>
<td>0.01–10⁻³</td>
<td>10⁻⁶–10⁻⁴</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FOG</td>
<td>0.01–10⁻⁴</td>
<td>10⁻⁴–10⁻⁶</td>
<td></td>
</tr>
<tr>
<td>Navigation grade</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Floated Gyro.</td>
<td>10⁻³–10⁻⁴</td>
<td>10⁻⁴–10⁻⁶</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ESG</td>
<td>10⁻⁴–10⁻⁵</td>
<td>10⁻⁶–10⁻⁷</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SERF Gyro.</td>
<td>~10⁻⁴</td>
<td>10⁻⁶–10⁻⁷</td>
<td>&lt;10⁵</td>
</tr>
<tr>
<td></td>
<td>AIG</td>
<td>&gt;10⁻⁵</td>
<td>~10⁶</td>
<td>10⁻³–10⁶</td>
</tr>
</tbody>
</table>

FIG. 1. Theoretical and technological developments for the atomic spin-based rotation measurement. The left column lists the progress on spin-rotation coupling theories—from early semi-classical theories to the geometric phase theory. The right column lists developments of atomic spin gyroscope technologies—from the NMRG to the latest nuclear spin gyroscope based on NV⁺ centers in diamond.
that mechanical rotation is coupled into spin dynamics, and they explained the phenomena from the viewpoint of angular momentum conservation.\(^3\) In 1956, Pancharatnam discovered a geometric phase theory that explained the geometrical phenomenon in an adiabatic cyclical evolving system.\(^4\) However, the work did not draw much attention until Berry rediscovered the geometric phase in 1984.\(^5\) In 1987, Aharonov and Anandan generalized the theory and pointed out that the adiabatic condition is not essential in geometric phase accumulation.\(^6\) During the same year, Suter et al. ran NMR experiments to explore the Berry phase,\(^7\) and Tycko demonstrated the Berry phase effect in nuclear quadrupole resonance (NQR) using a single crystal sample.\(^8\) Since then, various experiments have measured the geometric phase in different quantum systems.\(^9\)–\(^14\) Recently, Maclaurin et al. proposed a geometric phase measurement using a single negatively charged nitrogen vacancy (NV\(^-\)) center in a spinning diamond;\(^15\) the idea was extended to a nuclear spin gyroscope based on NV\(^-\) spin ensembles in diamond.\(^16\) This was the first time that the geometric phase theory was proposed as the principle of atomic spin-based rotation sensing.

In 1955, after Rabi and Bloch’s works on magnetic resonance,\(^17\)–\(^19\) Leete invented the first NMRG based on angular rate superposition.\(^20\) In the following years, Litton Corporation and Singer Corporation both designed navigation-grade NMRGs.\(^21\)–\(^22\) However, the NMRG lost its competitiveness in the 1980s to both the RLG and the FOG due to higher cost and larger package size.\(^23\) Thus, beyond the work by Appelt et al. on the deviation induced by the Berry phase in \(^{125}\)Xe and \(^{131}\)Xe NMRGs in 1994 and 1995,\(^24\)–\(^26\) the NMRG drew little attention for decades. In the 2000s, with the rapid development of laser technology, MEMS technology and atomic physics, atomic spin gyroscope studies experienced another boom. In 2005, Kornack et al. proposed a high sensitivity atomic spin gyroscope in the SERF regime based on a comagnetometer.\(^27\) In addition, more work was done to miniaturize the NMRG.\(^28\)–\(^30\) The atomic spin gyroscopes have shown their potential as either small robust gyroscopes or high-precision gyroscopes.

Due to its simplicity and practicality, spin precession is often used to describe principles of atomic spin-based measurements of inertial rotation. However, the semi-classical picture does not fully describe the dynamics of atomic spin evolution during rotation. In comparison, the geometric phase theory successfully explains the principle of a gyroscope based on a NV\(^-\) center ensemble.\(^31\)–\(^33\) It is expected that full quantum theories will help us further understand atomic spin gyroscopes and inspire new developments in inertial rotation sensing technologies. Since the atomic spin gyroscope is an important branch of inertial navigation technology, the interaction between spin and mechanical rotation must be discussed. In this review, various gyroscopes based on atomic spins and relative experiments are reviewed, focusing particularly on principle descriptions. We will also discuss the mechanisms of inertial rotation atomic spin-based measurement, aiming to improve understanding of atomic spin gyroscopes.

## II. SEMI-CLASSICAL THEORIES IN ATOMIC SPIN GYROSCOPE APPLICATIONS

The Einstein-de Haas effect and the Barnett effect were the earliest explorations of the relationship between rotation and atomic spin. The principle of angular momentum conservation is used to explain both effects. In the 1950s, an atomic spin gyroscope was demonstrated using a vapor cell comagnetometer containing alkali atoms and noble gas atoms. Both the NMRG and the SERF gyroscope are explained using the Larmor precession model, which phenomenologically describes the principle of the rotation measurement with a classical coordinate transformation of the angular moment. To further understand atomic spin gyroscopes, this section reviews the Einstein-de Haas effect, the Barnett effect, the NMRG, and the SERF gyroscope.

### A. Einstein-de Haas effect and Barnett effect

The Einstein-de Haas effect, discovered by Einstein and de Haas in 1915, is a physical phenomenon demonstrating that a spin angular momentum is of the same nature as the angular momentum induced by a mechanical rotation.\(^3\) In a typical experimental setup—as shown in Fig. 2(a)—a cylindrical ferromagnetic material is suspended with a string

![FIG. 2. (a) Experimental setup for the principle verification of the Einstein-de Haas effect. The sample is rigidly hung with a mirror attached. A strong magnetic impulse is generated by the coils and drives the sample to rotate an angle, \(\phi\). The slight rotation can be detected by measuring the reflection angle of the probe laser. (b) Experimental setup for Barnett field detection. The sample with atomic spin is placed on a high speed rotor. The detection coils and one of the coupling coils rotate together with the sample. Using the stationary coupling coils, the rotation-induced magnetic moment can be detected.](image)
inside a coil. When a magnetic field impulse is generated by the coil, an instantaneous rotation can be detected using a probe beam reflected by a mirror on the string.\footnote{50,64} By the law of angular momentum conservation, there must be a corresponding angular momentum inside the material to compensate for the rotation. It is now known that the extra angular momentum originates from polarized spins. Thus, the experiment can be used to determine gyromagnetic ratios, and the measurement results were detailed by Scott in 1962.\footnote{62}

Perhaps more meaningful for rotation sensing is the inverse of the Einstein-de Haas effect, named the Barnett effect. This effect was discovered by Barnett in 1915. The Barnett effect phenomenologically describes the magnetization of an uncharged spinning sample. The rotation of a sample containing spin ensembles will induce an emergent magnetic momenta, namely, the Barnett field.\footnote{33} Fig. 2(b) shows the experimental setup for Barnett field measurement.

From the classical mechanics view, the sample can be regarded as rigid with a sum of internal small angular momenta. Therefore, in the rotating frame, the classical Hamiltonian of the sample is given by

\[
H = \sum_i \left( r_i \times p_i + I_i \right) \cdot \omega - \frac{p_i^2}{2m_i} + \gamma B \cdot I_i ,
\]

where \( I_i \) is the internal angular moment, \( r_i \times p_i + I_i \) is the total angular momentum in the rotating frame, \( \frac{p_i^2}{2m_i} \) is the Lagrangian of the system, \( \omega \) is the angular velocity of the rotation, and \( \gamma \) is the gyromagnetic ratio. Combining the two parts that contain \( I_i \), we obtain

\[
H' = \gamma (B + \Delta B(\omega)) \cdot I ,
\]

\[
\Delta B(\omega) = \omega / \gamma ,
\]

where \( \Delta B(\omega) \) is the Barnett field and \( I \) is the total spin angular momentum. Obviously, through the measurement of the Barnett field, the angular velocity of the sample can be calculated. As a result, high-precision rotation sensing based on the Barnett field measurement requires a high magnetic field sensing sensitivity. With the development of cryogenic techniques, superconductors have been used for highly sensitive magnetic field detection. In 1989, Vitale \textit{et al.} proposed a gyroscope using a superconducting quantum interference device (SQUID) to detect the Barnett field generated by electron spins, and they demonstrated the principle verification experiment.\footnote{63} The technique was successfully employed in the gyroscopes for the Gravity Probe B (GPB) experiment.\footnote{64-66} The gyroscope drift is less than \( 1.6 \times 10^{-11} \text{rad/h} \), making the gyroscopes on the GPB satellite the most stable in the world yet.\footnote{1,67}

For decades, researchers have explored the nature of the Barnett field and the new frontier of related techniques. Progress on nuclear magnetic resonance theory and techniques has revealed that Barnett field detection should be accomplished via nuclear spin measurements, rather than electron spins, which are easily disturbed by external field fluctuations. In 1993, Frohlich and Studer developed a quantum Larmor precession theory, from which the Barnett field arises as a Zeeman term.\footnote{68} Recently, Matsuo \textit{et al.} investigated the problem based on the Bloch theory.\footnote{59} The results indicate that the Barnett field—as a natural behavior of the atomic spin in a non-inertial frame—is observable by monitoring resonance frequency shift. Applying the solid-state nuclear magnetic resonance (SSNMR) technique, Chudo \textit{et al.} observed the Barnett field of various nuclear spins at room temperature.\footnote{70-73} In the experiment, powdered samples were driven by a high-speed rotor in a cylindrical tube. The free induction decay (FID) signals clearly showed linear NMR spectrum shifts induced by the rotation. With a longer coherence time than that of the electron spin, a nuclear spin gyroscope based on the SSNMR technique is purportedly more sensitive and stable. However, the technique requires a Tesla grade external magnetic field to hyperpolarize the nuclear spin, which limits the detection sensitivity. In order to enhance the polarization of the nuclear spin ensembles under the influence of a weak magnetic field, the dynamic nuclear polarization (DNP) method was proposed. Diamond was the first solid-state material proposed for atomic spin gyroscopes, since it is a good system for polarizing and detecting nuclear spin ensembles.\footnote{45} Theories and experiments based on the Barnett field indicate that all forms of internal spin angular momentum can be used for rotation sensing, and various types of atomic spin gyroscopes are worth exploring. Rotation sensing based on the detection of the Barnett field is a promising technique for both high-precision gyroscopes and small, solid-state gyroscopes.

\[ \text{B. Nuclear magnetic resonance gyroscope} \]

When Leete proposed the first NMRG in 1955, he described the principle through angular velocity superposition.\footnote{50} The atomic spin magnetic moment precesses along the direction of the external magnetic field. This is called the Larmor precession, and its precession frequency depends only on the magnitude of the external field. The measurement principle of a NMRG is shown in Fig. 3(a). An observer in the rotating coordinate frame will acquire the sum of the original Larmor frequency and the rotation rate as

\[
\omega_o = \gamma B \pm \omega_r ,
\]

where \( \omega_o \) is the observed frequency, \( \omega_L = \gamma B \) is the Larmor precession frequency, and \( \omega_r \) is the rotation angular velocity.\footnote{74} The sign before \( \omega_o \) in Eq. (4) depends on the direction of the rotation. If the magnitude of the external field is known, the angular velocity of the carrier can be easily calculated. In a typical NMRG measurement setup, the nuclear spins are polarized along the direction of the external field by spin exchange collisions with continuously pumped alkali electron spins. In addition, a continuous linearly polarized RF-field is applied in the transverse plane to flip the nuclear spin. The precession frequency is monitored by a pair of sampling coils or an atomic magnetometer. Fig. 3(b) shows a typical NMRG research setup. The NMRG signal is detected using an atomic magnetometer via the balanced polarimetry technique to probe the nuclear spin precession.
A semi-classical theory for the NMRG theory was presented by Rabi et al. in 1954.\(^{75}\) With an external magnetic field, \(B\), the dynamics of the nuclear angular momentum in an inertial coordinate system are given by

\[
\hbar \frac{dI}{dt} = M \times B = \gamma \hbar I \times B,
\]

where \(I\) is the nuclear angular momentum and \(M = \gamma \hbar I\) is the nuclear magnetic moment. Meanwhile, in the rotating coordinate system, the nuclear angular momentum follows:

\[
\frac{dI}{dt} = \frac{\partial I}{\partial t} + \omega \times I.
\]

With Eqs. (5) and (6), the dynamics of \(I\) observed in the rotating frame are given by

\[
\hbar \frac{\partial I}{\partial t} = \gamma \hbar I \times \left( B + \frac{\omega}{\gamma} \right) = \gamma \hbar I \times \left( B + \Delta B(\omega) \right),
\]

where \(\Delta B(\omega) = \omega / \gamma\) is the same as the expression from the Barnett field. The extra effective field can also be understood as the rotation compensation, and Eq. (7) indicates that the observed NMR frequency shift caused by the rotation has the same nature as the Barnett field. Based on the fact that the NMRG signal is equivalent to an external magnetic field, the fundamental sensitivity is given by

\[
\delta \omega = \gamma_n \left( \frac{1}{\gamma_n} \sqrt{\frac{1}{NT_n^2 t^2}} \right) = \gamma_n \delta B,
\]

where \(\gamma_n\) is the nuclear gyromagnetic ratio, \(N\) is the number of the detected spins, \(T_n^2\) is the transverse relaxation time of the nuclear spin, \(t\) is the measurement time, and \(\delta B\) is the nuclear spin ensembles’ magnetic field sensitivity. As a result, efforts have been made to improve the stability of the external field and the RF field sensing sensitivity for decades. Magnetic field shielding has become one of the most important components in a NMRG for reducing the noise floor. Multi-species nuclear systems have been implemented for magnetic field fluctuation suppression,\(^{76}\) and the application of high sensitivity atomic magnetometers has significantly improved NMRG performance.\(^{77}\)

NMRG development has experienced two stages. During the first stage—from the 1960s to the early 1980s—the drift and the angular random walk (ARW) performance were the main research targets. Cryogenic sampling coils were used to pick up the NMR signal with high sensitivity. Although Happer and Mathur proposed the use of off-resonance light for polarized atom state probing as early in 1967,\(^{78}\) it was not until the early 1980s after further development of lasers that the technique is used in a NMRG.\(^{79}\) Optical probing combined with an atomic magnetometer greatly improves the sensitivity of NMR signal detection at room temperature. However, superconducting shields and coils are still needed to generate a stable homogenous field and to suppress the magnetic field noise floor for high performance NMRG applications.\(^{80,81}\) As a result, NMRG cost and size rapidly increased in comparison to the RLG and the FOG at the time, while the performance remained nearly the same. This led to nearly 30 years of NMRG development stagnation. The second stage of NMRG development began in the 2000s, with the advent of the chip-scale NMRG.\(^{58,76}\) With the latest vertical-cavity surface-emitting laser (VCSEL) and micro-vapor cell fabrication techniques, the NMRG package size has been tremendously reduced. Research groups at Northrop Grumman and the National Institute of Standards and Technology (NIST) made great

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**FIG. 3.** (a) Principle of the NMRG. The red sphere represents the atom ensembles in the cell (the blue circle). The nuclear magnetic moment (red arrow) precesses around the direction of the stationary magnetic field (blue arrow) with Larmor frequency in the inertial frame and emits a RF signal. The signal is detected by sampling coils or an atomic magnetometer. When the carrier frame rotates, the observed NMR signal will be the sum of the Larmor frequency and the rotating frequency. (b) Experimental setup for a NMRG with an atomic magnetometer. The comagnetometer cell containing alkali atoms and noble gas atoms is placed in a nonmagnetic oven and heated with high frequency current. The bias field coils offer the external magnetic field and the RF coils generate a RF field in the transverse plane. The nuclear spin is polarized through spin exchange collisions with the electron spin, and the electron spin is polarized by the circularly polarized laser beam. The balanced polarimetry technique is used to probe the electron spin precession, which is modulated by the nuclear spin precession.

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Larmor frequency \(\omega_L\) and observed NMR frequency \(\omega_0 = \omega_L + \omega_r\).
progress in the development of navigation grade micro-NMRGs. With comparable volumes and cost potentials between MEMS gyroscopes and optical gyroscopes, the NMRG is expected to fulfill the needs for tactical-grade navigation systems and civil applications in the near future.

C. Comagnetometer Gyroscope in the Spin Exchange Relaxation Free Regime

When exploring CPT (charge conjugation, parity transformation, and time reversal) symmetry with a K-3He comagnetometer, the Romalis group found that the comagnetometer had very high sensitivity to the earth’s rotation. This discovery indicates a new regime for high performance atomic spin gyroscopes based on a comagnetometer in the SERF regime. In the SERF regime, a vapor cell is heated to nearly 200°C in a near-zero magnetic field environment, so that the spin exchange relaxation is eliminated and an approximately 1 Hz electron spin magnetic resonance linewidth is acquired. The SERF magnetometer has achieved a sensitivity better than 1 fT/Hz^1/2, which corresponds to a ARW of a gyroscope better than 1 Hz. In contrast, the external field is eliminated in a NMRG. In addition to magnetic shields, an external field is applied to cancel out the polarized nuclear magnetic field sensitivity and the nuclear gyromagnetic ratio.

Therefore, to acquire a high-sensitivity rotation measurement, the external field should be either well-quantified or totally eliminated. In a NMRG, the former is adopted and the sensitivity is limited by the magnetic shield performance. In contrast, the external field is eliminated in a SERF gyroscope. In addition to magnetic shields, an external magnetic field is applied to cancel out the polarized nuclear spin magnetic field. As a result, the polarized nuclear spin ensembles adiabatically follow the external magnetic field change. This nuclear magnetic field self-compensation mechanism ensures the electron spin of the alkali atom evolves in a zero-field environment. Therefore, the polarized electron magnetic moment is static in the inertial coordinate system. When the apparatus coordinate system—formed by the pumping laser and the perpendicular probe laser—rotates at an angular velocity, \( \omega \), the dynamics of the electron magnetic moment are described by the Bloch equation

\[
\frac{dP}{dt} = \omega \times P + \gamma_e (B_{\text{ext}} + B_{\text{nuc}} + L) \times P - R_{\text{ext}} P, \tag{9}
\]

where the \( P \) is the electron spin magnetic moment, \( \gamma_e \) is the electron gyromagnetic ratio, \( B_{\text{ext}} \) is the external field, \( B_{\text{nuc}} \) is the effective field of the nuclear magnetic moment, \( L \) is the effective field of the light-shift (spectrum shift induced by laser due to the Stark effect), and \( R_{\text{tot}} \) is the total relaxation rate. The optical pumping is not included because it does not contain a polarization term, \( P \). In the SERF gyroscope, the effective magnetic field, \( B_{\text{eff}} = B_{\text{ext}} + B_{\text{nuc}} + L \), is set to 0, and the solution of the equation is

\[
P_x = \frac{P_z}{R_{\text{tot}}} \omega_x. \tag{10}
\]

During rotation, the electron magnetic moment, \( P \), lags behind the pumping laser because of the relaxation. Thus, the projection \( P_x \) along the probe beam direction is proportional to the rotation speed. \( \gamma_e, \omega_x, R_{\text{tot}} \) Fig. 4 describes the SERF gyroscope principles and the nuclear magnetic field self-compensation mechanisms.

With the coupled Bloch equations for the electron and nuclear spin dynamics, the shot noise-limited sensitivity of a gyroscope based on a comagnetometer can be expressed as

\[
\delta \omega = \frac{\gamma_n}{\gamma_e} \sqrt{\frac{1}{NT_{\text{eff}}^2}} = \gamma_n \delta B, \tag{11}
\]

where \( T_{\text{eff}}^2 \) is the transverse relaxation time of the electron spin. Eq. (11) shows that the fundamental sensitivity of the SERF gyroscope is limited by the SERF comagnetometer nuclear magnetic field sensitivity and the nuclear gyromagnetic ratio. The nuclear gyromagnetic ratio represents the precision of the nuclear magnetic field self-compensation, and it determines the residual magnetic field. With \( \gamma_n/\gamma_e \approx 10^{-3}, \sqrt{T_{\text{eff}}^2/T_2^2} \) ranges from 10 to a few hundred for different alkali atoms and noble gas atoms. As a result, the SERF gyroscope has a better fundamental sensitivity than the NMRG. In addition, because the SERF gyroscope has no magnetic field bias, the error accumulation induced by the magnetic field drift is suppressed. Moreover, the use of an all-optical

FIG. 4. (a) SERF gyroscope principle. The arrows in the cell (blue circle) represent electron spin magnetic moment and projection. The pump beam drives the magnetic moment, \( P \), along the \( z \)-axis as the light red arrow shows, while \( P \) falls behind in the rotation. The projection \( P_x \) detected by the rotating probe beam is proportional to the angular velocity. (b) The nuclear magnetic field self-compensation mechanism. The nuclear magnetic moment (blue spheres with arrows) automatically compensates for changes to the external magnetic field (gray dotted arrows). Thus, the electron spins (red spheres with arrows) evolve in a zero-field environment.
comagnetometer avoids RF coil noise entirely. As a result, both the stability and the sensitivity of the SERF gyroscope are improved in comparison to a NMRG. In 2005, Knoack et al. first demonstrated a K-3He gyroscope with a sensitivity of $5 \times 10^{-7}$ rad/s/Hz$^{1/2}$ and a drift of 0.04°/h at low frequencies. By applying $^{21}$Ne (with a smaller gyromagnetic ratio than $^{3}$He) Ghosh estimated that the rotational sensitivity could achieve $1 \times 10^{-9}$ rad/s/Hz$^{1/2}$ for a 6 x 15 x 15 mm$^3$ cell. In 2011, Smiciklas et al. experimentally demonstrated that a K-Rb-$^{21}$Ne comagnetometer sensitivity is better than $1 \times 10^{-8}$ rad/s/Hz$^{1/2}$, which corresponds to a drift better than 0.001°/h.

From the viewpoint of inertial navigation applications, there is another major difference between the NMRG and the SERF gyroscope: the NMRG is a rate gyroscope sensing the angular rate of change in the external field direction; and the SERF gyroscope is a kind of position gyroscope, since the electron spins evolve in a zero-field environment. The polarized electron spin is similar to a rotor in a mechanical gyroscope. Without a pump laser to force the spin precess following rotation, the electron spin becomes a “free rotor gyroscope” and the sensitivity is limited by the longitudinal relaxation time, $T_1$. Currently, the SERF gyroscope works with a continuous pump laser in a rate sensing mode similar to a mechanical rate gyroscope, and the effective field induced by the light-shift couples the angular rate of the two axes together. With a pulsed pump laser, the measurement can be carried out without pump laser interference so the light shift is eliminated. Mounted on a stabilized platform, as shown in Fig. 5, a pulse-pumped SERF gyroscope with two perpendicular probe lasers in the transverse plane acts as a double axis position gyroscope. With a longer coherence time limit, $T_1$, and the light shift elimination, higher sensitivities are expected.

One of the challenges for a SERF gyroscope is a narrow measurement bandwidth due to the slow nuclear magnetic self-compensation process. As a result, the gyroscope cannot be used in highly dynamic situations; it is only suitable as a platform inertial navigation system. In addition, the comagnetometer gyroscope takes several hours to polarize $^{3}$He or $^{21}$Ne. The start-up time can be shortened to 30 min using a Cs-$^{129}$Xe comagnetometer, and the dynamics of a Cs-$^{129}$Xe SERF gyroscope are analyzed. The drawbacks of the Cs-$^{129}$Xe SERF gyroscope is a relatively low sensitivity compared to the K-$^{3}$He and the K-Rb-$^{21}$Ne SERF gyroscopes. Nevertheless, with a lower cost, smaller size potential, and comparably low drift, the SERF gyroscope is competitive with conventional high-performance gyroscopes in strategic navigation applications.

### III. QUANTUM PHASE THEORY IN ATOMIC SPIN GYROSCOPE APPLICATIONS

Geometric phase theory provides another way to study atomic spin dynamics in a rotating frame. According to the theory, any time-dependent spin state accumulates a geometric phase during rotation; this principle can be applied to rotation measurement. This part of the paper reviews the geometric phase theory and its applications in NMR experiments. A nuclear spin gyroscope based on a NV center ensemble in diamond is also discussed.

### A. Geometric phase theory and applications in NMRG

In classical mechanics, when a macroscopic object rotates in a conservative force field, nothing is changed after a rotation cycle. This assumption is not true, however, in the microscopic world. In 1984, Berry found that during a cyclic adiabatic process, the quantum system wave function accumulates a geometric phase.

![FIG. 5. (a) Position-mode SERF gyroscope experimental setup. A comagnetometer cell is placed in an oven heated by high frequency current. An external magnetic field is applied by the coils to fulfill the nuclear spin magnetic field self-compensation conditions. The circular polarized pump beam—as well as the linearly polarized probe beam—is synchronous chopped by the acoustic-optical modulator for pulsed pumping. The two orthogonal probe beams in the transverse plane detect rotation along both axes. (b) Picture of a SERF gyroscope on a single axis stabilized servo platform. (c) Experimental setup for a position mode SERF gyroscope. The gyroscope is placed on a stabilized servo platform. The biaxial frame is driven with the feedback of the SERF gyroscope and the rotation of the frame is read as the rotation angle of the carrier in the inertial frame.](image-url)
accumulates a phase memorializing the Hamiltonian motion in the parameter space.\(^\text{35}\) For example, suppose that the Hamiltonian evolves adiabatically as it is subjected to a parameter \(R(t)\) via an enclosed circuit \(C\), then the instantaneous eigenstates of the Hamiltonian would acquire an extra phase factor, \(e^{\gamma(C)}\). Through Berry’s calculations, the “geometric phase change” \(\gamma(C)\) is proportional to the solid angle, \(\Omega\).\(^\text{99, 100}\) Based on this work, Aharonov and Anandan developed the geometric phase theory without adiabatic conditions.\(^\text{36, 102, 103}\) In a cyclic evolving system, the instantaneous state \(|\psi(t)\rangle\) satisfies

\[
|\psi(\tau)\rangle = e^{i\phi}|\psi(0)\rangle. \tag{12}
\]

By applying the transformation

\[
|\psi(t)\rangle = e^{i\gamma(t)}|\xi(t)\rangle, \tag{13}
\]

to the time-dependent Schrödinger equation, the solution of the phase factor is

\[
\phi = -\frac{1}{\hbar}\int_0^\tau \langle\psi(t)|[H, \psi(t)]|\xi(t)\rangle dt + \int_0^\tau \langle\xi(t)|\frac{\partial}{\partial t}|\xi(t)\rangle dt = \alpha(\tau) + \gamma(\tau), \tag{14}
\]

where \(\phi = f(\tau) - f(0), |\xi(\tau)\rangle = |\xi(0)\rangle\), and \(\alpha(\tau) = 1/\hbar - \int_0^\tau \langle\psi(t)|[H, \psi(t)]|\xi(t)\rangle dt\) is the dynamic phase, and \(\gamma(\tau) = \phi - \alpha(\tau)\) is the geometric phase or the AA phase.\(^\text{104}\) When the state \(|\psi(t)\rangle\) is a stationary state, \(\Phi(\tau) \equiv 0\). The AA phase accumulation does not require the Hamiltonian to cyclically change. As shown in Eq. (14), the phase factor consists of a dynamic phase and a geometric phase, where the dynamic phase is determined by the Hamiltonian of the spin system. In a NMRG, the acquired signal consists of a magnetic field portion and a rotation portion. Eqs. (2), (7), and (9) are similar in form to that of the magnetic field portion of the signal corresponding to the dynamic phase in Eq. (14). As a result, we can infer that the rotation signals of different atomic spin gyroscopes originate from the geometric phase. In the spin precession model, the first principle is that the spin is treated as a small magnetic needle aligned with the inertial frame, and the rotation difference observed in the rotating frame is read as the angular velocity. This model does not explain how the rotation is coupled into the spin state evolution. Meanwhile, the geometric phase is derived from the spin dynamics in a rotating frame by solving the time-dependent Schrödinger equation, and the rotation angular velocity is coupled into the spins state evolution as a phase factor. Thus, it is convenient and fruitful to explore geometrical-related problems from quantum physics by using the observable phase factor. This method paves a new way for the measurement of rotation using atomic spins.

In atomic spin gyroscope applications, the spin state cyclically evolves through interactions with the external field (magnetic and electric), the spin dipole, the spin quadrupole, and the rotation. Although the eigenvalues of the system remain unchanged, the geometric phase accumulates. The geometric phase was originally considered an error source in NMRG experiments. In 1994, Appelt et al. explored the nonlinear phenomenon induced by the geometric phase of the \(^{131}\)Xe NQR.\(^\text{34, 35}\) In their \(^{131}\)Xe NMRG setup, the principle axes of the \(^{131}\)Xe electric field gradient tensor were chosen based on the cylindrical cell. The experiment results showed a deviation from the adiabatic Berry phase at high angular velocity. Appelt et al. later confirmed the experiment based on \(^{129}\)Xe NMRG.\(^\text{44}\) According to their result, the mis-alignment of the NMRG quantum principle axis and the rotation axis led to a measurement error induced by the geometric phase. On the other hand—with a known quantum principle axis—the geometric phase can also be used for rotation sensing. In 1987, Tycko demonstrated the geometric phase measurement in a \(^{35}\)Cl NQR experiment with a single rotating NaClO\(_3\) crystal.\(^\text{38}\) With \(I = 3/2\) and \(H = \hbar gQI_z\), the Berry phase shift was obtained by rotating NMR sampling coils. Although the experiment was not intended to measure rotation, the setup essentially acted as a single-axis rotation sensor. Recently, a NV\(^-\) center in diamond with its quantum axis aligned along the N-V axis was proposed as a gyroscope based on the geometric phase theory.\(^\text{45}\) Although the principle verification experiment has not been completed, it inspires us to develop new kinds of solid-state atomic gyroscopes that have small package size and high sensitivity.

### B. Atomic spin gyroscope based on nuclear spin ensembles in diamond

A negatively charged nitrogen-vacancy center in a diamond is an atom-like system with an electron spin \(S = 1\) and a \(^{14}\)N nuclear spin \(I = 1\). As shown in Fig. 6, the NV\^-\) center in diamond is constituted of a nitrogen atom substituting a carbon and a vacancy substituting the carbon adjacent to the nitrogen. The N-V axis determines the principal quantum axis. The zero-field splitting (ZFS) of the triplet ground state is approximately 2.87 GHz, so microwaves are used to manipulate the electron spin population.\(^\text{105–107}\) The phonon sideband (PSB) makes it easy to initialize the electron spin with a 532 nm laser and to detect the electron spin population by monitoring the 600 nm–800 nm fluorescence signal.\(^\text{108}\) The Hamiltonian of the NV\^-\) center electron spin and the \(^{14}\)N nuclear spin is given by

\[
\mathcal{H} = DS^2 + \gamma_e B \cdot S + QI^2 + \gamma_n B \cdot I + AS \cdot I, \tag{15}
\]

where \(D\) is the ZFS line, \(Q = -4.95\) MHz is the intrinsic quadrupole interaction, \(\gamma_e = 2\) kHz/G, and \(A \approx 2.2\) MHz is the hyperfine constant.\(^\text{109}\) The \(^{14}\)N nuclear spin ensembles can be efficiently polarized through excited state level anti-crossing (ESLAC) or a dynamic nuclear polarizing scheme. The state can be read by mapping the population onto the electron spin.\(^\text{110–114}\) Therefore, it is possible to measure geometric phase with both the electron spin and the nuclear spin of the NV\^-\) center.

In 2012, Maclaurin et al. proposed using a single NV\^-\) center in a spinning diamond to measure the geometric phase accumulated by the electron spin.\(^\text{45}\) Ramsey geometry and spin echo geometry were suggested for the geometric phase measurement, and the experiments were demonstrated in 2014.\(^\text{115}\) New paths for the accumulation of the non-Abelian geometric phase were later designed to improve the
measurement sensitivity. \(^{59}\) Although a single NV\(^-\) center in diamond has a long coherence time, \(^{116}\) the measurement sensitivity is limited by the single photon signal. It is also difficult to integrate a single photon counting system into a compact diamond gyroscope. Therefore, spin ensembles in diamond have been used in place of the single NV\(^-\) center in diamond. According to the fluorescence signal, the shot noise sensitivity limit of the rotation measurement is

\[
\delta \omega = \frac{1}{R\eta N T_c t},
\]

where \(R\) is the signal contrast, \(\eta\) is the fluorescence collection efficiency, \(N\) is the number of the sensing spins, \(T_c\) is the coherence time, and \(t\) is the measurement time. \(^{20}\) Nuclear spin has a much longer coherence time than the electron spin so the fundamental sensitivity can be significantly improved by applying the \(^{14}\)N nuclear spin. Meanwhile, increasing the NV\(^-\) center density also improves the sensitivity. Taking these into consideration, Ledbetter et al. proposed the idea of developing a gyroscope based on nuclear spin ensembles in diamond. \(^{46}\) At the same time, Cappellaro proposed the use of nuclear spin ensembles in diamond to realize a “three-axis diamond gyroscope.” \(^{47}\) The “three-axis diamond gyroscope” includes a pair of rotating RF coils, on-chip coils, and MW waveguides. The geometric phase is accumulated from the relative rotation between the RF field and the diamond chip. The current projected sensitivity is \(5 \times 10^{-4} \text{ s}^{-1/2} \text{ Hz mm}^3\) and the ARW is \(0.03 \text{ s}^{-1/2}\), which makes the diamond gyroscope a high-performance, rate-grade gyroscope.

To demonstrate the nuclear spin diamond gyroscope principle, we present the Ramsey sequence to calculate the geometric phase accumulation during rotation. After initialization, the electron spin state and the nuclear spin state, \(|m_s, m_I\rangle\), are polarized to \(|0, 0\rangle\). RF pulses that resonant with the nuclear quadrupole, \(Q\), are applied to manipulate the nuclear spin state. After a \(\pi\)-pulse, the state of the system turns to

\[
|\psi\rangle = \frac{1}{\sqrt{2}} (|0, 1\rangle + |0, -1\rangle).
\]

Then, since \(m_s = 0\) only the nuclear spin terms are left in the Hamiltonian

\[
\mathcal{H}_{\text{nuc}} = Q I_z^2 + \gamma_B \mathbf{B} \cdot \mathbf{I}.
\]

Fig. 6(b) shows the rotation measurement process, where the external field rotates around the N-V axis. The instantaneous Hamiltonian is described in the static frame as

\[
\mathcal{H} = \mathcal{H}_0 + \mathcal{H}' = Q I_z^2 + \gamma_B |I_z| + \gamma \times \begin{pmatrix} 0 & \sqrt{2} B_\perp e^{-i\omega t} & 0 \\ \sqrt{2} B_\perp e^{i\omega t} & 0 & \sqrt{2} B_\perp e^{-i\omega t} \\ 0 & \sqrt{2} B_\perp e^{i\omega t} & 0 \end{pmatrix},
\]

where \(B_\parallel\) is the magnetic field parallel to the N-V axis, \(B_\perp\) is the transversal field, and \(\mathcal{H}_0 = Q I_z^2 + \gamma B_\perp |I_z|\). By applying the perturbation theory, the instantaneous eigenvalues with the second order approximation are given as \(E_+ + \Delta_+\), \(E_- + \Delta_-\), \(-\Delta_+ - \Delta_-\), where \(E_+ = Q + \gamma B_\parallel\), \(E_- = Q - \gamma B_\parallel\), \(\Delta_+ = \gamma^2 B_\perp^2 / 2 E_+\), and \(\Delta_- = \gamma^2 B_\perp^2 / 2 E_-\). The corresponding instantaneous eigenstates (without normalization) are

\[
|\psi^{+1}\rangle = \begin{pmatrix} 1 \\ \Delta_+ \sqrt{E_+} e^{2i\omega t} \\ 2\gamma B_\parallel E_+ e^{2i\omega t} \end{pmatrix}, \quad |\psi^0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |\psi^{-1}\rangle = \begin{pmatrix} -\Delta_- \sqrt{E_-} e^{-2i\omega t} \\ \Delta_- \sqrt{E_-} e^{-2i\omega t} \\ 0 \end{pmatrix}.
\]

FIG. 6. (a) Energy level diagram of the NV\(^-\) center spins in diamond. With the phonon sideband (PSB) and singlet states, the electron spin can be initialized to the \(m_s = 0\) ground triplet state using a 532 nm laser. The electron spin state population is detected by monitoring the 600-800 nm fluorescence signal. The zero phonon line (ZPL) is at 637 nm and the zero-field splitting (ZFS) of the triplet ground state is at 2.87 GHz. The inset shows the crystal structure of the NV\(^-\) center in diamond. Two adjacent carbon atoms (blue spheres) are substituted by a nitrogen atom (red sphere) and a vacancy (white sphere). (b) Principle of rotation sensing with nuclear spin of the NV\(^-\) center in diamond. The \(^{14}\)N nuclear spin is polarized along the N-V axis. The external field rotates together with the diamond, and the rotating transverse magnetic field induces Berry phase accumulation.
Under a weak external field $\gamma B_1 \ll Q$. Thus, the accumulated geometric phase can be calculated according to the Berry phase theory

$$\Phi^n = \int_0^t (\psi^n | \dot{\psi}^n) dt' = -m \int_0^t \alpha dt'.$$

Therefore, after free evolution time, the state of the system is

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( e^{-i(\phi_x+\omega t)} |0, 1\rangle + e^{i(\phi_x-\omega t)} |0, -1\rangle \right),$$

where $\phi_x$ is the dynamic phase dependent on the external magnetic field. By applying another $\pi$-pulse, the population of state $|0, 0\rangle$ becomes $\cos (\phi_x + \omega t)$. Through the nuclear spin and electron spin interaction, the nuclear spin state population is transferred to the electron spin state so that the rotation rate can be measured by collecting the fluorescence.

With the application of a high density NV$^-$ center ensemble in diamond, highly sensitive rotation measurements can be carried out using a micron-size diamond granule. PSB excitation allows the use of the green light emitting diode (LED) to initialize electron spins. Integrated RF-microwave circuits can be used to manipulate the spins, and all operations are performed at room temperature. By adopting the MEMS techniques, the diamond gyroscope has the potential of being the smallest navigation-grade gyroscope.

However, there are still several challenges for the diamond gyroscope to face. The first challenge is the low polarization efficiency of the $^{14}$N nuclear spin ensembles in a weak field. Although nuclear spin ensembles polarization is highly efficient using the ESLAC method (at about 500 G), it is difficult to integrate such a strong magnetic field into the diamond gyroscope. In addition, such a high external field also limits the sensitivity. Dynamic nuclear polarization is used to initialize the nuclear spins at a few micro-Tesla, so the efficiency is rather low. Another challenge is the conflict between the strength and the uniformity of the manipulation field. Near-field microwaves lack uniformity, which leads to linewidth broadening during spin manipulation. Far-field microwaves are more uniform and homogenous, but the field is usually too weak to flip the spins within the coherence time. Therefore, new techniques for generating strong, uniform manipulation fields are under development.

The principle verification experiment for the diamond gyroscope is ongoing. We propose a device for the principle verification experiment of the diamond gyroscope, as shown in Fig. 7. Currently, a $3\,\text{nm} \times 3\,\text{nm} \times 0.3\,\text{nm}$ high pressure high temperature (HPHT) diamond plate (Element Six Company) is used. The sample is irradiated with $1 \times 10^{18}$ e/cm$^2$ electrons and is annealed at $800^\circ\text{C}$ for 1 h. The final NV$^-$ density is $1.78 \times 10^{17}$ cm$^{-3}$, according to ESR spectrum estimation. In order to improve manipulation field homogeneity, the microwave waveguide and the RF coils are fabricated on both sides of the diamond plate. A 1 W green LED (Longji, 520–525 nm) is used to initialize the electron spins of the NV$^-$ center. Control pulse overshoot can reshape the slow slope of the light pulses, so the control pulses must be precise and stable to maintain power stability. Avalanche photo diodes (APDs) are used for the fluorescence detection. In the proposed setup, only the NV$^-$ centers in the center part of the diamond can be excited and emit fluorescence through irises on both sides of the diamond plate. By adjusting the diameter of the irises and the power of the green light, we ensure that the APD is not saturated. Currently, the fluorescence detection contrast ratio is rather low ($R \approx 0.03$), and the collection efficiency $\eta$ is lower than 0.05 (depending on the iris on top of the diamond). In order to estimate the ARW of the device, we take the active volume of the diamond as 0.03 mm$^3$ (with 1 mm$^2$ irises). The dephasing time of the nuclear spin ensembles can be as long as 1 ms. Thus, the ARW is about $3.6/\sqrt{t}$. By optimizing the microwave waveguide and the RF coils, we can enlarge the sensing volume of the diamond. The use of an optical cavity and edge-cut diamond can improve the collection efficiency to over 90% by collecting the fluorescence emitted from the sides and edges of the diamond. With these techniques, the gyroscope ARW is projected to be better than 0.1°/\sqrt{t}.

IV. CONSIDERATIONS FOR ATOMIC SPIN GYROSCOPES FROM THE VIEWPOINT OF QUANTUM PHASE THEORY

The semi-classical theory regards spin ensembles as a group of magnetic needles maintaining a constant precession frequency in the inertial frame. Although the precession theory is simple and practical, many details of the spin evolution are not considered in the description and errors might be ignored in the rotation measurement process. Meanwhile, the geometric phase theory studies spin state evolution in a...
rotating frame by solving the time dependent Schrödinger equation, so that the interactions between spin and rotation are fully considered. As is well-known, \( e^{i\mathbf{k}\cdot\mathbf{r}} \) is typically used in mathematics to represent a coordinate system rotating at an angle, \( \varphi \), around an axis, \( \mathbf{k} \); this expression has the same form as a wave function phase factor. Therefore, it can be inferred that geometric phase detection is the essence of inertial rotation measurement with atomic spins. In Section II, angular momentum conservation and spin precession models are used to explain the Barnett effect and the two atomic spin gyroscopes based on a comagnetometer. In this section, principles of different atomic spin gyroscopes will be reconsidered within a unified quantum phase picture.

In the Barnett field experiments and the NMRG implementation, FID is used to detect the NMR frequency shift. In both experiments, the rotation axis is along the direction of the external field and the Berry phase is not accumulated. However, when a resonant RF field is applied in the transverse plane, the initial state is not a static state so that the AA phase is accumulated in the spin free evolution time. A two-level spin system is considered as follows. When an external magnetic field is applied to split the Zeeman state into two-level spin system is considered as follows. When an external magnetic field is applied to split the Zeeman state.

The dynamic phase is
\[
\alpha(t) = -\int_0^t \left\langle \hat{H} \right\rangle \left\langle \psi(t') | \psi(t') \right\rangle dt' = \cos \theta \frac{\omega_L t}{2}.
\]

In the FID measurement scheme, the rotation angle, \( \theta \), depends on the RF field pulse width. In the NMRG continuous measurement scheme, \( \theta \) depends on the amplitude of the RF field. During the free evolution time, a phase difference accumulates between the two eigenstates in the spin state wave function. Therefore, at time \( t \), the spin state can be written as
\[
|\psi(t)\rangle = \cos \frac{\theta}{2} e^{-i\omega_L t/2} |\uparrow\rangle + \sin \frac{\theta}{2} e^{-i\omega_L t/2} |\downarrow\rangle,
\]

where \( \omega_L \) is the Larmor frequency. With the interaction of the transverse RF field, the initial state is
\[
|\psi(0)\rangle = \cos \frac{\theta}{2} |\uparrow\rangle + \sin \frac{\theta}{2} |\downarrow\rangle.
\]

The dynamic phase is
\[
\alpha(t) = -\int_0^t \left\langle \hat{H} \right\rangle \left\langle \psi(t') | \psi(t') \right\rangle dt' = \cos \theta \frac{\omega_L t}{2}.
\]

The detected NMR frequency is \( \omega_L \), and the AA phase of the system is
\[
\gamma(t) = |\psi(t)\rangle \langle \psi(t)| = \cos \theta \frac{\omega_L t}{2}.
\]

From Eqs. (27) and (30), the inertial rotation measurement sensitivity can be considered.

1. When \( \omega_L \approx \omega \), \( \theta \neq 0 \), \( \pi/2 \), a static magnetic moment projection can be detected in the transverse plane, and \( \gamma(t) \) grows very quickly. Therefore, the measurement point is near zero and the NMRG sensitivity is improved.

2. When \( \theta = \pi/2 \), \( \omega_L \gg \omega \), the geometric phase is the max and \( \gamma = \pi \), as well as the transverse plane magnetic moment projection as shown in Eq. (27). The sensitivity is higher than the situation where \( \theta = \pi/2 \).

Therefore, the NMRG measurement scheme can be modified to improve the sensitivity.

A pulsed FID measurement scheme may have a better sensitivity than the continuous measurement scheme according to discussion 2. The spin moment is flipped into the transverse plane with a \( \pi/2 \) RF pulse to ensure \( \theta = \pi/2 \). However, the FID scheme suffers from low measurement frequency due to a long nuclear polarization time. To account for this, the measurement time is shortened and the nuclear spin ensembles are repolarized for a while following the \( -\pi/2 \) RF pulse. Before the \( -\pi/2 \) RF pulse is applied, an echo pulse is used to refocus the spins to the initial phase. The pulse sequence is shown in Fig. 8(c). As the spin momentum evolves in the transverse plane, the geometric phase factor accumulation is doubled and the NMR signal amplitude is improved. The spin echo pulse also extends the coherence time from \( T_2^* \) to \( T_2 \). As a result, the noise can be suppressed by extending the measurement time for integration.

When \( \theta = 0 \) and \( \omega_L = 0 \), the condition is identical to that of a SERF gyroscope; the electron spins evolve in a zero period \( \tau \), the observed phase factor must be 0 to satisfy the cyclic evolution condition. In a NMR experiment, the \( x \)-axis is determined by the direction and the initial phase of the transverse RF field.

In a NMR experiment, only the observer rotates around the precession axis. A continuous RF field is applied and the \( x \)-axis rotates in the transverse plane with an angular velocity, \( \omega \). After a cycle time, \( \tau \), the spin state phase evolution is shown in Fig. 8(a), where
\[
\phi = \omega \tau; \quad \omega_L \tau = 2\pi + \phi|_{\omega},
\]
\[
\tau = \frac{2\pi}{\omega_L - \omega}.
\]
proposed to improve the inertial rotation measurement number. Based on the discussion, new schemes have been frame and the solid angle depends on the spin quantum accumulated when the spin system rotates in the inertial no external field rotating around the spin, the AA phase is of the atomic spin inertial rotation measurement signal. With the geometric phase theory. The quantum phase is the origin inertial rotation measurement methods can be realized using back signal for the stabilized platform in the inertial frame.

In the SERF gyroscope measurement scheme, \( I_x \) is the detected quantity and the signal represents the rotation angle, \( \phi \), between the spin momentum and the reference frame. In the continuous pumping scheme, \( I_x \) follows the carrier frame and \( \phi \) is read as the rotating angular velocity. In the pulsed pumping scheme described in Section II, \( \phi \) is read as the rotating angle in the inertial frame and it is used as the feedback signal for the stabilized platform in the inertial frame.

In conclusion, the above discussions show that different inertial rotation measurement methods can be realized using the geometric phase theory. The quantum phase is the origin of the atomic spin inertial rotation measurement signal. With no external field rotating around the spin, the AA phase is accumulated when the spin system rotates in the inertial frame and the solid angle depends on the spin quantum number. Based on the discussion, new schemes have been proposed to improve the inertial rotation measurement sensitivity. Generally, by exploring the spectral shifts or spin precession induced by the “non-inertial motion field” within the context of the geometric phase theory, the performance of the atomic spin inertial sensor will be greatly improved.

V. CONCLUSION AND OUTLOOK

In this review, various atomic spin gyroscopes and their principles were presented. Both semi-classical theories and quantum phase theories and their applications to inertial rotation measurement were reviewed. From the viewpoint of the geometric phase theory, the microscopic mechanisms of various inertial rotation measurement experiments can be understood in a unified frame. As a result, more error sources in current NMRG and SERF gyroscopes can be traced; dynamic decoupling techniques commonly used in solid-state spin magnetic resonance experiments can be used to develop more complex measurement schemes for cell NMRGs to improve measurement sensitivity. New spin evolution paths will be designed to further improve the geometric phase signal so that both the drift and the sensitivity of current atomic spin gyroscopes will be improved. On the other hand, it is expected that different spin systems may be optimized for inertial rotation sensing with geometric phase accumulation. For example, many other types of color centers (e.g., \( V_{\text{Al}} \) in AlN) are similar to the NV\(^-\) center in diamond \(^{120}\) and efforts are being made to find more suitable spin systems in solid-state materials for rotation sensing. \(^{121–123}\) Atomic spin gyroscopes have demonstrated potentials in both strategic-grade INS and chip-scale navigation-grade INS applications. Though there are many challenges to overcome before wide application, a complete

![Diagram of SERF gyroscope measurement scheme](image1)

![Diagram of NMRG zero-control system](image2)
understanding of inertial rotation measurements with atomic spin and new techniques will undoubtedly lead to a cheap, robust atomic spin gyroscope in a small package with high performance in the near future.

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