An adaptive backpropagation neural algorithm for limited-angle CT image reconstruction

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ABSTRACT

The proposed system for CT image reconstruction is structured with three layers of neurons. In our previous work, we used the resilient backpropagation (Rprop) instead of the straight BP to modify the network weights. The basic idea is to minimize the error between the projections of the original image and of the reconstructed image. We noticed that the system performance depends on the initial status of the network. Based on this observation, we propose a novel approach for choosing optimal values of the connection weights. The experimental results indicate that the new method can find a satisfactory solution despite that only a few projections are available.

Keywords: Image reconstruction, Neural Networks, Resilient Backpropagation

1. INTRODUCTION

In computed tomography (CT) applications, some projection data of an unknown multi-dimensional distribution (target image) are assumed to be given. Several techniques are known for reconstructing the original image by manipulating the projection data (Figure 1). When the number of projection directions is limited to three or four due to an experimental environment, Genetic Algorithms (GAs), Algebraic Reconstruction Technique (ART), Simulated Annealing (SA), and Back-Propagation (BP) algorithm have been used for CT image reconstruction to date.

Here, a neural network model is applied to gray CT image reconstruction from only four projection directions, and is based on the previously proposed backpropagation algorithm based method. The previous and newly proposed models use the resilient backpropagation (Rprop) to modify the weights instead of the straight BP. The Rprop method, introduced by M. Riedmiller in 1992, which is derived from BP, can find global minima faster than BP and effectively avoid local minima. It is demonstrated that the precision of the reconstructed images varies depending on the initial values of the connection weights of the system, and that optimal choices of the initial weights can be made.

2. THE IMAGE RECONSTRUCTION SYSTEM

2.1. Structure of the Network

The system for the reconstruction of the images which we use is structured with three layers of neurons, as is depicted in Figure 2. The number of the neurons of the first layer is equal to the projection data which we get from an original image. The next layer is the hidden layer and has a same number of the neurons as the pixels of the image. The number of neurons of the output layer is the same as that of the input layer. The output from the output layer gives the projection data from a reconstructed image.

The connection weights between the first and the second layer are denoted by $w_{ji}$, which is adapted through the Rprop learning process. The weights between the second and the third layer are denoted by $w_{kj}$, which are kept constant through the learning process. The weights $w_{kj}$ are determined by the method explained in section 2.2, and the projection data can be computed from the second layer, and entered to the third layer.

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Projection data obtained from different directions

Use only these projection data

Reconstruction from projection data

Figure 1: Reconstruction of an Image from projection

This reconstruction system is based on a simple idea that if an error between the projections of the original image and of the reconstructed image is small, the reconstructed image will be similar to the original image. The input to the system is the projection data of the original unknown image as an input to the first layer.

2.2. Weight Initialization

The weights \( w_{kj} \), which connect the second and the third layer, are used for computing the projection data of the reconstructed image as input to the output layer. The method for initialization of the weights is explained with Figure 3. Suppose that 25(5×5) squares give one image, each square representing one pixel. The image

Figure 2: The Structure of the Network
on the left is before a rotation and the one on the right is after the rotation. For 0 degree projection, the coordinates (4.0, 2.0) of the pixel marked do not change and remain on the same point at 0 degree, and this pixel will project the entire area (1.0) to the 4th box under the image and this value is taken as a weight for the 0 degree projection.

After the 45 degree rotation, the pixel (4.0, 2.0) moves to the coordinates (4.4, 3.0). This pixel is projected into two separate boxes with area ratio 0.6 and 0.4. That is to say, the pixel on the coordinate (4.0,2.0) is related to the pixel on \( R(45^\circ, 4), R(45^\circ, 5) \), these representing the 4th and the 5th projection data for 45 degree projection direction, respectively, with weights 0.6 and 0.4. Likewise, weights of all the pixels are determined and computed for all directions.

### 2.3. The Learning Method of the Network

The network is to renew the weights with Rprop. Suppose that \( y_i \) is the output of the \( i \)-th unit of the input layer and \( w_{ji} \) is the connection weight between nodes \( i \) and \( j \), then the output \( y_j \) of the \( j \)-th unit in the hidden layer is given by

\[
y_j = \frac{1}{1 + e^{x_j}}, \quad x_j = \sum_i y_i w_{ji}
\]  

(1)

The output \( y_k \) of the \( k \)-th unit in the output layer becomes

\[
y_k = \sum_j y_j m_{kj}
\]  

(2)

and, the corresponding error function \( E_k \) is given by

\[
E_k = \frac{1}{2}(y_k - d_k)^2
\]  

(3)

and, the total error becomes

\[
E = \sum_k E_k
\]  

(4)

![Figure 3: The Initialization of the Weights](http://proceedings.spiedigitallibrary.org/)
Using the total error $E$, the weights $w_{ji}$ are renewed with the Rprop algorithm:

$$\frac{\partial E}{\partial w_{ji}} = \sum_k [(y_k - d_k) w_{kj}] y_j (1 - y_j) y_i$$  \hfill (5)

$\forall i, j : \Delta_{ji}(t) = \Delta_0$

$\forall i, j : \frac{\partial E}{\partial w_{ji}}(t - 1) = 0$

Repeat

Compute Gradient $\frac{\partial E}{\partial w_{ji}}(t)$

For all weights and biases:

if $(\frac{\partial E}{\partial w_{ji}}(t - 1) * \frac{\partial E}{\partial w_{ji}}(t) > 0)$ then {

$\Delta_{ji}(t) = \text{minimum}(\Delta_{ji}(t - 1) * \eta^+, \Delta_{\text{max}})$

$\Delta w_{ji}(t) = - \text{sign} \left( \frac{\partial E}{\partial w_{ji}}(t) \right) * \Delta_{ji}(t)$

$w_{ji}(t + 1) = w_{ji}(t) + \Delta w_{ji}(t)$

$\frac{\partial E}{\partial w_{ji}}(t - 1) = \frac{\partial E}{\partial w_{ji}}(t)$

}$

else if $(\frac{\partial E}{\partial w_{ji}}(t - 1) * \frac{\partial E}{\partial w_{ji}}(t) < 0)$ then {

$\Delta_{ji}(t) = \text{maximum}(\Delta_{ji}(t - 1) * \eta^-, \Delta_{\text{min}})$

$\frac{\partial E}{\partial w_{ji}}(t - 1) = 0$

}$

else if $(\frac{\partial E}{\partial w_{ji}}(t - 1) * \frac{\partial E}{\partial w_{ji}}(t) = 0)$ then {

$\Delta w_{ji}(t) = - \text{sign} \left( \frac{\partial E}{\partial w_{ji}}(t) \right) * \Delta_{ji}(t)$

$w_{ji}(t + 1) = w_{ji}(t) + \Delta w_{ji}(t)$

$\frac{\partial E}{\partial w_{ji}}(t - 1) = \frac{\partial E}{\partial w_{ji}}(t)$

}$

Until (converged)

2.4. Examination of the parameters

A problem of the existing systems is that the accuracy of gray scale image reconstruction is not as good as that of binary image reconstruction. We improved the system for gray scale image reconstruction by introducing a method to determine the initial values of connection weights.

In our proposed method, the initial value of weight $W_{ji}$ is determined by using the weights $M$ as in equation 6.

$$W_{ji(\text{init})} = M_{kj} * M2W_{\text{Bias}},$$  \hfill (6)

where $M2W_{\text{Bias}}$ is a magnification parameter. We set the optimal $M2W_{\text{Bias}}$ empirically. For this, the system makes about 100 trials by varying the parameter values. An example of $M2W_{\text{Bias}}$ and $\text{ProjectionError(PE)}$, $\text{ImageError(IE)}$ is shown in Figure 4. The system with $M2W_{\text{Bias}}$ has high accuracy and can do without median filters (Figure 5), however, the search for optimal parameters takes much time.

We set the initial values of the weight $W_{ji}$ by equation 7.

$$W_{ji(\text{init})} = \frac{1}{P_i},$$  \hfill (7)

when $P_i$ is the number of the pixels that are projected to the $i$-th element of the projection. The role of equation 7 is like normalization and the purpose is to smoothen the initial reconstruction image.

The effectiveness of the determined parameters is demonstrated by experiments in the next section.
3. EXPERIMENTS AND THE RESULTS

We show several reconstruction results. The number of the projections is four at angles 0°, 45°, 90°, and 135°.

Figures 5 and 6, 7 show reconstruction results, and the simulation was done with the parameter values in Table 1. Some results by the previous method are shown in Figure 5. Comparison of results by proposed method and the previous method is depicted in Figures 6, 7, 8, 9.

We find that the proposed system reconstructs the images satisfactorily despite the very small number of projection directions.

Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0$</td>
<td>$5.0e^{-3}$</td>
</tr>
<tr>
<td>$[\Delta_{\text{min}}, \Delta_{\text{max}}]$</td>
<td>$[0.5, 5.0e^{-3}]$</td>
</tr>
<tr>
<td>$[\eta^-, \eta^+]$</td>
<td>$[0.5, 1.2]$</td>
</tr>
</tbody>
</table>

Figure 4: $M2W_{bias}$ and Errors

Figure 5: Results of the previous method(Gauss:32x32, A:32x32)

4. CONCLUSIONS

Based on the previous work where we introduced a magnification parameter to improve the reconstruction accuracy, we propose a novel approach for choosing the optimal initial connection weights of the Rprop network.
to speed up the reconstruction processing. The experimental results indicate that the new method can find a satisfactory solution despite that only a few number of projections are available.

REFERENCES
Figure 6: Results (Gauss: 128x128)
Figure 7: Results (A: 128x128)
Figure 8: Results (Okinawa: 128x128)
Figure 9: Results (Square: 128x128)
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