Performance of a Ring Laser Strapdown Marine Gyrocompass

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Received June 9, 1981

ABSTRACT

This paper describes an accuracy performance analysis of a ring laser (RL) strapdown marine gyrocompass. Theoretically derived relationships and covariance analysis results are presented to indicate the effect of ring laser errors on heading accuracy and on settling time. Test results obtained with a pre-production model of the RL strapdown gyrocompass are also shown, and they illustrate the capability of this system to meet U.S. Navy Type I at-sea performance requirements.

INTRODUCTION

The modern version of the marine gyrocompass provides both heading and attitude information for use in ship navigation and in radar/sonar/weapon systems control. And these “stabilized” gyrocompasses are more compact and accurate than their predecessors. Evolutionary improvements in the size, weight, and power requirements of these systems reflect the significant state-of-the-art advances in electronics, components, and materials of the last two decades. But these current systems also share a major disadvantage of previous designs, viz. the cost/reliability/maintainability constraints associated with the use of a gim-balled platform.

In many aircraft, missile, and marine applications, the limitations of electro-mechanical gimbaled systems are now being bypassed by use of strapdown configurations—so called because the acceleration and rate sensors are mounted directly (strapped down) to the vehicle. Implementation of a strapdown system requires inertial sensors with a wide dynamic range, and a high speed computational capability. These requirements derive from the need to measure rates and accelerations in a vehicle frame, and to transform the data into an earth-fixed frame. The transformation operation requires a high speed update of the direction cosine matrix relating the two frames.

Computational requirements for strapdown systems are well within the capabilities of the state-of-the-art digital computer, and currently available rate/acceleration sensors can provide the necessary accuracy and dynamic range for a wide spectrum of applications. Conventional (spinning wheel) gyros have been used in strapdown systems; but the ring laser, an unconventional gyro, is especially compatible with the strapdown environment. Briefly, the ring laser gyro is an optical oscillator in which the oscillation frequencies are related, in a known way, to the input angular rate. The ring laser consists of a closed, ring-type optical cavity and a gain medium. Lasing action produces two independent, contra-
tating, traveling waves which traverse the same path. The oscillation frequency of each wave is dependent on the optical length of the path. Ideally, in the absence of rotation, the clockwise and counterclockwise waves have the same oscillation frequency. Rotation of the ring about an axis normal to the lasing plane increases the path length in the direction of rotation and decreases the path length in the opposite direction. But the conditions for lasing require that the length of the cavity equal an integral number of wavelengths at any instant of time, i.e., the returning wave must be in phase with the initiating energy (and that the round trip gain exceed the losses). Therefore, in order to maintain lasing in the presence of the path length changes induced by rotation rates, the cavity must select the appropriate wavelengths for the cw/ccw waves as required to provide the “in-phase” condition. Because the velocity of light is constant, a change in optical frequency is associated with the change in wavelength \( \nu = c/\lambda \), where \( \nu \) is frequency, \( c \) is the velocity of light and \( \lambda \) is wavelength. Measurement of the difference (beat) frequency between the two waves provides a precise indication of the input angular rotation rate.

In general, the ring laser acts as an integrating rate gyro; it has the capability for accurately measuring angular rates over a wide dynamic range, and inherently provides a direct digital output. Its operation does not depend on mass and momentum, but rather on the relativistic properties of light. In contrast to spinning wheel gyros, ring laser gyro drift is insensitive to vehicle acceleration, its scale factor characteristic is intrinsically symmetric, and the gyro responds only to rotations about an axis normal to the lasing plane. The absence of moving parts implies high reliability and low cost. Because of these features, ring laser strapdown systems have been developed and evaluated in a variety of applications.\(^3, 4, 5, 10\)

This paper provides analytical and system evaluation test results pertinent to the accuracy performance characteristics of a ring laser strapdown marine (stabilized) gyrocompass. The specific system which is considered includes the use of an indexer assembly to periodically rotate the inertial sensor cluster (3 gyros/3 accelerometers) between four orthogonal dwell positions in the deck plane. This configuration promotes accurate calibration/averaging of equivalent east axis gyro and accelerometer bias drift; and this leads to an associated significant improvement in heading accuracy performance.

Among the key results presented for the RL strapdown gyrocompass are the following:

- Ring laser white noise drift results in a steady state rms heading error. This is caused by the required trade-off between the influence of vertical (Z) axis and east axis white noise drift. At 45 degrees latitude, a white noise drift of 0.01 deg/\( \sqrt{\text{hr}} \) on each gyro axis yields a steady-state rms heading error of about 1.5 arc-min; this assumes use of optimal processing of velocity reference data.
- This rms heading error varies as the square root of the secant (latitude).
- The time needed to settle within a given heading accuracy is dependent on both Z axis gyro turn-on bias drift and white noise random drift. The latter error source limits the rate at which accurate estimates of Z-axis gyro bias drift can be developed.
- Gyro scale factor error of 100 PPM is compatible with the heading accuracy obtained with a gyro white noise level of 0.01 deg/\( \sqrt{\text{hr}} \).
Sperry Gyroscope has fabricated/tested a pre-production model of an RL strapdown Marine Gyrocompass that satisfies U.S. Navy Type I at-sea requirements. This was primarily accomplished by adding an indexer assembly to the previous Sperry-developed laser gyro strapdown MK 16 MOD 11 Stable Element; the MK 16 MOD 11 is currently operational and in production for the U.S. Navy. In this system a fourteen-state Kalman Filter provides optimal processing of available velocity reference data. This implementation of an RL strapdown Marine Gyrocompass is briefly described and sample performance data is presented. The test results generally confirm the analytical performance predictions.

Because of the use of ring lasers and the elimination of gimbals and gimbal servos, the RL strapdown marine gyrocompass achieves a significantly higher reliability and lower life-cycle cost than conventional gyrocompasses. With very little modification—primarily in the software—the gyrocompass can be converted to a marine navigator. Development of a RL marine inertial navigator is now in process at Sperry.

PRINCIPLES OF THE GYROCOMPASS

The principles of gyrocompass operation are discussed in a number of papers and texts. The concepts are of course equally applicable to the synthesis/analysis of strapdown and gimballed systems; these concepts are briefly reviewed below.

Designate the vehicle-fixed cartesian coordinate frame \( (n, e, k) \) with the symbol \( n \); this frame has its origin at the vehicle location and its axes are directed north, east, and along the local vertical. Define a vector, \( \Omega \), directed northward along the earth's axis of rotation with norm, \( \| \Omega \| \), equal to the earth's inertial rotation rate; it is normal to the equatorial plane. Further, define a vector, \( g \), directed down along the vertical with norm, \( \| g \| \), equal to the gravity force (earth's gravitational acceleration minus earth's centripetal acceleration). Let the normalized earth rate and gravity vectors be designated by the unit (polar axis) vector, \( p \), and the unit (local vertical) vector, \( k \), i.e.,

\[
\begin{align*}
p &= \frac{\Omega}{\| \Omega \|} \\
k &= \frac{g}{\| g \|}
\end{align*}
\]

When the polar axis and local vertical vectors are non-collinear, they define the "meridian" plane. Local latitude, \( L \), is the angle between the local vertical vector and the equatorial plane. The unit vector, \( e \), which defines "east", is normal to the meridian plane (and pointed in the direction of earth's rotation).

Let \( x^i \) designate the vector, \( x \), coordinatized in the basis of the \( i \) frame. Then the following mathematical expressions formalize the above statements:

\[
\begin{align*}
p &= \{ \cos L, 0, -\sin L \} \\
k &= \{ 0, 0, 1 \} \\
\sin L &= -(k \cdot p); \text{ scalar product} \\
e \cos L &= (k \times p); \text{ vector product}
\end{align*}
\]

A. Fixed Base: For illustrative purposes, consider an arbitrarily oriented triad of orthogonal gyro's and orthogonal accelerometers mounted on a stationary base. These sensors provide a measure of the earth rate vector and of the
gravity vector, respectively. Designate the vectors computed from the measurements as \( \Omega_c \) and \( g_e \), and the corresponding normalized vectors as \( p_c \) and \( k_c \). Suppose that an east vector, \( e_e \), is computed using the cross-product operation, i.e.,

\[
e_e \cos L_e = k_c \times p_c
\]  
(5)

This is equivalent to the basic operation performed in a gyrocompass implementation. Because of measurement errors, the computed east vector will not, in general, be aligned with the direction of true east. The heading error, \( \delta \psi \), is defined as the component of the misalignment about the \( k \) axis. Therefore, using small angle assumptions

\[
\delta \psi = k \cdot (e_e \times e)
\]  
(6)

Substituting from equations (4) and (5) results in

\[
\delta \psi = k \cdot \left( \frac{k_c \times p_c}{\cos L_c} \right) \times \left( \frac{k \times p}{\cos L} \right)
\]  
(7)

Using vector algebra

\[
\delta \psi \cos L \cos L_c = \text{DET}(k_c^a, p_c^a, p^a) + \text{DET}(k_c^a, p_c^a, k^a) \sin L
\]  
(8)

The notation \( \text{DET}(X^i_1, X^i_2, X^i_3) \) designates the determinant of a \( 3 \times 3 \) matrix; the elements of each row of the matrix are the components in a common basis (the \( i \) frame) of the \( X_1, X_2, X_3 \) vectors. Of course, if two or more rows of these matrices are equal, the value of the determinant is zero. Certain fundamental error characteristics of the gyrocompass are implicit in equation (8); and these apply regardless of the implementation. This is illustrated further by the following special cases.

(1) Assume that there are no errors in the measurement of the normalized gravity vector \( (k_c = k) \), but that the computed earth rate vector is in error because of gyro drift. Then

\[
\Omega_c = \Omega + \xi_G
\]  
(9a)

where \( \xi_G \) is the gyro drift vector

This can be expressed in geographic coordinates as

\[
\Omega^a = \{ (\Omega \cos L + \xi_{Ga}), \xi_{Ca}, \}
\]

\[
(-\Omega \sin L + \xi_{Gk})
\]  
(9b)

and \( (\xi_{Ga}, \xi_{Ge}, \xi_{Gk}) \) represent the equivalent gyro drifts in the \( (n, e, k) \) axes after transformation from gyro axes. Substituting in the above heading error equation results in:

\[
\delta \psi = -\xi_{Ge} / \| \Omega_c \| \cos L_c
\]  
(10)

This demonstrates a well-known performance characteristic of a gyrocompass, \textit{viz.} heading accuracy is limited by the effective gyro drift in the east axis. At a latitude of 48.3° (\( \Omega \cos L = 10^2/\text{hr} \)), an east gyro bias drift of 0.01°/hr results in a heading offset error of 1 mrad. On a fixed base gyrocompass, any unknown rate about the east axis, e.g., due to settling, is equivalent to an east gyro drift.
(2) Assume that there are no errors in the measurement of the normalized earth rate vector \( \mathbf{p}_e = \mathbf{p} \), but that accelerometer errors cause the computed gravity vector to be in error. Then

\[
\mathbf{g}_c = \mathbf{g} + \xi_A
\]

\[
\mathbf{g}_c' = \{\xi_{An}, \xi_{An}, \xi_{Ah} + \mathbf{g}\}
\]

where \( \xi_A \) is the acceleration error vector and \( (\xi_{An}, \xi_{An}, \xi_{Ah}) \) represent the equivalent acceleration errors in the \((n, e, k)\) axes. The heading error under these conditions is:

\[
\delta \psi = \frac{-\xi_{An}}{\| \mathbf{g}_e \|} \sin L \frac{\sin L}{\cos L_c}
\]

But define “tilt” about the north axis as \( \delta \theta_n = \frac{\xi_{An}}{\| \mathbf{g}_e \|} \)

Heading error can then be written as:

\[
\delta \psi = -\delta \theta_n \tan L
\]

For example, at 45° latitude, an effective “tilt” about the north axis of 1 arc-min results in a heading error of 1 arc-min.

B. Moving Base: In the particular case of interest in this paper, the strapdown gyrocompass is, of course, required to operate on a moving base, viz. a marine vehicle. Under these circumstances, the uncorrected gyro and accelerometer measurements do not directly define the meridian plane since (a) the gyros measure vehicle rate in inertial space and (b) the accelerometers measure specific force (the difference between vehicle inertial acceleration and gravitational acceleration). The necessity to generate corrections to the measurements in order to establish the meridian plane implies that performance will be configuration-sensitive. The most desirable type of system configuration has the property of dynamic exactness, i.e., a system in which the errors are independent of vehicle motions.

In general, the error equations for a dynamically exact low speed cruise system show that behavior is primarily described by the combination of Schuler loop (84 minute period) and earth loop (24-hour period) effects. It is well-known that a Schuler-tuned vertical loop configuration will provide a dynamically exact indication of the vertical, i.e., in the absence of inertial sensor errors and regardless of vehicle accelerations, the computed vertical will track the true vertical. In a conceptually similar manner, the earth loop is associated with the dynamically exact indication of a vector fixed in inertial space, e.g., a unit vector directed along the polar axis of the earth.

The ideal Schuler-tuned loop is an undamped system with 84-minute period; initial conditions and uncorrelated bias error sources would cause only bounded oscillations to occur in the vertical error. But random error sources are also present, and these would result in unbounded rms vertical errors. For this reason, and because of the extended operating times, undamped vertical loops do not provide satisfactory performance in most marine applications. A variety of damped vertical loop configurations have, however, been developed which preserve a certain type of dynamic exactness. These configurations require a continuous damping reference, and use only the difference between the inertial and the
reference data to develop the damping signals. Most commonly, a velocity reference is used (EM log, doppler, or, in the future, possibly GPS velocity) though continuous position references such as Omega (others might include LORAN or, in the future, GPS position) can also serve this role. Dependent on the details of the damping configuration, random errors in the reference and in the gyros/accelerometers will cause associated (but bounded) rms errors in the indicated vertical. Bias components of acceleration measurement errors in the horizontal plane cause a correlated tilt error.

Vertical loop damping can be accomplished without influencing the undamped structure of the earth loop. This particular combination (damped schuler-tuned vertical loops and undamped earth loop) is commonly implemented in accurate marine inertial navigators such as SINS. Use of high quality gyros—which generally requires operation of the gyros in a controlled benign environment—is, of course, implicit to the design of inertial navigators. But even in an inertial configuration, any “east” gyro bias will result in a corresponding average heading error.

In less demanding applications, an alternate implementation which involves damping of the earth loop as well as of the vertical loops is possible. This is the traditional approach used in marine stabilized gyrocompass designs, and it can relieve requirements on gyro drift stability. Again, a damping reference is necessary; and the generation of earth loop damping corrections is based only on the difference between inertial and reference data. The use of a damped earth loop configuration leads to bounded rms errors in response to random gyro drift; but performance now becomes dependent upon the low frequency error characteristics of the damping reference. The effect of this implementation on heading error may be illustrated by noting that the inertial rate associated with a stabilized east pointing gyro can be generally represented as \( \dot{V}_n / R + \xi_{CE} \), where \( V_n \) is vehicle north velocity, and \( R \) is earth’s radius. Because the low frequency information in the reference is necessarily used for earth loop damping, then any (effective) reference error in north groundspeed is indistinguishable from east gyro drift. And heading error is therefore directly affected by such errors in the reference.

For example, assume that damping is accomplished with a waterspeed sensor. A north ocean current is a direct error in north groundspeed, and a relatively constant north ocean current of 0.6 knots would cause a heading error equivalent to a 0.01 deg/hr east gyro bias drift, viz. 1 mrad at \( L = 48.3^\circ \). The use of groundspeed sensors, such as position references, GPS velocity, or a bottom reflecting Doppler sonar would eliminate the dependency of heading error on ocean currents.

It is clear from the above that heading accuracy performance in a stabilized gyrocompass is inherently limited by east gyro drift and by vertical tilt about a north axis. Additional error sources are present, but their effect depends on the particular implementation. In order to provide an acceptable level of heading/attitude accuracy at sea, use of damped schuler-tuned vertical loops is necessary. A damped earth loop configuration is also normally used. But performance is then dependent on the low frequency error characteristics of the damping reference. Use of a waterspeed sensor for vertical and earth loop damping provides the advantages of an autonomous shipboard system; however, errors in the north groundspeed component of the damping reference act as equivalent east gyro drift errors, and this can be a significant at-sea error source.
SYSTEM CONFIGURATION

An east gyro bias drift of 0.01 deg/hr causes a heading offset of 2.3 sec L arc-min, and an east accelerometer bias of 100 μg results in a heading error of 0.35 tan L arc-min. Component performance at this level is generally compatible with the U.S. Navy Type I at-sea heading accuracy requirement of 4. sec L arc-min. The medium grade spinning wheel gyros used in current marine stabilized gyrocompasses routinely provide 0.01 deg/hr turn-off-to turn-on repeatability; however, long term drift effects eventually cause the gyro drift magnitude relative to the fixed drift compensation to exceed an acceptable level. The conventional approach to bounding this error buildup is to periodically recalibrate the gyros (and accelerometers) using platform rotation techniques. The calibrations are usually conducted dockside at time intervals of about three months. In at least one marine gyrocompass, the need for such special calibration techniques is eliminated by use of a gyro case reversal arrangement (180° rotations about a vertical axis at approximate 3 min intervals) during normal system operation. This technique not only averages case fixed gyro bias drift, but it further improves accuracy by shifting the spectrum of random gyro drift well beyond the pass band of the system. The use of rotation techniques to improve system performance has, in fact, been successfully applied to many gimbaled systems. Even in the case of an accurate, inertial navigator such as MK 3 SINS, a rotation technique involving a fourth gyro, mounted on an indexing table, is used to bound heading offsets, and to monitor horizontal gyro drifts. Rotation techniques have also been studied and applied in strapdown systems, and the approach is pertinent to the practical implementation of an accurate ring laser strapdown marine gyrocompass.

To consider this further, note that the drift of a ring laser can be characterized by the combination of turn-on bias, white noise, and temperature-sensitive components. Compensation has been effective in minimizing the temperature-related errors. However, the current state-of-the-art for turn on repeatability of a medium grade, low cost ring laser can be in the order of 0.10 deg/hr. Therefore, this class of ring laser is appropriate for use in an accurate marine gyrocompass only if a system technique capable of reducing and maintaining the effective east axis bias below 0.01 deg/hr can be defined. Rotation of the system gyros about the vehicle yaw axis provides a feasible method for accomplishing this objective in marine applications. There is a variety of ways in which such rotation can be implemented. For example, the rotation can be continuous, it can be unidirectional or reversing, or it may comprise discrete rotations-with intervening dwell periods-to indexed directions. An alternate, but higher cost, configuration could use an additional ring laser mounted on an indexing monitor table. The particular technique discussed in this paper involves periodic high rate rotation of the gyro-accelerometer cluster about the yaw axis of the ship to four discrete orthogonal positions symmetrically arranged with respect to the ship's roll axis. The rotation is followed by a dwell period (on the order of ten minutes) at each index point. This type of implementation is especially compatible with the strapdown concept in that relative gyro/accelerometer alignments are unaffected. The index positions can be computer-controlled to provide desirable flexibility. The particular configuration requires accurate measurement only of the rotation angle, i.e., the angle (about the yaw axis) between the rotator reference axis and the longitudinal axis of the vehicle. This angle defines the transformation between the gyro frame and
the vehicle frame and is used in conjunction with the direction cosine matrix relating the geographic frame to the gyro frame to compute vehicle orientation.

Figure 1 provides a general block diagram of this RL strapdown gyrocompass. The basic elements of the system are the inertial measurement unit (IMU), the velocity reference unit (an EM Log in this case), a digital computer, a control/display panel, and input/output (I/O) interface equipment. The IMU comprises a gyro-accelerometer sensor package mounted on a structure which is free to turn through a limited angle about the yaw axis of the vehicle. Flex leads, rather than slip rings, are used. Rotator position is accurately measured using a two speed synchro system. The sensor package consists of three ring laser gyros and three accelerometers with input axes pointed in orthogonal directions designated (A, B, C). The input axes of the “C” ring laser and of the “C” accelerometer are directed down along the rotation axis, while the A and B input axes are in the plane perpendicular to that axis, i.e., (A, B) are in the vehicle deck plane. The gyros measure angle increments and the accelerometers measure velocity increments in the gyro/accelerometer (A, B, C) coordinate frame, and this data as well as the rotator angle measurements and the gyro/accelerometer temperature measurements are transmitted to the digital computer. The temperature readings are used to correct gyro and accelerometer bias and scale factor. The corrected gyro/accelerometer data, and the rotator angle measurements, are processed with an appropriate set of strapdown algorithms and then combined with the reference data in a Kalman Filter to generate optimal estimates of vehicle attitude/heading. Other navigational data (velocity/latitude) as well as body rates and accelerations are also available for use.

A modern state-of-the-art digital computer is required in order to meet the computational rate and accuracy requirements of the strapdown algorithms. Such computers generally provide a capacity in excess of that required to perform control, display, I/O, and strapdown tasks. As a result, it is quite feasible to implement relatively sophisticated algorithms for reference data processing. A Kalman Filter is used in the system discussed in the paper. This filter operates on the difference between (a) inertial velocity along the longitudinal axis and EM Log, and (b) inertial velocity along the transverse axis and a computed transverse waterspeed reference (assumed zero except for sideslip during turns). A latitude fix capability is also provided. Filter outputs constitute estimates of the modeled states, and are used to correct system error sources and outputs. The system

![Fig. 1—RL gyrocompass block diagram](image-url)
includes provision for power-off storage of updated calibration data, latitude, etc.; this capability minimizes turn-on settling time. Vertical velocity is computed in a separate digital filter.

STRAPDOWN GYROCOMPASS: LEVEL/ALIGN

The operations involved in self-leveling and self-aligning (gyrocompassing) are conceptually identical in a strapdown system to those performed in a gimballed gyrocompass. In the strapdown system, an analytic platform exists in the software attitude matrix. To facilitate the description, a simplified (least squares filter) mechanization of the strapdown leveling and align loops is shown in Figure 2.

The computed direction cosine matrix can be thought of as an analytic platform. If the values of its elements are not correct, then the analytic platform is tilted off the vertical and misaligned in azimuth. Since this matrix is used to transform the body velocity increments into a local level reference frame, then a “tilt” will cause a component of vertical acceleration (g) to appear on a horizontal axis; and this will integrate into a horizontal velocity error. This inertial velocity is compared to a velocity reference and the difference is fed back to modify the direction cosine matrix until the “tilts” are corrected. The leveling operation is computationally equivalent to that in a gimballed system.

Any tendency of the tilts to continue increasing is assumed to be due to gyro drifts or, in the case of the east axis, to a coupling of the north component of earth rate caused by an azimuth misalignment. In the configuration of Figure 2, the tilt corrections are summed up over some time period and divided by that time period to yield average gyro drifts. In the case of the east axis, this drift is
divided by the north component of earth rate to provide an estimate of the azimuth misalignment of the analytic platform. The azimuth correction is used to adjust the elements of the direction cosine matrix. These azimuth corrections are in turn summed up over a selected time period to obtain a calibration of the azimuth gyro drift. This basic gyrocompassing operation is equivalent to that performed in a conventional gimballed system. As in such systems, any true east axis gyro drift will cause a corresponding heading offset error. As previously noted, the use of the indexer minimizes this error in the RL Marine Gyrocompass discussed in this paper.

**RL GYROCOMPASS PERFORMANCE ANALYSES**

It was previously shown that gyrocompass heading accuracy is limited by east axis gyro/accelerometer bias drift. However, the RL gyrocompass described in this paper uses an indexer assembly to periodically rotate the inertial sensors, resulting in zero average residual drift in the east (and north) axes. Therefore, (A, B) gyro/accelerometer bias does not cause a heading error offset, though variations in gyrocompass errors will occur during the rotation cycle. In general, indexer rotation may be considered to modulate drift components fixed to the sensor axes, essentially shifting the center of the drift spectrum from zero frequency to the rotation frequency. If the gyro/accelerometer drift is solely a bias, then the shifted spectrum is a delta function at the rotation frequency (and at its integer multiples). The shifted drift spectrum will result in zero-mean heading errors with rms magnitude dependent on the particular configuration parameters; these include, of course, the rotation frequency and the indexer rotation angle sequence.

These heading error variations can be minimized if an in-system calibration capability (analogous to synchronous demodulation) is implemented. The RL strapdown gyrocompass considered in this paper includes such a capability in the form of a Kalman Filter. The Kalman Filter operates on the differences between inertial and reference velocity to generate optimal estimates of all observable modeled states. In general, north axis gyro drift is among the “observables” while east axis gyro drift is not; only the linear combination of east axis gyro drift and heading error (multiplied by north earth rate) can be observed using velocity measurements. However, the periodic repositioning of the sensors to the selected index directions causes different combinations of (A, B) gyro drift to appear as east and north axis drift. To the extent permitted by other error sources, indexer angles, dwell time, etc., the Kalman filter will use available measurements of the associated velocity error pattern to estimate, correct, and therefore, to minimize the effects of (A, B) bias drifts.

In addition to (A, B) gyro/accelerometer bias drift, gyrocompass accuracy is also influenced by a number of other error sources. These are not necessarily affected by indexer rotations. They include white noise drift in the (A, B, C) gyros (this is an important drift component for the ring laser class of interest), C gyro bias, reference velocity errors, etc. A series of analyses were conducted to study the influence of these various errors on the accuracy performance of the RL strapdown gyrocompass. This has included the derivation of closed form expressions showing the relationship of heading accuracy to certain of the error sources as well as the conduct of computer-aided covariance analyses. A detailed presentation of the covariance analysis results is provided below. This is followed by a
brief description of the derived closed form expressions with details given in Appendices A, B and C.

Covariance Analysis: Description and Results

Conduct of an accuracy analysis for the RL strapdown gyrocompass requires definition of a mathematical model describing the characteristics of the system and of the error sources. The following pair of vector equations comprise an appropriate means for representing the required information:

State Space Equation \[ \dot{X} = FX + G\eta \]  
Measurement Equation \[ Z = HX + v \]

where \( X \) is the state vector (dim \( n \)), \( Z \) is the observation vector (dim \( r \)), \( F \) is the \( n \times n \) system matrix, \( H \) is the \( r \times n \) measurement matrix, \( G \) is the \( n \times n \) distribution matrix, \( \eta \) is the white plant noise vector (dim \( n \)), and \( v \) is the white measurement noise vector (dim \( r \)). The state vector can be considered to consist of three partitions, viz. (1) states describing the unforced dynamical behaviour of the plant, i.e., the Schuler loop and earth loop dynamics, (2) states describing the dynamical characteristics of the plant error sources (gyro drift and acceleration errors), and (3) states describing the errors in the references. A fifteen-state structure was used in the covariance analyses. The states include inertial velocity errors (2), tilts (2), inertial heading error (1) and latitude error (1), to represent the plant dynamics; gyro drifts (3), acceleration errors (2), and C gyro scale factor error (1) in the category of internal error sources; ocean currents (2), and velocity reference (EM Log) bias error (1) in the reference error partition. The vector, \( Z \), describes the “noisy” available measurements of linear combinations of the states. In this case, \( Z \) represents the errors associated with the “measurements” of inertial longitudinal and transverse velocity.

The state equations for the strapdown system plant are shown in Figure 3. Six states are involved, and they define the dynamics of strapdown system errors in the horizontal plane. These equations are assumed to be decoupled from those describing vertical velocity/altitude errors. Gyro and accelerometer errors are mapped into the geographic frame using appropriate elements of the direction cosine matrix; this transformation generally involves vehicle pitch, roll, and heading as well as the indexer angle.

For these analyses, ring laser drift was modeled as the combination of a random bias (turn-on repeatability) and a white random process. It is particularly noted that the effective drift in a geographic frame due to white noise in the sensor frame is also a white random process. Further, assuming that the white noise in the sensor axes is independent with equi-variances, then these conditions also apply to the associated effective drift in the geographic frame. Also included in the state vector is a bias representation of c gyro scale factor error; the associated c axis drift is essentially a pulse during rotation between index points. Acceleration errors are modeled as the combination of a random bias and a random walk (approximating vertical deflections, etc.). A more complex model for the sensors would include all scale factor errors and misalignments. The influence of such errors in state-of-the-art sensors is relatively small; Appendix C indicates the interaction of the indexer with this category of error sources.

The measurement equations characterize the errors in the differences between
inertial and reference velocities in the longitudinal/transverse axes; the transverse waterspeed is assumed to be zero except during turns. The model used for these errors in the analyses includes white noise, EM Log bias, and random north/east ocean currents.

A generalized error analysis computer program was used to generate the covariance results presented in this paper. The program assumes that an optimal (Kalman) filter is used to process the available measurements and develop estimates of the modeled system states. The covariance matrix of the estimation errors is computed based on the assumption that the system is Gauss-Markov, and that the error models used to structure the filter match the real world. As indicated in a subsequent section, the pre-production RL strapdown gyrocompass includes a Kalman filter so that the covariance results which are presented are representative of expected system performance.

The at-sea covariance analysis results are presented in Figures 4-8. The index angle sequence (relative to the ships longitudinal axis) used in the simulation was \((-135^\circ, +45^\circ, +135^\circ, -45^\circ, \ldots\). For purposes of the analyses, the dwell time at each index position was set to 21.24 m, and the time to rotate between dwell positions was 21.6 s; the filter iteration interval was also 21.6 s. The “baseline”
**INITIAL VALUE = 2°**

**BASELINE MODEL CONDITIONS**
- TURN-ON GYRO BIAS REPEATABILITY 1.0 DEG/H
- OCEAN CURRENTS 0.42 kt/AXIS; 0.75 H
- WHITE VEL MEAS NOISE 0.1 kt/AXIS
- DWELL PERIOD 0.354 H
- INDEX SEQUENCE -135°, +45°, +135°, -45°
- C SCALE FACTOR ERROR 100 PPM
- V = 20 kt
- L = 45°N
- CONSTANT HEADING (±0°)

**RMS HOG ERROR VS TIME**
- GY WN = 0.015 DEG/H\(^{1/2}\); LOG BIAS = 0.5 kt
- GY WN = 0.010 DEG/H\(^{1/2}\); LOG BIAS = 0.5 kt (BASELINE MODEL)
- GY WN = 0.010 DEG/H\(^{1/2}\); LOG BIAS = 0.0 kt
- GY WN = 0.005 DEG/H\(^{1/2}\); LOG BIAS = 0.5 kt

**Fig. 4**—Effect of white noise gyro drift and EM log bias

**INITIAL VALUE = 2°**

**BASELINE MODEL**
- 0.010 DEG/H\(^{1/2}\) WHITE GYRO NOISE
- LATITUDE = 45°
- CONSTANT HEADING (±0°)

**RMS HEADING ERROR VS TIME**
- GY WN = 0.015 DEG/H\(^{1/2}\); LOG BIAS = 0.5 kt
- GY WN = 0.010 DEG/H\(^{1/2}\); LOG BIAS = 0.5 kt (BASELINE MODEL)
- GY WN = 0.010 DEG/H\(^{1/2}\); LOG BIAS = 0.0 kt
- GY WN = 0.005 DEG/H\(^{1/2}\); LOG BIAS = 0.5 kt

**Fig. 5**—Effect of operating latitude

**INITIAL VALUE = 2°**

**RMS HEADING ERROR VS TIME**
- BASELINE MODEL 0.010 DEG/H\(^{1/2}\) WHITE GYRO NOISE
- CONSTANT HEADING (±0°)
- L = 75°
- L = 45°
- L = 90°

**Fig. 6**—Effect of C gyro SF error

**INITIAL VALUE = 2°**

**RMS HEADING ERROR VS TIME**
- BASELINE MODEL 0.010 DEG/H\(^{1/2}\) WHITE GYRO NOISE
- LATITUDE = 46°
- CONSTANT HEADING (±0°)
- INITIAL C SF ERROR = 500 PPM
- INITIAL C SF ERROR = 100 PPM (BASELINE MODEL)

**Fig. 7**—Effect of ship maneuvers
model used for gyro drift includes a turn-on bias repeatability of 1.0 deg/h and an rms white noise magnitude of .010 deg/h^{1/2}. A C gyro scale factor error of 100 PPM is modeled. Acceleration errors include a turn-on bias of 115 μg, and a random walk of 35 μg/h^{1/2}. The baseline model for the (waterspeed) velocity reference includes an EM Log bias of 0.5 kt., and random north/east ocean currents of 0.42 kt. rms/axis with 15 nmi correlation distance. Velocity measurements are made (in ship axes) at 21.6 s intervals; white measurement noise of 0.1 kt/axis is included. Initial rms tilt is modeled as 10 deg/axis, and the initial rms heading error is 2 deg.
Figure 4 provides a plot of rms heading error vs time for several levels of white noise gyro drift and EM Log bias error. In all cases, the operating latitude is 45°N, and the ship is assumed to be traveling on a fixed heading (due north). The Type I heading accuracy specification at this latitude is 5.7 arc-min. (4. sec L). Figure 4 shows that the rms heading error is within the specification value in 2.5 h with the baseline model; by 4.5 h after start-up, the rms heading is 4.4 arc-min. The plots also show that increasing white noise gyro drift to 0.015 deg/h\(^{1/2}\) increases the heading error at 4.5 h to 5.7 arc-min while a decrease to 0.005 deg/h\(^{1/2}\) decreases the heading error at 4.5 h to 3.2 arc-min. Additional improvement
in heading accuracy with time is indicated for each of these conditions. The corresponding steady-state rms vertical errors range from 0.13–0.22 arc-min, dependent on the white noise level. Because of the north ship heading in these runs, EM Log bias acts as a north velocity reference error; the heading error associated with 0.5 kt north velocity reference bias at 45° latitude is 2.7 arc-min. Elimination of EM Log bias would then be expected to result in an improvement in heading accuracy. Figure 4 substantiates this conjecture; the rms heading error at 4.5 h with EM Log bias set to zero improves from 4.4 arc-min to 3.9 arc-min.

The occurrence of ship turns provides information which can be used to calibrate EM Log; the particular structure used in the Kalman Filter is such that calibration of the EM log is accomplished automatically.

Figure 5 indicates the effect of operating latitude on rms heading accuracy performance with the baseline model. At 4.5 h after startup, the rms heading error at latitudes of 0°, 45°, 75° is 3.3, 4.4, and 11.6 arc-min, respectively. These are within the (4. sec L) Type I specification. It is noted that the trend for the 75° latitude case implies a substantial further improvement in accuracy performance with time, and significantly less than a sec L relationship. Appendix A projects a (sec L)\(^{1/4}\) dependency of steady state heading error (due to white noise gyro drift—a major error source) with operating latitude.

In the above cases, a C gyro scale factor error of 100 PPM was assumed. The effect on heading error of an increase in C scale factor error to 500 PPM is shown in Figure 6. The time to settle to within the specification is extended somewhat from 2.5 h to 3.0 h after startup. The indicated performance generally shows a convergent trend toward the behavior of the baseline model; by 4.5 h, the C scale factor has been estimated by the Kalman filter to within 270 PPM. The ring laser
class of interest is expected to provide scale factor turn-on repeatability and stability well within 100 PPM; as a result, it is probably unnecessary to include this state in the Kalman filter structure since computed data show that it is not strongly observable at that level.

Vehicle maneuvers are of special interest in strapdown systems since, in contrast to gimballed systems, the geographic frame errors associated with sensor frame errors are a function of heading. Because of the use of the indexer, such effects are not expected to be significant in the RL strapdown gyrocompass. This is illustrated by the data in Figure 7 which indicates rms heading error behavior with the baseline model and a single 90° turn at 2.186 h after startup; the turn rate is about 1 deg/s. Rms heading error at 4.5 h (from startup) after a single 90° turn at 2.186 h is 3.9 arc-min in contrast to 4.4 arc-min w/o the turn. The 90° turn changes the ship track from northerly to easterly. Therefore, residual EM Log bias (after calibration associated with the turn) would not affect heading error subsequent to the maneuver in this run. However, for the case of an arbitrary ships track, the calibrated log will lead to improved heading accuracy performance. Ship maneuvers will result in a residual EM Log bias of less than 0.1 kt.

Figure 8 provides a time history of the estimation errors in heading/latitude, pitch/roll, gyro drift, accelerometer bias, EM Log bias, and n/e ocean current; these apply for an 0.005 deg/h½ white noise level and with multiple 90° turns (at 0.72 hr intervals commencing at 2.186 h). The plots show that (A, B) gyro drifts are estimated to 0.017 d/h in 4.5 hr. The drift estimation is not significantly affected by the turns. A large part of the (A, B) gyro and accelerometer bias calibration is accomplished during the first few indexer rotations.

Figures 9A and 9B provide information regarding heading accuracy performance under land-based operating conditions. The state structure is similar to that previously described except that the EM Log bias and north/east ocean current states are omitted. In addition, the white noise in the velocity measurements is changed from 0.1 kt rms/axis in the at-sea cases to 0.01 kt rms/axis.

Figure 9A is a time history of rms heading error for a gyro white noise of 0.007 deg/h½, 45° latitude, C scale factor error of 100 PPM, and an indexer dwell period of 21.24 m. The effect of various levels of gyro bias turn-on repeatability (0.01 deg/h—1.0 deg/h) on heading error transient behaviour is indicated. Since the indexing arrangement results in rapid estimation of (A, B) gyro bias, the
critical parameter is essentially the bias repeatability of the C gyro. At 4 h after
turn-on, the rms heading error is between 1.7 arc-min and 2.4 arc-min, dependent
on the indicated value of gyro turn-on bias. But, by 9 h after turn-on, the rms
heading error is approximately 1.6 arc-min, and the variation with the initial gyro
bias is minor. The results generally indicate that C gyro turn-on bias repeatability
limits the heading error transient performance. Settling time is improved if C
gyro turn-on bias repeatability is reduced below a “threshold” (which depends on
the white noise drift). This threshold is near the level to which C bias drift would
be estimated at that time in the settling transient. Figure 9B further illustrates
heading error transient behaviour under somewhat different conditions. The
white noise gyro drift level is 0.010 deg/h, latitude is 45°, the indexer angle
sequence is changed to (0°, 180°, 0°, . . .), the dwell period is changed to 10.8 m,
and C gyro scale factor error is not included in the model. For initial gyro bias
drifts of 0.01 deg/h - 1.0 deg/h, the heading error at 4 h is between 2.1 and 3.3
arc-min. By 9 h after turn-on, the rms heading error is about 2.1 arc-min. Figure
9B also indicates that the rms heading error at 9h for zero turn-on bias drift is 1.6
arc-min. This is generally consistent with the analytical prediction for the steady-
state rms heading error derived in Appendix A (viz. 1.4 arc-min for these
conditions).

**Theoretical Performance Analyses**

As is true in many ring laser gyro applications, one of the dominant error
sources in the RL Gyrocompass is that due to white-noise random drift. This
error source is not averaged out nor minimized by the indexer sequencing.

A derivation is presented in Appendix A for an optimal system which shows
that the white-noise random drift yields a steady-state heading error given by the
following simple expression:

\[ \delta \theta_{2} \text{(rms-steady state)} = \sigma_{WN}(\sec L/\Omega_{E})^{1/2} \]  

(16)

where \( \delta \theta_{2} \) is rms heading error

\( \sigma_{WN} \) is rms white noise random drift (rms value

assumed to be equal on all axes)

\( L \) is Latitude

\( \Omega_{E} \) is Earth-Rate
This steady-state heading error results from a balancing between white noise drift effects occurring on the East axis and the Azimuth axis.

It is noted that this particular heading error varies as the square root of secant Latitude, unlike many other error sources which vary directly as the secant Latitude. The ratio of this heading error at $L = 75^\circ$ to that at $L = 0^\circ$ is 1.97; the ratio for those errors dependent on sec $L$ is 3.86.

As an example of equation (16), a white noise drift of 0.01 deg/h$^{1/2}$ at a latitude of 45$^\circ$ yields a steady-state heading error of 1.4 minutes of arc.

This basic relationship has been generally confirmed by the covariance analysis results described in the previous section.

To provide additional insight into the balancing of East axis and Azimuth axis effects, a derivation of the steady-state heading error due to white-noise random drift is presented in Appendix B for a special fixed-parameter third-order gyrocompass configuration. The steady state rms heading error relationship for this configuration is shown to be similar to that derived for the optimum case; it is identical in form to equation (16) but larger by a factor of 1.22.

In addition to averaging out various gyro bias drift and accelerometer bias effects, the indexer sequencing also averages out certain gyro scale-factor and misalignment errors. These indexer relationships are derived in Appendix C where it is shown that the following gyro error sources are averaged:

1. A and B axis gyro bias drifts
2. Misalignment of the A gyro about the B axis and the B gyro about the A axis.
3. Misalignments of the C gyro about A and B axes.
4. C gyro scale-factor operating on indexer rotation rate (due to indexer reversal)
5. A and B axis scale-factor asymmetry
6. A and B axis scale-factor errors become an effective north axis drift and average out on the east-axis (and in heading error)

It is also shown that, as might be expected, misalignments of the A and B gyros about the C axis, become direct heading errors.

PRE-PRODUCTION RL MARINE GYROCOMPASS: Description and Test Results

Sperry has fabricated and laboratory-tested a laser gyro strapdown marine gyrocompass that satisfies U.S. Navy Type I at-sea requirements. This RL Marine Gyrocompass is shown in Figure 10. Its design is an evolutional extension of the prior Sperry-developed laser gyro strapdown MK 16 MOD 11 Stable Element. The MK 16 MOD 11 is currently operational and in production for the U.S. Navy. In the gyrocompass, the inertial sensors are mounted on an indexer assembly with the axis of rotation perpendicular to the deck plane. The addition of the indexer is the primary difference in the mechanical design of the RL Gyrocompass relative to that of the RL Stable Element. The gyrocompass also incorporates a backup battery, and certain synchro output amplifiers and bus circuits required to satisfy U.S. Navy Type I specifications. Software changes include the deletion of the ship's heading input used in the stable element configuration, and some associated modifications in the Kalman Filter. There is 90% commonality between the gyrocompass and the stable element.
As shown in Figure 10, the inertial cluster—consisting of three SLG-15 laser gyros and three Q-Flex accelerometers—is mounted on the indexer assembly. The indexer is automatically sequenced between four orthogonal dwell positions (−45°, −135°, +45°, +135°) relative to the ship's longitudinal axis. The dwell time in each position is about 11 minutes, and the rotation rate between dwell positions is approximately 10 deg/sec so that the indexer is in motion only 3% of the time. Slip rings are not required since the indexer angular travel is restricted to ±135 deg. A multipole two-speed synchro continuously measures the angle of the indexer relative to the reference axis. Implementation of this indexing arrangement provides an in-system capability for accurately averaging/calibrating the equivalent east axis gyro drift and the horizontal accelerometer biases.

Figure 11 is a functional block diagram of the laser gyro marine gyrocompass. Solid-state synchro-to-digital converters are used to supply ship speed and indexer angle data to the SP-1000 microprocessor (DMP). The DMP also receives body angular increments from the strapdown laser gyros, and body velocity increments from the strapdown accelerometers via pulse rate converters. Inertial sensor temperatures are also monitored and provided to the DMP for use in the computation of scale factor/bias compensation. A control and display panel provides an operator interface. In addition to the required strapdown and data processing computations, the DMP also performs self-test and performance monitoring/fault localization functions. The DMP supplies output pitch, roll, and heading data to users via solid state digital-to-synchro converters. A digital interface can also be provided, if desired.

The microprocessor performs the basic functions shown in Figure 12. Body angular increments are appropriately integrated and used to compute a direction cosine matrix; the elements of this matrix contain the desired information regarding vehicle attitude with respect to an earth reference frame. Pitch, roll, and heading, i.e., the Euler angles, are subsequently extracted from the direction cosines. The direction cosine matrix is also employed to transform the body axis velocity increments into earth reference frame velocity increments. These are integrated to provide velocity (for comparison to reference velocity and for coordinate-frame-rate "torquing") and latitude (for the earth-rate portion of the coordinate frame rate).

The attitude information is leveled, aligned, and maintained to the required
accuracy level by performing optimal Kalman Filter in multiplications on the differences between reference velocity and inertial velocity. Kalman Filter corrections are obtained and applied to correct inertial attitude, gyro biases, accelerometer biases, velocity, and latitude data in a feed back closed-loop manner that yields minimum variance errors in the various states. The Kalman Filter contains 14-states and has a basic iteration time of 2.56 seconds.

The RL Gyrocompass has been tested in the Sperry laboratory and at the Naval Air Development Center (NADC), Warminster, Pa., under a variety of static and dynamic conditions. Some of the results are shown in Figures 13 and 14.

Figure 13 displays a typical settling run taken at NADC. In this case, the system is started under large angle Scorsby motion. At the end of 2 hours, the...
Scorsby motion is stopped; the time required to settle from this condition meets the 4-hour Type I specification. Figure 14 provides some typical dynamic and static performance obtained at Sperry in November, 1980. The particular test for which this data applies involved prior settling under static conditions. Large-angle Scorsby motion (with the indicated amplitudes and periods) was then applied for 8 hours, with measurements at approximately one hour intervals. The Scorsby motion was then stopped, and data collected during a subsequent 16 hour static run. The steady state static performance is close to that predicted for the white noise random drift levels (about 0.015 deg/√hr) of the particular laser gyro's.

A series of runs were performed at Sperry in November, 1980 (witnessed by U.S. Navy personnel) that included large angle Scorsby motion (3 eight hour runs at 0°, 45°, and 270° headings), small angle Scorsby motion (4 eight hour runs at 90°, 180°, 225°, 90° headings) and static runs at 0°, 45°, and 270° headings of 10 to 16 hours duration. The overall absolute mean heading error of these runs was 0.49 arc-minutes (0.37 sec L), indicating the general effectiveness of the indexer configuration. Heading error variation at different heading angles was not significant. The overall large angle Scorsby motion rms error was 4.48 arc-minutes (3.39 sec L). Table A is an at-sea error budget which includes the contribution of reference velocity and gravity anomaly errors to heading accuracy. The table shows that the above large angle Scorsby performance of the RL Gyrocompass is consistent with the Type I at-sea heading accuracy requirement of 4 sec L arc-
Table A—RL gyrocompass at-sea heading accuracy budget

<table>
<thead>
<tr>
<th>Major Error Contributors</th>
<th>ARC-MIN RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Large-Angle Scorsby performance in factory (Nov. 1980 status)</td>
<td>3.39 sec $L^*$</td>
</tr>
<tr>
<td>• EM Log Bias (0.5 knots)</td>
<td>1.35 sec $L$</td>
</tr>
<tr>
<td>• Ocean current variations (0.6 kts, 15 nm corr. dist. @ 20 knots)</td>
<td>1.65 sec $L$</td>
</tr>
<tr>
<td>• Gravity anomalies (10 arc-sec, 20 nm corr. dist. @ 20 knots)</td>
<td>0.10 set $L^{**}$</td>
</tr>
</tbody>
</table>

RSS 4.0 set $L$

Type I Spec 4.0 set $L$

* White noise gyro drift contributions varies as $\sqrt{sec \ L}$
** Tilt about north contribution varies as $\tan L$

Table B—RL Gyrocompass Requirements vs Status

<table>
<thead>
<tr>
<th>U.S. Navy Type I RQMT</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (Ft$^3$)</td>
<td>10</td>
</tr>
<tr>
<td>Weight (lbs)</td>
<td>650</td>
</tr>
<tr>
<td>Power (watts)</td>
<td>600</td>
</tr>
<tr>
<td>MTBF (hrs)</td>
<td>3000</td>
</tr>
<tr>
<td>MTTR (hrs)</td>
<td>0.5</td>
</tr>
<tr>
<td>Performance</td>
<td>4 sec $L$</td>
</tr>
<tr>
<td>Hdg (at-sea) (arc-min rms)</td>
<td>1.75</td>
</tr>
<tr>
<td>Pitch &amp; roll (at-sea) (arc-min rms)</td>
<td>4</td>
</tr>
<tr>
<td>Settling time (hrs)</td>
<td>Synchro</td>
</tr>
</tbody>
</table>

* Dominant error source varies as $\sqrt{sec \ L}$

min. It is also noted that the pitch and roll error during each of the tests was under 1 arc-min rms.

Table B summarizes the favorable characteristics of the current RL Marine Gyrocompass vis-a-vis the Type I specification.

Sperry has developed several laser gyro improvements since November, 1980. These include the addition of an aperture which has reduced SLG-15 white noise random drift to 0.007 deg/$\sqrt{hr}$, as well as certain path-length control loop modifications which decrease the sensitivity of the laser gyro to dynamic effects. Sperry is also currently converting the laser gyro marine gyrocompass into a marine navigator via appropriate software changes and the addition of a thermally-controlled inertial assembly enclosure. This system retains a marine gyrocompass mode of operation. It is planned to conduct additional system tests with the improved laser gyros in the very near future: significant improvements in gyrocompass performance are anticipated.

CONCLUSIONS

It has been clearly established, by both analysis and actual test data, that an accurate strapdown marine stabilized gyrocompass can be synthesized with the use of ring laser gyros. This configuration offers the potential of lower cost-of-
ownership, with greater reliability and simpler maintainability than that presently being achieved by conventional gimbaled systems.

Error relationships have been derived and performance analysis results have been presented that should prove useful in the analysis of any ring laser gyrocompass.

With minor software and hardware modifications, the ring laser gyrocompass described herein can be converted into a cost-effective strapdown marine navigator.

ACKNOWLEDGEMENTS

The laser gyro marine gyrocompass technology described in this paper is the result of contributions made by many individuals at Sperry Gyroscope. Acknowledgement and appreciation is hereby given to all those people.

The authors wish to specifically note the major contributions of Dr. Bernard Schwartz, John Moryl, Richard Krivin, and Noel Maniere in the successful development and test of the RL Marine Gyrocompass.

Also acknowledged are personnel at the Naval Air Development Center, particularly R. Hibbard, who monitored the RL Marine Gyrocompass effort and performed various performance tests of the system.

Appendix A—White Noise Gyro Drift Effects in an Optimal Gyrocompass

The effect of white noise gyro drift on heading accuracy performance in an "optimal" gyrocompass is described in this appendix. This is especially pertinent since white noise is the dominant random drift characteristic of the ring lasers under consideration, and the gyrocompass described in the paper includes an optimal (Kalman) filter to process reference velocity data.

Figure A-1 provides a simplified mathematical model for the gyrocompass plant and the associated error sources. The state space and measurement equations for this dynamical system can be represented in the following vector format:

State space equation: \[ \dot{X} = FX + \eta \] (A-1a)

Measurement equation: \[ Z = HX + \nu \] (A-1b)

Fig. A-1—Simplified system error block diagram
Based on figure A-1, these equations can be expanded as:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 & -g & 0 \\ 1/R_E & 0 & -\Omega_z \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix}$$

(A-2a)

$$Z_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \nu_1$$

(A-2b)

where Figure A-1 identifies $X_1$, $X_2$, $X_3$ as the north inertial velocity error, the vertical error about the east axis, and the heading error respectively. All the error sources are represented as zero mean, independent white noise processes with $\eta_1$, $\eta_2$, $\eta_3$, $\nu_1$, designating north acceleration error, $y$ gyro drift, $z$ gyro drift, and north reference velocity error, respectively.

Assume that continuous measurements are made of the difference between north inertial velocity and north reference velocity ($Z$), and that these measurements are processed in a Kalman Filter to generate estimates ($\hat{X}$) of the modeled states. Designate the error in these estimates as $\hat{X} = (\hat{X} - X)$. Then the covariance matrix of the estimation errors $[P = E(\hat{X}\hat{X}^T) = \{p_{ij}\}]$ is the solution to the following Riccati equation:

$$P = FP + PF^T - PHR^{-1}HP + Q$$

(A-3)

where

$$Q = E(\eta \eta^T) \quad \text{and} \quad R = E(\nu \nu^T)$$

In this case, let

$$Q = \text{diag}(q_{11}, q_{22}, q_{33}); \quad R = r$$

(A-4)

i.e., $q_{11}$, $q_{22}$, $q_{33}$, $r$ are the white noise variances for the acceleration error, $y$ gyro drift, $z$ gyro drift, and reference velocity respectively.

The scalar equations for the steady-state solution ($\dot{P} = 0$) are as follows:

$$p_{11}/r = q_{11} - 2gp_{12}$$

(A-5a)

$$gp_{22} = p_{11}(1/R_E - p_{12}/r) - \Omega_x p_{13}$$

(A-5b)

$$gp_{23} = -p_{11}p_{13}/r$$

(A-5c)

$$p_{32}/r = 2p_{12}/R_E - 2\Omega_x p_{23} + q_{22}$$

(A-5d)

$$(\Omega_x)p_{33} = p_{13}(1/R_E - p_{12}/r)$$

(A-5e)

$$p_{33}/r = q_{33}$$

(A-5f)

where $p_{11}, p_{22}, p_{33}$ are the variances of the velocity, vertical, and heading error, respectively.

The solution must generally satisfy constraints imposed by the characteristics of a covariance matrix. From equation (A-5f), note that

$$p_{13} = \pm \sqrt{q_{33}r}$$

(A-6)

(with the appropriate sign to be selected). Equations (A-5b), (A-5e), and (A-5c) provide a means of computing $p_{22}$, $p_{33}$, and $p_{23}$—given $p_{11}$, $p_{12}$, and $p_{13}$. The
solution for $p_{11}$ and $p_{12}$ is given by the following pair of simultaneous quadratic equations:

\[ p_{11}/r = q_{11} - 2gp_{12} \]  
\[ p_{12}/r = 2p_{12}/R_E + 2(\Omega_X/g)(p_{13}/r)p_{11} + q_{22} \]  
\[ (A-7a) \]
\[ (A-7b) \]

The parameters $(q_{11}, q_{22}, q_{33}, r, \Omega_X)$ could be varied over a representative range, and the associated numerical solution to these equations computed. An (approximate) closed form solution is, instead, provided in the sequel.

In order to simplify the solution conditions somewhat, assume that the white acceleration noise can be neglected. Then, $q_{11} = 0$, and note from equation A-7a that the only valid solution requires that $p_{12} \leq 0$. The following fourth order equation for $p_{11}$ (normalized) can be formed:

\[ y^4 + 4y^2 - 8aby - 4c = 0 \]  
\[ (A-8a) \]

where

\[ y = p_{11}/(\omega_r r) \quad a = (\Omega_X/\omega_r) \]
\[ b = \pm \sqrt{R^2_E q_{33}/r} \]
\[ \omega_r^2 = g/R_E \quad c = R^2_E q_{22}/r \]  
\[ (A-8b) \]

For the primary cases of interest, the solution to this fourth order equation can be approximated as:

\[ y^4 \equiv (4c) \]  
\[ (A-9a) \]

or

\[ p_{11} = \sqrt[4]{4r^3 q_{22} g^2} \quad \text{(vel. error variance)} \]  
\[ (A-9b) \]

Using this solution in equations (A-7a), (A-6), and (A-5e) results in the following:

\[ p_{12} = -\sqrt{rq_{22}}; \quad p_{13} = +\sqrt{rq_{33}} \]  
\[ (A-10) \]
\[ p_{33} = \frac{\sqrt{q_{33}}}{\Omega_X} \left( \sqrt{q_{22}} + \sqrt{r/R_E^2} \right) \]  
\[ (A-11) \]

\[ \equiv \sqrt{q_{22}q_{33}}/(\Omega \cos L) \]  
\[ (A-12a) \]

Let $q_{22} = \sigma_y^2 \triangleq y$ gyro white noise variance

$q_{33} = \sigma_z^2 \triangleq z$ gyro white noise variance

\[ E(\delta \theta_2^2) = p_{33} \triangleq \text{heading error variance} \]

Then $E(\delta \theta_2^2) = (\sigma_y \sigma_z / \Omega) \sec L$ \hspace{1cm} (A-12b)

If $\sigma_y = \sigma_z = \sigma_{WN}$

then $\delta \theta_1 \text{ (rms-steady state)} = \sigma_{WN} \sqrt{\sec L / \Omega}$ \hspace{1cm} (A-13)

Equation (A-13) shows that the steady state rms heading error, over the range of white noise $y, z$ gyro drift of interest, is proportional to the white noise magnitude and the square root of secant (latitude).

For the conditions given, viz. all error sources can be represented as white noise, the optimal gyrocompass is a third order configuration. The Kalman gain
matrix is

\[ K = PH^T R^{-1} \]  
\[ K^T = [p_{11}/r; p_{12}/r; p_{13}/r] \]

In the absence of z gyro white noise, the steady-state rms heading error is zero. The presence of a steady-state rms heading error is therefore the result of the need to balance the effects of y and z gyro white noise.

Appendix B—White-Noise Gyro Drift Effects in Lumped-Parameter Third-Order Gyrocompass

An analysis of white-noise gyro drift effects was performed for a special lumped parameter third-order closed loop gyrocompass configuration. The steady state rms heading error relationship in this case is shown to be similar to that derived in Appendix A for the optimal Kalman Filter system. This approach provides additional insight into the necessity to select loop parameters which balance the effects of white noise drift in the East (Y) and Azimuth (Z) axes.

Figure B-1 is a block diagram of the intercoupled North accelerometer/East axis gyro vertical loop and the Azimuth loop for a general closed-loop gyrocompass configuration (for the case of a constant small velocity). One set of transfer functions \( G_1(S), G_4(S), G_5(S), \) and \( G_K(S) \)) which leads to a third order gyrocompassing loop is:

\[ G_1(S) = K_1; \quad G_4(S) = 0; \quad G_5(S) = K_5; \]

\[ G_K(S) = K_H \sec \theta \]

Also, let

\[ K_5 = (K_1^2/(2g/R)) - 1; \quad K_H = K_1/(4\Omega_E) \]
Then the third-order gyrocompass is stable with a damping ratio for the complex roots of 0.5, and performance is dependent on only a single design parameter ($K_1$). For this special case, the transfer functions relating heading error to East axis and Azimuth axis gyro drift are (neglecting certain small coupling terms):

$$\frac{\theta_K(S)}{\epsilon_{GY}(S)} = \frac{K_1^{3/8}}{\Omega_E \cos L(S^3 + K_1S^2 + K_1^3S/2 + K_1^3/8)}$$

$$\frac{\theta_K(S)}{\epsilon_{GZ}(S)} = \frac{S^2 + K_1S + K_1^3/2}{S^3 + K_1S^2 + K_1^3S/2 + K_1^3/8}$$

The steady-state mean-square output error can be generally expressed as:

$$(\text{RMS}_y)^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 \Phi_{xx}(\omega) \, d\omega$$

where $G(\omega)$ is the amplitude frequency response of the transfer function. $\Phi_{xx}(\omega)$ is the power-spectral density function of the input error source.

For white-noise drift $\Phi_{xx}(\omega) = A_n^2$. Then;

due to East axis white noise drift,

$$\theta_{KRMS}(\text{STEADY-STATE}) = \frac{A_{NY}}{\Omega_E \cos L} \sqrt{\frac{K_1}{6}}$$

due to azimuth axis white-noise gyro drift,

$$\theta_{Kazi}(\text{STEADY STATE}) = A_{NZ} \sqrt{\frac{10}{3K_1}}$$

It is noted that as small a gain $K_1$ as possible is desired to minimize heading error due to East axis white-noise drift, while as large a gain $K_1$ as possible is desired to minimize heading error due to azimuth axis white-noise drift. There is thus an optimum value of gain $K_1$ that will minimize the total mean square steady-state heading error; this value of $K_1$ balances the contributions of the $Y$ and $Z$ gyro white-noise drift.

Assuming the same white noise gyro drift for both $Y$ and $Z$ axes ($A_{NY} = A_{NZ} = A_n$), then the total mean square heading error is given by:

$$\overline{\theta_k} (\text{TOTAL}) = A_n^2 \left[ \frac{K_1}{6\Omega_E^2 \cos^2 L} + \frac{10}{3K_1} \right]$$

For minimum mean square error, select $K_1$ such that

$$\frac{\delta \overline{\theta_k}}{\delta K_1} = A_n^2 \left[ \frac{1}{6\Omega_E^2 \cos^2 L} - \frac{10}{3K_1^2} \right] = 0$$

$$K_{1\text{opt}} = \sqrt{20 \Omega_E \cos L}$$

Substituting this result into (B-8) yields

$$\theta_{Kazi, \text{RMS}} = A_n \sqrt{\frac{\sec L}{\Omega_E}} \times \frac{\sqrt{20}}{3}$$
It is noted that the steady-state rms heading error in this case is only 1.22 times greater than the Kalman Filter minimum derived in Appendix A.

Appendix C—Gyro Bias, Scale Factor, Misalignment Effects with Indexing

The effect of gyro bias, scale factor errors (symmetrical and unsymmetrical), and misalignments in a strapdown system have been generally discussed elsewhere. As indicated below, certain of these effects tend to be "averaged" by use of the indexer.

Consider the cartesian coordinate frames designated by the symbols $i$, $s$, $b$, and $n$. The $i$, inertial, frame has its origin at the earth’s center. The $s$, sensor, frame is fixed to the indexer assembly with axes directed along the nominal (A, B, C) gyro input axes (A, B axes are in the nominal deck plane, and the C axis is directed down along the indexer axis of rotation). The coordinates of the $b$, body, frame are defined by the vehicle roll, pitch, yaw axes. The $n$, geographic, frame is fixed to the vehicle and has its axes directed north, east, and down along the local vertical. The effective gyro drift in the geographic frame associated with gyro bias and scale factor/misalignments can be generally expressed as

$$\xi^n = C^n_p \{ \xi^s + \Delta C^n_s \omega^s + S_u C^n_b \omega^n_b \}$$

where $C^n_p$ designates the transformation from the $p$ frame to $q$ frame

$\xi^s$ is the gyro drift vector in the geographic frame

$\xi^s$ is the gyro bias drift vector in the sensor frame

$\Delta = (\Delta_{ij})$: matrix of gyro (symmetrical) scale factor errors and misalignments; diagonal terms are scale factor errors and off-diagonal terms are misalignments ($\Delta_{ij}, i \neq j$ is the misalignment of the $i$ gyro in the $i, j$ plane).

$S_u = \text{diag} \{ S_{uA}, S_{uB}, S_{uC} \}$, i.e., the asymmetrical (A, B, C) gyro scale factor errors.

$\omega_{is}$ is the rate of the $s$ frame relative to inertial space ($\omega^n_{is} = \omega^n_{in} + \omega^n_{ib}$)

$\omega^n_{is} = (\omega_n, 0, \omega_b)$ where $\omega_n = \Omega \cos L$ and $\omega_b = -\Omega \sin L$; $\Omega$ is earth rate

$\omega^n_{nb}$ comprises the vehicle roll, pitch, yaw rate vector.

Assume that vehicle nominal pitch/roll attitude is zero. Also designate vehicle heading by $\psi$, and designate the rotation angle of the indexer relative to the vehicle roll axis by $\alpha$. Then the following results can be obtained from the above equation.

Let $\xi^n$ represent the gyro drift (in the navigation frame coordinates) due to gyro bias, symmetrical scale factor errors, and misalignments. Then:

$$\xi_{1n} = \omega_n \left( \frac{\Delta_{11} + \Delta_{22}}{2} \right) + \omega_n \left( \frac{\Delta_{11} - \Delta_{22}}{2} \right) \cos 2\alpha -$$

$$\omega_n \left( \frac{\Delta_{13} + \Delta_{23}}{2} \right) \sin 2\alpha + \cos \alpha (\xi_A + \Delta_{13} \omega)$$

(C-2a)
\[
\sin \alpha (\bar{\xi}_0 + \Delta_{23} \omega_z)
\]
\[
\xi_{1e} = -\omega_0 \left( \frac{\Delta_{12} - \Delta_{21}}{2} \right) + \omega_0 \left( \frac{\Delta_{12} + \Delta_{21}}{2} \right) \cos 2\alpha + \omega_0 \left( \frac{\Delta_{11} - \Delta_{22}}{2} \right) \sin 2\alpha + \cos \alpha (\bar{\xi}_0 + \Delta_{23} \omega_z) + \sin \alpha (\bar{\xi}_4 + \Delta_{13} \omega_z)
\]
\[
\xi_{1k} = \omega_0 \Delta_{31} \cos \alpha - \omega_0 \Delta_{32} \sin \alpha + (\bar{\xi}_c + \Delta_{33} \omega_z)
\]

where: \( \omega_2 = \omega_k + \dot{\alpha}_e(t) \); \( \dot{\alpha}_e(t) \) is non-zero only during indexer rotation.

\( \alpha = \psi + \alpha_e \), i.e., \( \angle \) of sensor frame A axis relative to north.

The average value of these drift components over a full cycle of indexer assembly rotation (assuming the average value of \( \sin 2\alpha \) and \( \cos 2\alpha \) is zero) is:

\[
E(\xi_{1n}) = \omega_0 (\Delta_{11} + \Delta_{22})/2
\]

\[
E(\xi_{1e}) = \omega_0 (\Delta_{12} - \Delta_{21})/2
\]

\[
E(\xi_{1k}) = \bar{\xi}_c + \Delta_{33} \omega_z
\]

Note that the residual average east axis drift is due only to (A, B) gyro misalignments about the C axis. This results in a direct heading error; the effect is relatively small since misalignment stability errors of less than 6 arc-sec can be attained in the ring laser class of interest. The average scale factor error of the A and B gyros results in a residual average north drift. But the indexer rotations "average" the effects of A, B gyro bias drifts, and misalignments of the A gyro about the B axis and the B gyro about the A axis. The residual average drift in the \( k \) axis directly reflects C gyro bias and C scale factor error; however, the effects of C gyro misalignment about the A, B axes is averaged. Indexer rotation rate effects are also averaged as a result of the selected reversing sequence (-135°, 45°, 135°, -45°).

An approach similar to the above shows that indexer rotation also serves to average the effect of any scale factor asymmetries in the A, B gyros.\(^{19}\)

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