Fully Differential Single Resonator FM Gyroscope Using CW/CCW Mode Separator

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Abstract—This paper reports a fully differential frequency modulated (FM) gyroscope where the superposed clockwise (CW) and counter clockwise (CCW) modes are independently controlled on a single resonator. The CW and CCW mode detectors were developed to control both modes simultaneously on the resonator. A frequency and quality factor mismatch compensation method was developed to perfectly orthogonalize these two modes. A FM gyroscope operation with excellent linearity between applied angular rate and frequency difference was demonstrated. The temperature coefficient of scale factor was as small as $-52 \pm 136$ ppm/$^\circ$C. The scale factor and bias were stable over 700 hours with the drift less than $0.062 \pm 0.100$ ppm/hr and $31 \pm 121$ ppm/hr, respectively. [2018-0103]

Index Terms—FM Gyroscope, CW/CCW-mode separator, mode-matched MEMS resonator, mismatch compensation.

I. INTRODUCTION

MEMS gyroscopes have unique advantages such as low cost, small size and low power consumption, therefore they are used in many applications such as image stabilization of cameras, vehicle stabilization, motion detection of game controllers and attitude control of drones. Recently, a high precision MEMS gyroscope has become important for some new applications such as an automatic car driving system and a small inertial measurement unit, in which small temperature coefficient and wide bandwidth are required. Conventional MEMS vibrating structure gyroscopes based on amplitude modulation have a trade-off between the sensitivity and bandwidth, because a high quality factor resonator generally has a large time constant [1], [2]. Therefore, low quality factor or intentionally mode-mismatched resonators are used, which results in low performance.

One of the solutions is to increase the operation frequency around 10 MHz or higher [3]–[5], however a high-speed circuit is necessary. Another method is suppressing the sensing axis vibration using feedback control, i.e. force rebalancing. Because the oscillation state is steady, the response of the resonator does not depend on the quality factor. Therefore, a mode-matched high quality factor resonator can be used to improve both bandwidth and sensitivity [6]. However, the scale factor has temperature dependency, which should be removed for the practical gyroscope by using some temperature compensation methods [7]–[10]. Kline et al. [11] proposed a mode reversal technique to reduce the low frequency noise. The bias stability could be improved by using this technique. However, two resonators were used to compensate the frequency fluctuation.

A frequency modulated (FM) gyroscope is known to be theoretically insensitive to temperature [12], [13]. The FM gyroscope measures a frequency difference between two eigenmodes of the mode-matched resonator, of which the frequency difference is proportional to the applied angular rate. Both eigenfrequencies have the same temperature dependency, and thus the frequency fluctuation induced by temperature can be cancelled out for the frequency difference.

Zotov et al. [12] used a quad mass gyroscopes (QMG) as the FM gyroscope, and the frequency split of two modes during ring-down was measured by spectral analysis. They demonstrated the good temperature stability of the FM gyroscope, but the gyroscope worked only in its ring-down period. Kline et al. reported a more practical FM gyroscope, in which two ring resonators were used for different eigenmodes. Both angular rate and rotation angle were measured by frequency and phase difference between these resonators, respectively [14]. However, it requires two mode-matched resonators, which increases fabrication difficulty. Moreover, if the temperatures of these two resonators are different, the temperature induced frequency change cannot be completely cancelled.

Izyumin et al. [15] proposed a FM gyroscope using a single mode-mismatched resonator to alternatively generate two eigenmodes on it. In recent years, this type of FM gyroscope, called as Lissajous FM (LFM) gyroscope, has drawn much attention. Minotti et al. [16] reported a low power and low noise LFM gyroscope with large dynamic range and good linearity. Most of the FM gyroscopes mentioned detect Z axis rotation. Recently, Zega et al. [17] reported a pitch gyroscope using the LFM technique. Also, 3-axis FM gyroscopes were reported by Minotti et al. [18] and Zega et al. [19]. Similarly, Eminoglu et al. excited both eigenmodes on a single resonator with time-division drive [13]. However the bandwidth is theoretically limited by the frequency split or switching frequency.

Recently, we proposed a fully-differential single resonator FM gyroscope which independently controls both eigenmodes simultaneously on the resonator [20], [21]. Compared with the previous works, this method enables both CW and CCW modes continuously excited on the same resonator without interaction between these two modes. Thus the common mode...
frequency change, mainly caused by temperature fluctuation and ageing, can be perfectly removed. In addition, this method is free from band-width limitation caused by frequency mismatch or mode switching.

In this paper, we report a mismatch detection and compensation technique for a mode-mismatched resonator and the long-term stability of scale factor and bias.

II. WORKING PRINCIPLE OF FM GYROSCOPE

Equation of motions of a two-dimensional mode-matched resonator are written as

\[ \begin{align*}
\dot{x} + \frac{\omega_0}{Q_0} \dot{x} + \left( \omega_0^2 - \Omega^2 \right) x &= 2k\Omega \dot{y} + U_x \\
\dot{y} + \frac{\omega_0}{Q_0} \dot{y} + \left( \omega_0^2 - \Omega^2 \right) y &= -2k\Omega \dot{x} + U_y,
\end{align*} \]

(1)

(2)

where \( \omega_0, Q_0, \Omega \) and \( k \) are a natural frequency, a quality factor, an applied angular rate and an angular gain factor [6], respectively. The vibration state, \( |V]\), and the excitation force, \( |U]\), at the angular frequency of \( \omega \) are defined as

\[ \begin{align*}
|V \rangle & = a_x |x \rangle e^{i\omega t} + a_y |y \rangle e^{i\omega t} \\
|U \rangle & = u_x |x \rangle e^{i\omega t} + u_y |y \rangle e^{i\omega t},
\end{align*} \]

(3)

(4)

where \( |x \rangle \) and \( |y \rangle \) are the states of linear motion along with X and Y axis, respectively. Thus the amplitudes of oscillation, \( x \) and \( y \), can be expressed as

\[ \begin{align*}
x &= \langle x |V \rangle e^{i\omega t} = a_x e^{i\omega t} \\
y &= \langle y |V \rangle e^{i\omega t} = a_y e^{i\omega t}.
\end{align*} \]

(5)

(6)

The equations of motion (Eq. (1) and (2)) can be then re-written as

\[ \begin{align*}
A(\omega) |V \rangle e^{i\omega t} &= |U \rangle e^{i\omega t},
\end{align*} \]

(7)

where the matrix components of \( A \) are defined as

\[ \begin{align*}
\langle x | A(\omega) | x \rangle &= \omega_0^2 - \omega^2 - \Omega^2 + i\frac{\omega \alpha_0}{Q_0} \\
\langle x | A(\omega) | y \rangle &= -2i k\omega \Omega \\
\langle y | A(\omega) | x \rangle &= 2i k\omega \Omega \\
\langle y | A(\omega) | y \rangle &= \omega_0^2 - \omega^2 - \Omega^2 + i\frac{\omega \alpha_0}{Q_0}.
\end{align*} \]

(8)

(9)

(10)

(11)

The non-orthogonal terms, \( \langle x | A(\omega) | y \rangle \) and \( \langle y | A(\omega) | x \rangle \), couple the X and Y oscillations, thus the \( |x \rangle \) and \( |y \rangle \) are not eigenstates. Instead, the eigenstates of \( A \) can be obtained as

\[ \begin{align*}
|CW \rangle &= \frac{1}{\sqrt{2}} |x \rangle + i\frac{1}{\sqrt{2}} |y \rangle \\
|CCW \rangle &= \frac{1}{\sqrt{2}} |x \rangle - i\frac{1}{\sqrt{2}} |y \rangle,
\end{align*} \]

(12)

(13)

where “CW” and “CCW” mean the clock-wise and counter-clock-wise motions in the X-Y plane, respectively. When these eigenstates are chosen as basis, the non-orthogonal terms of \( A \) disappears as

\[ \begin{align*}
\langle CW | A(\omega) | CW \rangle &= \omega_0^2 - \omega^2 - \Omega^2 + i\frac{\omega \alpha_0}{Q_0} + 2k\omega \Omega \\
\langle CCW | A(\omega) | CCW \rangle &= \omega_0^2 - \omega^2 - \Omega^2 + i\frac{\omega \alpha_0}{Q_0} - 2k\omega \Omega \\
\langle CW | A(\omega) | CW \rangle &= 0 \\
\langle CCW | A(\omega) | CW \rangle &= 0.
\end{align*} \]

(14)

(15)

(16)

(17)

The vibration state and excitation force can also be expressed by these eigenstates as

\[ \begin{align*}
|V \rangle &= a_{cw} |CW \rangle + a_{ccw} |CCW \rangle \\
|U \rangle &= u_{cw} |CW \rangle + u_{ccw} |CCW \rangle.
\end{align*} \]

(18)

(19)

Then Eq. (7) becomes two independent equations as

\[ \begin{align*}
a_{cw} &= (\omega_0^2 - \omega^2 - \Omega^2 + i\frac{\omega \alpha_0}{Q_0} + 2k\omega \Omega)^{-1} u_{cw} \\
a_{ccw} &= (\omega_0^2 - \omega^2 - \Omega^2 + i\frac{\omega \alpha_0}{Q_0} - 2k\omega \Omega)^{-1} u_{ccw}.
\end{align*} \]

(20)

(21)

When the resonator has high quality factor and the applied angular rate is much smaller than the operation frequency, the eigenfrequencies of both CW and CCW modes can be simplified as

\[ \begin{align*}
\omega_{cw} &= 2\pi f_{cw} = \omega_0 + k\Omega \\
\omega_{ccw} &= 2\pi f_{ccw} = \omega_0 - k\Omega.
\end{align*} \]

(22)

(23)

Generally, the natural frequency, \( \omega_0 \), has a temperature dependency [22], but this temperature induced frequency change can be cancelled out by a differential method as

\[ \Delta \omega = \omega_{ccw} - \omega_{cw} = 2k\Omega. \]

(24)
A. Control System Based on CW/CCW Mode Separation

Figure 1 shows the principle of a proposed FM gyroscope. A single resonator is used for both CW and CCW mode oscillations. Superposed modes on the resonator can be separated by a CW/CCW mode separator, in which amplitudes and phases of these modes are detected by a signal processing method based on synchronous demodulation. The detailed signal processing procedure is described in the latter part of this section. These modes can be simultaneously but independently excited at their resonant frequencies by using two independent phase locked loops (PLLs), consisting of the mode separator, PI controllers and numerically controlled oscillators (NCOs). The angular rate can be measured by the frequency difference between these two NCOs.

Figure 2 shows the schematic diagram of the control system. The CW/CCW mode separator consists of CW and CCW detectors, of which the signal processing procedure is based on synchronous demodulation as illustrated in Fig. 3. To detect the CW component, the signal from the resonator, 

\[
\begin{bmatrix}
X \\
Y
\end{bmatrix}
\]

is multiplied by two sets of CW reference signals, 

\[
\begin{bmatrix}
X_{\text{ref}, I} \\
Y_{\text{ref}, I}
\end{bmatrix} = \begin{bmatrix}
\cos(\omega t) \\
-\sin(\omega t)
\end{bmatrix}
\]

and 

\[
\begin{bmatrix}
X_{\text{ref}, Q} \\
Y_{\text{ref}, Q}
\end{bmatrix} = \begin{bmatrix}
-\sin(\omega t) \\
\cos(\omega t)
\end{bmatrix},
\]

thus only the CW component contained in the input signal can be demodulated to the low frequency region, similar to the “conventional” synchronous demodulator. After removing the high frequency signals, the amplitude, \(r_{\text{CW}}\), and phase, \(\theta_{\text{CW}}\), of CW signal can be calculated as

\[
r_{\text{CW}} = \sqrt{x_{\text{CW}}^2 + y_{\text{CW}}^2}
\]

(25)

\[
\theta_{\text{CW}} = \tan^{-1}\left(\frac{y_{\text{CW}}}{x_{\text{CW}}}\right),
\]

(26)

where \(x_{\text{CW}}\) and \(y_{\text{CW}}\) are the low-pass-filtered signals. The CCW detector has essentially same structure, but the reference signals are 

\[
\begin{bmatrix}
X_{\text{ref}, 1} \\
Y_{\text{ref}, 1}
\end{bmatrix} = \begin{bmatrix}
-\cos(\omega t) \\
-\sin(\omega t)
\end{bmatrix}
\]

and 

\[
\begin{bmatrix}
X_{\text{ref}, Q} \\
Y_{\text{ref}, Q}
\end{bmatrix} = \begin{bmatrix}
\sin(\omega t) \\
-\cos(\omega t)
\end{bmatrix}.
\]

The phase of each modes, \(\theta_{\text{CW}}\) or \(\theta_{\text{CCW}}\), has the same frequency dependency as that of the normal spring-dumper-mass system, i.e. the phase delay at the resonance is \(-90^\circ\). Therefore, similar to the conventional resonator, the CW and CCW mode oscillations can be excited at their resonance by using the PI controller. To excite the CW or CCW mode, the driving signals, of which the phase differences between
Fig. 4. (a) Amplitude and (b) phase response of mode-mismatched resonator.

X and Y components is 90° or −90°, are generated by each NCO. These CW and CCW driving signals are superposed and introduced to the resonator.

B. Mode-Mismatch Compensation

A perfectly symmetric resonator has the same frequencies and quality factors along with both X and Y axes. However, a practical oscillator usually has the frequency and/or quality factor mismatch mainly caused by the fabrication imperfection. Figure 4 shows the typical amplitudes and phases of the mode-mismatched resonator. The frequency and quality factor mismatch induce the difference in phase delay (Fig. 4 (b)) and amplitude (Fig. 4 (a), respectively, near the resonance.

Figure 5 shows the driving signals and induced oscillations of a perfectly symmetric, a frequency mismatched and a quality factor mismatched resonators. The theoretical phase differences between X and Y amplitudes are 90° and −90° for the CW and CCW modes, respectively. The pure CW (or CCW) oscillation can be excited by the pure CW (or CCW) driving signal only when the resonator is perfectly symmetric (Fig. 5 (b)). When the resonator has the frequency mismatch, due to the different phase response in X and Y axis as shown in Fig. 4 (b), the oscillation induced by the pure CW driving signal does not have a phase difference of 90° between X and Y motion, i.e. it is not the pure CW oscillation (Fig. 5 (c)). In addition, when the resonator has quality factor mismatch, the amplitudes of X and Y motion becomes different, which also means the induced oscillation is not the pure CW mode (Fig. 5 (d)). Any two dimensional motion at resonance can be expressed by the linear combination of CW and CCW motions, because these two modes constitute the basis. Thus, both frequency and quality factor mismatch generate the CCW oscillation from CW driving signal, and vice versa. As a result, both CW and CCW PLL interferes each other.

For the correct operation, this interference must be removed. The basic idea is illustrated in [23, Fig. 6]. The phases, Δϕ_CW and Δϕ_CCW, and amplitudes, A_CW,X, A_CW,Y, A_CCW,X and A_CCW,Y, of the driving signals are adjusted to compensate the mismatch. Thus, the pure CW (or CCW) mode can be excited, i.e. the amplitudes of X and Y axis motion are identical, and the phase difference between X and Y axis is 90° (or −90°).

The proposed mismatch compensation technique does not depend on the driving and sensing method, therefore it can be applicable to wide variety of resonators.

III. EXPERIMENTAL RESULT

A. Experimental Setup

Figure 7 shows an experimental setup. A ring shape resonator (SGH03, Silicon Sensing Systems, UK), of which the frequency mismatch was about 2 ppm, was placed on a rotation table. The resonator was driven by the Lorentz force and
the motion was detected by the electromagnetic induction. The oscillation modes used here were two \( n = 2 \) wineglass modes as illustrated in Fig. 8 (a) and (b). By using the modal coordinate, these two modes can be treated as X and Y axis motions, \( |x\rangle \) and \( |y\rangle \). The X mode oscillation can be excited by the driving signal, \( X_d \), and the oscillation amplitude can be measured by the sensing signal, \( X_s \). Similarly, the Y mode can be driven by \( Y_d \) and sensed by \( Y_s \). The schematic illustrations of CW and CCW mode oscillations are shown in Fig. 8 (c) and (d), which are linear combinations of \( |x\rangle \) and \( |y\rangle \) according to Eq. (12) and (13), respectively.

Voltage-to-current converters and pre-amplifiers were placed on the rotation table, and the control signals were connected to control equipments through slip rings. The driving and sensing signals were connected to digital lock-in amplifiers (UHF-LI, Zurich Instruments Ltd., Switzerland), in which the CW and CCW demodulators and the PLLs were implemented. The temperature of resonator was controlled by a Peltier element.

**B. Mismatch Compensation**

To determine the tuning parameters of CCW driving signal, the CW component driven by the CCW driving signal was measured and minimized as follows. Only the CCW mode PLL was activated and the CW component contained in the oscillation was detected by the CW demodulator. Using \( \Delta \phi_{ccw} \), \( A_{ccw,X} \) and \( A_{ccw,Y} \) as the tuning parameters, the cross-coupling terms between CW and CCW, \( x_{cw} \) and \( y_{cw} \), was measured. Figure 9 shows the measurement result. The cross-coupling terms were normalized by the CCW oscillation amplitude, \( r_{ccw} \). As can be seen, both \( x_{cw} \) and \( y_{cw} \) can be controlled by the amplitude ratio, \( A_{ccw,Y}/A_{ccw,X} \), and phase.
difference, \( \Delta \phi_{ccw} \), respectively. A cross point of three planes, 
\( Z = x_{cw}, y_{cw} \), and 0, provides the optimal tuning point. The 
the CW component (\( x_{cw}, y_{cw} \)) and amplitudes ratio (\( A_{cw,Y}/A_{cw,X} \)) of the driving signal.

C. Excitation of CW and CCW Oscillations

Figure 11 shows the measured X and Y amplitudes of the resonator. When only the CW or CCW mode PLL was 
excited on the resonator. When only the CW or CCW mode PLL was 
activated with the same amount, the superposed 
oscillation (Fig. 11 (a) - (d)). When both CW and CCW mode 
PLL, (c, d) only CCW mode PLL and (e, f) both CW and CCW mode PLLs, 
were applied to the rotation table. Both CW and CCW mode 
frequencies were almost identical when no rotation applied 
excited the opposite frequency change. The 
angular rate induced the opposite frequency change. The 
frequencies slowly drifted mainly caused by the temperature 
change, however it could be removed by the differential 
method using Eq. (24), as shown in Fig. 12 (b).

\[ a_0 e^{i(\Delta \phi_t)} e^{i(\omega_0 t + k\Omega t)} |x| + a_0 e^{-i(\Delta \phi_t)} e^{i(-\omega_0 t - k\Omega t)} |y| \]  
\[ = a_0 e^{i\omega_0 t} \left\{ e^{i(\Delta \phi_t + k\Omega t)} |x| + e^{-i(\Delta \phi_t + k\Omega t)} |y| \right\} \]  
\[ = \frac{a_0 e^{i\omega_0 t}}{\sqrt{2}} \left\{ \cos \left( \frac{\Delta \phi}{2} + k\Omega t \right) |x| \right\} \]  
\[ + i \left( e^{i(\Delta \phi_t + k\Omega t)} - e^{-i(\Delta \phi_t + k\Omega t)} \right) \]  
\[ = \frac{a_0 e^{i\omega_0 t}}{\sqrt{2}} \right\} \right\} \]  
(27)

where \( \Delta \phi \) is the phase difference between CW and 
CCW mode oscillations. The experimental results 
(Fig. 11 (e) and (f)) well agreed with the above equation, 
indicating that the both modes could be simultaneously 
excited on the resonator.

D. Angular Rate Measurement

Figure 12 shows the measured frequencies of both CW and 
CCW modes without mismatch compensation and temperature 
stabilization, i.e. \( \Delta \phi_{cw} = \Delta \phi_{ccw} = 0 \) and \( A_{cw,X} = A_{cw,Y} = A_{ccw,X} = A_{ccw,Y} = 50 \) [mV]. The angular rates of \( \pm 100^\circ/s \) 
were applied to the rotation table. Both CW and CCW mode 
frequencies were almost identical when no rotation applied 
(\( \Omega = 0 \)). When a positive angular rate (\( \Omega > 0 \)) 
was applied, the CW and CCW resonant frequencies increased 
decreased by the same amount, respectively, and the negative 
angular rate induced the opposite frequency change. The 
frequencies slowly drifted mainly caused by the temperature 
change, however it could be removed by the differential 
method using Eq. (24), as shown in Fig. 12 (b).

The imperfection of MEMS resonator couples CW and 
CCW control loops as discussed in section II-B, which induces 
the interference between CW and CCW control loops. Thus 
the periodic fluctuation (beat of CW and CCW frequencies) 
was observed as shown in Fig. 12. This unnecessary frequency
Fig. 12. (a) Frequencies of both PLLs without and with rotation (±100°/s) and (b) frequency difference between CW and CCW modes. Mismatch compensation was not applied.

Fig. 13. Measured frequency difference with mismatch compensation.

fluctuation can be removed by the mismatch compensation as introduced in III-B. Figure 13 shows the measurement result with the mismatch compensation.

Figure 14 shows the relationship between the frequency difference, $\Delta \omega = \omega_{cw} - \omega_{ccw} = 2\pi (f_{cw} - f_{ccw})$, and the applied angular rate, $\Omega$. The frequency difference linearly changed with the applied angular rate, of which the slope (i.e. the scale factor) was 0.724. Figure 15 shows the temperature dependency of the scale factor. The scale factor had an almost constant value in a range from 25°C to 75°C without any temperature compensation technique. From the linear fitting, the temperature coefficient of scale factor was as small as $-52 \pm 136$ ppm/°C.

The CW and CCW mode frequencies at the stationary state ($\Omega = 0$) are shown in Fig. 16 (a). The measurement was done without temperature stabilization, therefore the frequencies were not constant mainly caused by the temperature fluctuation. However, as shown in Fig. 16 (b), this temperature fluctuation can be removed by the differential method as

$$\Omega_{\text{meas}} = \frac{2\pi (f_{ccw} - f_{cw})}{2k}. \tag{28}$$

The power spectral densities (PSDs), $S^2$, of the angular rates obtained from only the CW frequency ($\Omega_{\text{meas,cw}} = \frac{2\pi}{k} (f_{cw} - \langle f_{cw} \rangle$), the CCW frequency ($\Omega_{\text{meas,ccw}} = \frac{2\pi}{k} (f_{ccw} - \langle f_{ccw} \rangle$) and the differential method (Eq. (28)) are shown in Fig. 17. Both $\Omega_{\text{meas,cw}}$ and $\Omega_{\text{meas,ccw}}$ have large noise at
the low frequency region of which the frequency dependency is \( S^2 \propto 1/f^2 \). On the other hand, the PSD calculated from the differential method has no frequency dependency, which means the low frequency noises of the CW and CCW modes have strong correlation, and thus can be completely removed by the proposed differential method. The white noise density was about \( 0.25^\circ/\sqrt{\text{s}} \), which is consistent with the white noise density obtained from the PSD.

### E. Long Term Stability

To measure the long term stability, the scale factor and bias were measured repeatedly. In each measurement, the frequency difference between CW and CCW modes was recorded at the applied angular rates of \( \Omega_1 = 0, \pm 30, \pm 60, \pm 90 \) and \( \pm 120^\circ/\text{s} \). The scale factor was obtained from the linear fitting between the measured frequency difference and the applied angular rate. The bias was obtained from the frequency difference at \( \Omega_1 = 0^\circ/\text{s} \).

Figure 19 shows the long term stability of the scale factor. The non-zero bias value at about \( 0.157^\circ/\text{s} \) was observed, however it can be removed because it is almost constant. From the linear fitting, the drift rate of the scale factor was within \( 0.062 \pm 0.100 \) ppm/hr.

Figure 20 shows the long term stability of the output bias. The non-zero bias value at about \( 0.157^\circ/\text{s} \) was observed, however it can be removed because it is almost constant. From the linear fitting, the drift rate of the bias was within \( 31 \pm 121 \) ppm/hr.
IV. CONCLUSION

A FM gyroscope using independently controlled CW and CCW mode oscillations on a single resonator was proposed. A mismatch compensation technique by adjusting the amplitudes and phases of driving signals was proposed and experimentally confirmed. The proposed method could compensate both frequency and quality factor mismatches, and made the CW and CCW mode oscillations perfectly orthogonal. The modulated frequencies were well proportional to the applied angular rate. The proposed gyroscope could measure the frequency difference of two eigenmodes on the same resonator at the exactly same time, thus the common-mode frequency fluctuation could be perfectly removed. As a result, the differential angular rate. The proposed method could compensate both frequency and quality factor mismatches, and made the CW and CCW mode oscillations perfectly orthogonal. The modulated frequencies were well proportional to the applied angular rate. The proposed gyroscope could measure the frequency difference of two eigenmodes on the same resonator at the exactly same time, thus the common-mode frequency fluctuation could be perfectly removed. As a result, the differential angular rate. The proposed method could compensate both frequency and quality factor mismatches, and made the CW and CCW mode oscillations perfectly orthogonal. The modulated frequencies were well proportional to the applied angular rate. The proposed gyroscope could measure the frequency difference of two eigenmodes on the same resonator at the exactly same time, thus the common-mode frequency fluctuation could be perfectly removed. As a result, the differential angular rate.

REFERENCES


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