Distributed Robust Finite-Time Secondary Voltage and Frequency Control of Islanded Microgrids

Nima Mahdian Dehkordi, Nasser Sadati, Member, IEEE and Mohsen Hamzeh, Member, IEEE

Abstract—This paper presents a distributed, robust, finite-time secondary control for both voltage and frequency restoration of an islanded microgrid with droop-controlled inverter-based distributed generators (DGs). The distributed cooperative secondary control is fully distributed (i.e., uses only the information of neighboring DGs that can communicate with one another through a sparse communication network). In contrast to existing distributed methods that require a detailed model of the system (such as line impedances, loads, other DG units parameters, and even the microgrid configuration, which are practically unknown), the proposed protocols are synthesized by considering the unmodeled dynamics, unknown disturbances and uncertainties in their models. The other novel idea in this paper is that the consensus-based distributed controllers restore the islanded microgrid’s voltage magnitudes and frequency to their reference values for all DGs within finite time, irrespective of parametric uncertainties, unmodeled dynamics, and disturbances, while providing accurate real power sharing. Moreover, the proposed method considers the coupling between the frequency and voltage of the islanded microgrid. Unlike conventional distributed controllers, the proposed approach quickly reaches consensus and exhibits a more accurate robust performance. Finally, we verify the proposed control strategy’s performance using the MATLAB/SimPowerSystems toolbox.

Index Terms—Distributed cooperative secondary control, feedback linearization, finite-time control, microgrids, multi-agent systems, power sharing, robust control.

I. INTRODUCTION

Microgrids as modern, small-scale conventional power systems consist of renewable energy sources, such as wind generators (WTs), photovoltaics (PVs), and micro turbines (MTs); energy storage systems; and local loads that can operate in both grid-connected and islanded operating modes. Microgrids offer more efficiency and reliability than conventional power grids. In the grid-connected mode, while the microgrid is connected to the main grid, the main grid determines the microgrid’s voltage and frequency. In this case, the microgrid delivers the preplanned scheduling real and reactive powers to the main grid. In the islanded mode operation, unpredictable disturbances or preplanned scheduling cause disconnection of the microgrid from the main grid. As a result, the islanded microgrid’s pre-islanding control strategy can make it unstable.

To maintain the voltage and frequency stability of distributed generators (DGs) in the microgrid, an effective control strategy, called primary, should be employed [1], [2]. However, because primary control has some drawbacks, such as the voltage and frequency deviation caused by droop technique, hierarchical control has been presented to standardize the microgrid operation [3]–[6]. Hierarchical control consists of three layers: primary, secondary, and tertiary control. The secondary control compensates for the primary control’s deviation, while the tertiary control is responsible for economic dispatch and power flow optimization issues.

This paper focuses on the secondary control of an islanded microgrid, presenting a distributed robust finite-time control structure for an islanded microgrid system. The traditional secondary control uses a centralized structure that requires all the information of the individual DGs and a central computing and communication unit. These requirements reduce the overall system reliability and increase its sensitivity to failures that can lead to a single point of failure [4], [5], [7]. To overcome the aforementioned drawbacks, several recent studies have proposed the distributed secondary control as a promising approach [8]–[21]. The distributed cooperative secondary control uses only the information of neighboring DGs that can communicate with one another through a sparse communication network. The distributed secondary control has the advantages over a central control structure of increasing system reliability, decreasing its sensitivity to failure, and eliminating the need for a central computing and communication unit.

Moreover, because of plug-and-play capability of microgrids, microgrid’s physical and communication structures can be time-varying. The distributed control structure provides a robust secondary control framework that properly works irrespective of time-varying, restricted, and unreliable communication networks. Centralized control structure requires a complex communication network with two-way communication link [7], [10], [11], [17]–[20]. This requirement reduces the overall system reliability and increases its sensitivity to failures that can lead to a single point of failure. However, the distributed controllers communication network is sparse, and each agent communicates with local neighbors. Unlike a fully connected network, this sparse network reduces the communication infrastructure costs and makes it scalable and reliable.

Because more extensions increase the controller complexity, scalability of central controllers is not straightforward. Moreover, the failure of any unit can shut down the whole system. In summary, a distributed control strategy has the advantages of surviving uncertainty and disturbances. It also improves plug-and-play capability, security, and reliability. Moreover, it allows you to have easier scalability, simpler communication network, and fully distributed data updating leading to efficient information sharing, and finally to make faster decisions and operations [14]–[21].
However, existing distributed secondary control methods have the following shortcomings:

- Many control methods are usually synthesized in the small-signal equations or suffer from incomplete plant dynamics because they ignore the impact of the inner controllers on the secondary controller [8], [11], [13], [14], [17], [20]. Despite their simplicity, such controllers lack global stability, which is a necessary requirement in complex networks.
- Despite considering a complete nonlinear model of the system [10], [12], [13], [16], [19], existing distributed control methods require a detailed model of the system. However, in practice, the microgrid parameters, such as line impedances, loads, other DG units parameters, and even the microgrid configuration, are unknown. In other words, unmodeled dynamics are not considered, as they include unknown disturbances and uncertainties in their models. This is problematic because it can create the coupling between the frequency and voltage control loop. Therefore, these methods cannot guarantee robust stability when faced with parametric uncertainties, unmodeled dynamics, and disturbances, especially in high-power converters.
- From a practical point of view, sensitive loads in a microgrid require operation at the nominal voltage and frequency. Therefore, it is particularly desirable to speed up the synchronization process and guarantee the consensus in a finite time. Moreover, the finite-time control methods provide robust performance and stability, higher accuracy, and disturbance rejection properties along with a finite settling time. However, existing distributed secondary control methods reach consensus over an infinite-time. The finite-time control methods have already been presented in the literature to control robot manipulators and spacecraft [22]–[28].

Recently, researchers have recommended some types of distributed finite-time secondary control of microgrids to accelerate the synchronization process and improve the rate of convergence of both voltage and frequency control [10], [29], [30]. Key drawbacks of existing distributed finite-time secondary control methods are the following: 1) they cannot guarantee robust stability when faced with parametric uncertainties, unmodeled dynamics, and disturbances from unknown frequency and amplitude; 2) some methods restore either voltage or frequency in a finite time; and 3) the synchronization is achieved via discontinuous protocols.

To overcome the aforementioned difficulties, this paper proposes a fully distributed, consensus-based, robust, finite-time secondary control for the voltage and frequency restoration of an islanded microgrid that includes an arbitrary number of DGs. First, we use the input-output feedback linearization to transform the complete nonlinear dynamics of DGs with unknown dynamics and uncertainties into linear dynamics. The microgrid is considered a multi-agent system in which inverter-based DGs serve as agents and DGs communicate with one another through a sparse communication network, thus transforming the secondary control into a second-order tracking synchronization problem. In this paper, we use a weighted undirected graph to model the communication topology among DGs. Next, we design robust distributed controllers to restore the voltage magnitudes and frequency of the islanded microgrid to their reference values for all DGs within finite time, irrespective of the parametric uncertainties, unmodeled dynamics, and disturbances. In the case of frequency restoration, the controller should also meet the power sharing property. We can summarize the main features of the proposed method as follows:

1. To the best of the authors’ knowledge, a fully distributed, consensus-based, robust, finite-time secondary control for both voltage and frequency restoration of an islanded microgrid that includes an arbitrary number of DGs is first proposed irrespective of the parametric uncertainties, unmodeled dynamics, and disturbances.
2. From a practical perspective, the microgrid parameters, such as line impedances, loads, other DG units parameters, and the microgrid configuration are unknown. Unlike existing distributed methods that require a detailed model of the system, the proposed protocols are designed by considering unmodeled dynamics, unknown disturbances and uncertainties in their models, as additive disturbance terms. Therefore, this modeling leads to controllers being independent of the DG parameters and considers the coupling between the frequency and voltage control loop.
3. In the case of frequency restoration, the distributed consensus-based control meets the power sharing accuracy in a finite time as well.
4. Unlike a central control structure, the distributed cooperative secondary control uses only the information of neighboring DGs which can communicate with one another through a sparse communication undirected network.
5. In comparison with conventional distributed controllers, the proposed method rapidly reaches consensus and shows a more accurate robust performance for controller activation, structure reconfiguration, and load changes. Also, it is fully independent of DG parameters (line impedances, loads, and other DG units parameters).

Finally, we verify the proposed method’s performance using the MATLAB/SimPowerSystems software environment.

II. MICROGRID SYSTEM LARGE-SIGNAL DYNAMICAL MODEL

Fig. 1 shows the schematic diagram of an inverter-based DG in islanded-mode operation. The DG is represented by the primary energy source, the voltage source converter (VSC), the series LCL filter, the output connector, and the power, voltage, and current control loops. The control loops use a PI controller to regulate the VSC’s output voltage and frequency. It is worth mentioning that each DG’s nonlinear large-signal model is presented in its own $d-q$ (direct-quadrature) reference frame. We assume that the reference frame of one DG is designated the common reference frame and that the dynamics of other
DGs are transformed to the common reference frame with the rotating frequency of $\omega_{\text{com}}$. The primary controller is modeled by considering the dynamics of the real and reactive power droop techniques. The primary controller provides the voltage references $v^*_{\text{adi}}$ and $v^*_{\text{oji}}$ as well as the operating frequency $\omega_i$ for the VSC. The real and reactive power of the $i^{th}$ DG are passed through two first-order low-pass filters with the cutoff frequency of $\omega_{ci}$ to obtain the measured $P_i$ and $Q_i$. Moreover, according to the circuit principle, the $i^{th}$ inverter’s real and reactive power can be represented according to Fig. 1 as follows:

$$P_i = \frac{R_{ci}}{R_{ci}^2 + X_{ci}^2} (v_{b_2}^2 - v_{b_1}v_{b_2} \cos \delta_i) + \frac{X_{ci}}{R_{ci} + X_{ci}} (v_{b_1}v_{b_2} \sin \delta_i)$$

$$Q_i = \frac{X_{ci}}{R_{ci}^2 + X_{ci}^2} (v_{b_2}^2 - v_{b_1}v_{b_2} \cos \delta_i) - \frac{R_{ci}}{R_{ci}^2 + X_{ci}^2} (v_{b_1}v_{b_2} \sin \delta_i),$$

where $v_{b_1}$ and $v_{b_2}$ represent the amplitude of the $i^{th}$ bus voltage and the $i^{th}$ inverter output voltage, respectively; $\delta_i$ is the power angle difference; and $R_{ci}$ and $X_{ci}$ are the resistive and inductive output impedance components, respectively. Equations (1) and (2) represent $P_i$ and $Q_i$, which are both dependent on the $i^{th}$ DG frequency and voltage. Therefore, the voltage and frequency droop characteristics for the $i^{th}$ DG (the primary control locally implemented at the $i^{th}$ DG) are presented by the following equations:

$$\omega_i = \omega_{ni} - m_P P_i,$$

$$v^*_{\text{adi}} = V_{ni} - n_Q Q_i,$$

$$v^*_{\text{oji}} = 0,$$

where $m_P$ and $n_Q$ are the frequency and voltage droop gains, respectively, and $V_{ni}$ and $\omega_{ni}$ are the desired frequency and voltage, respectively. We represent large signal state space model of the $i^{th}$ DG, shown in Fig. 1, as the following form of multi-input multi-output (MIMO) nonlinear system:

$$\dot{x}_i = f_i(x_i) + g_{i1}(x_i)u_{i1} + g_{i2}(x_i)u_{i2} + k_i(x_i)D_i,$$

$$y_{i1} = v_{odi} = h_{i1}(x_i),$$

$$y_{i2} = \omega_i = \omega_{ni} - m_P P_i = h_{i2}(x_i) + d_i u_{i2},$$

where $x_i = [\omega_i, P_i, Q_i, i_{Ld}, i_{Lq}, v_{odi}, v_{oji}, \omega_{di}, \omega_{qi}]^T$, $\omega_i$ is the angle of the DG reference frame with respect to the common reference frame. $i_{Ld}$, $i_{Lq}$, $v_{odi}$, $v_{oji}$, $\omega_{di}$, and $\omega_{qi}$ are the direct and quadrature components of $i_{Li}$, $V_{odi}$, and $\omega_i$ in Fig. 1. $y_1 = [y_{i1}, y_{i2}]^T = [v_{odi}, \omega_i]^T$, $u_1 = [u_{i1}, u_{i2}]^T = [V_{ni}, \omega_{ni}]^T$, and $D_i = [\omega_{com}, v_{odi}, v_{oji}]^T$ are considered the outputs, inputs, and known disturbance, respectively. Also, $f_i(x_i)$, $g_{i1}(x_i)$, $g_{i2}(x_i)$, and $k_i(x_i)$ are defined in [12]. The purpose of the secondary control is to select $V_{ni}$ and $\omega_{ni}$ so $v_{odi}$ and $\omega_i$ are regulated to the desired values.

III. PRELIMINARY OF GRAPH THEORY

The microgrid is usually recognized as a multi-agent system in which DGs play the role of agents. DGs can communicate with one another through a sparse communication network. In this paper, we model the communication topology among DGs using a weighted undirected graph $G = (V_G, E_G, A_G)$ with a set of $N$ agents $V_G = \{1, 2, ..., N\}$, a set of edges $E_G \subset V_G \times V_G$, and a weighted adjacency matrix $A_G = (a_{ij} \geq 0) \in R^{N \times N}$, $(i, j) \in E_G \Leftrightarrow a_{ij} = a_{ji} = 1$; otherwise $a_{ij} = a_{ji} = 0$, $a_{ii} = 0$ for all $i \in V_G$ because of $(i, i) \notin E_G$. The result in $A_G$ is symmetric. The set of neighbors of node $i$ is denoted by $N_i = \{j \in V_G : (i, j) \in E_G, j \neq i\}$. For the leader-following multiagent system including $N$ followers with the leader, the leader adjacency matrix is defined as $B = [b_1, b_2, ..., b_N]^T \in R^N$. If agent $i$ is a neighbor of the leader, $b_i = 1$; otherwise, $b_i = 0$. It is worth mentioning that the edges of the corresponding undirected graph of the communication network denote the communication links among DGs. Moreover, for an undirected graph, if node $i$ is a neighbor of node $j$, then node $j$ can get information from node $i$ and vice versa [31].

IV. DISTRIBUTED ROBUST FINITE-TIME SECONDARY VOLTAGE CONTROL

In this section, a distributed robust finite-time cooperative control selects proper control inputs $V_{ni}$ in (3) to synchronize the voltage magnitude of DGs, $v_{odi}^{magi}$, to the reference voltage $v_{ref}$. According to the $d-q$ transformation, the magnitude of the DG output voltage is

$$v_{odi}^{magi} = \sqrt{v_{odi}^2 + v_{oji}^2}.$$ 

From (3) and (5), the synchronization of the voltage magnitude of DGs, $v_{odi}^{magi}$, is equal to the synchronization of the $d$ component of output voltages $v_{odi}$. We assume that DGs can communicate with one another through the prescribed communication undirected graph $G$. Therefore, the secondary voltage control selects $u_{i1}$ in (4) so that $y_{i1} \rightarrow v_{ref}, \forall i$. As we can see from (4), a MIMO nonlinear system represents the dynamics of DGs. Inspired by the mathematical model of the open loop system (4), we explain the overview of the input-output partial feedback linearizing stabilization scheme by following [32].

Fig. 1. Block diagram of an inverter-based DG.
A. Overview of Partial Feedback Linearizing Stabilization Scheme

Equation (4) represents the MIMO nonlinear state space model of the $i^{th}$ DG. We consider the following nonlinear coordinate transformation
\[
z = [h_{11} L_{f1} h_{11} \ldots L_{f1}^{r_1-1} h_{11} h_{12} L_{f1} h_{12} \ldots L_{f1}^{r_2-1} h_{12}]^T,
\]
(6)
where $L_{f1} h_{11} = \frac{\partial h_{11}}{\partial x}$ and $L_{f1} h_{12} = \frac{\partial h_{12}}{\partial x}$ are the Lie derivative [32] of $h_{11}(x)$ and $h_{12}(x)$ along $f_i(x)$, respectively. Therefore, the nonlinear system (4) has been transformed from $x$ to $z$ coordinates, provided that the following conditions are established:
\[
L_{g1} L_{f1}^{r_1-1} h_{11} \neq 0, \quad L_{g2} L_{f1}^{r_2-1} h_{12} \neq 0 \quad k < r_j - 1, \quad j = 1, 2, \quad n = r_1 + r_2,
\]
(7)
where $r_1$ and $r_2$ are the relative degree [32] of the system corresponding to output functions $h_{11}(x_i)$ and $h_{12}(x_i)$, respectively. We can represent linearized system as follows:
\[
z = Az + Bv,
\]
(8)
where $A$ is the system matrix, $B$ is the input matrix, and $v$ is the new linear control input for the feedback linearized system. Under the condition that $(r_1 + r_2) < n$, the partial feedback linearization method can be used. In this case, we may write the transformed states $z$ as
\[
z = [z_p \ z_n]^T,
\]
(9)
where $z_p$ represents the states transformed through nonlinear coordinate transformation up to the order $r_1 + r_2$, and $z_n$ represents the states related to the remaining $n - (r_1 + r_2)$ order. The dynamics of $z_p$ (states that are not observable from the output of the system) are called the internal dynamics of the system whose stability must be investigated before designing the linear controller for the following partially linearized system:
\[
\dot{z}_p = A_p z_p + B_p v_p,
\]
(10)
where $A_p$ is the system matrix, $B_p$ is the input matrix, and $v_p$ is the new linear control input for the partially linearized system.

B. Control Law

The input-output partial feedback linearization for the system with uncertainties and disturbances (4) can be obtained as
\[
\dot{y}_i = L_{f1}^{2} h_{11} + L_{g1} L_{f1} h_{11} u_{i1} + w_i
\]
\[
\dot{\mu}_i = W_i(y_i, \mu_i), \forall i,
\]
(11)
where $\mu_i \in \mathbb{R}^{n_i}$ represents the set of internal dynamics. We assume that internal dynamics are asymptotically stable [12], [33]. In practice, the microgrid parameters, such as line impedances, loads, other DG units parameters, and even the microgrid configuration are unknown. Moreover, the coupling between the voltage and the frequency control loop can degrade the performances of both controllers. Therefore, the term $u_i$ is added to (11) to represent unmodeled dynamics of DG, including unknown disturbances and uncertainties as well as the coupling between two control loops. Equation (11) can be written as the following second order system:
\[
\dot{\bar{y}}_i = y_{i1,2} + w_i, \forall i.
\]
(12)
We define $v_i = f_i(x_i) + g_i(x_i) u_{i1} + w_i$, so we have:
\[
\dot{y}_i = y_{i1,2} + v_i, \forall i.
\]
(13)

Assumption 4.1: [34] The leader is globally reachable.
Assumption 4.2: [34] There exist constants $L > 0$ and $\eta > 0$, such that $\|w_i\|_{\infty} < L$ and $\|w_i\|_{\infty} < \eta, \forall i$.
We note that just one DG has access to the reference voltage value. Therefore, the local neighborhood’s tracking error voltage is defined as
\[
e^v_i = \sum_{j \in N_{ci}} (y_{i1} - y_{j1}) + g_i(y_{i1} - y_{01}),
\]
(14)
in which $g_i$ is the voltage pinning gain, which is nonzero for the DG that has access to the reference voltage value. $N_{ci}$ represents the communication neighborhood set of the $i^{th}$ controller.

Lemma 4.3: [34] If Assumption 4.1 is established, and it is assumed that $w_i = 0$, the consensus tracking can be achieved in a finite time $T > 0$ by the homogeneous protocol
\[
v_i = k_1 r_1 v_i + k_2 r_2 v_i,
\]
where $v_i = v_{i1}^{nom}$, $\alpha_1 \in (0, 1)$, and $\alpha_2 = \frac{2 \alpha_1}{1 + \alpha_1}$, i.e.,
\[
\lim_{t \to T} |y_i - y_0| = 0,
\]
y_i = y_0, $\forall t \geq T, i = 1, 2, ..., N$.

We note that $\text{sgn}(x)^\alpha = |x|^{\alpha} \text{sgn}(x)$, where $\text{sgn}(.)$ is the sign function. Motivated by [35], we follow the design procedure for system (4) under the Assumptions 4.1 and 4.2. The design’s main purpose is to minimize the error between the desired reference value $y_{i1}$ and its actual value $y_{i0}$ so that the system remains globally stable. First, we can define the sliding surface as
\[
s_i = y_{i1,2} - y_{i1,2}(0) - \int_0^t v_i^{nom},
\]
(16)
where $y_{i1,2}(0)$ is the initial value of $y_{i1,2}$. By differentiating (16) and substituting from (13) and (15), we have
\[
\dot{s}_i = v_i - v_i^{nom} + w_i, \quad i = 1, 2, ..., N.
\]
(17)
Now, we define the super-twisting control $v_i^{\ast}$ as follows:
\[
v_i^{\ast} = -k_1 \text{sgn}^{\frac{1}{2}}(s_i) + \rho_i
\]
\[
\dot{\rho}_i = -k_2 \text{sgn}(s_i), \quad i = 1, 2, ..., N,
\]
(18)
where \( k_1 > 0 \) and \( k_2 > \eta \). We define the intermediate variable 
\[
\dot{z}_i = \rho_i + w_i
\]
and substitute (18) in (17), obtaining (19).
\[
\begin{align*}
\dot{s}_i &= -k_1 \text{sign}(s_i) + z_i \\
\dot{z}_i &= -k_2 \text{sgn}(s_i) + \dot{w}_i, \quad i = 1, 2, ..., N.
\end{align*}
\]
(19)

To stabilize (19), we consider the positive definite Lyapunov function as
\[
V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \zeta_i^T P_i \zeta_i,
\]
where \( P_i \in \mathbb{R}^{2 \times 2} > 0 \) and \( \zeta_i = [\text{sign}(s_i) \ 1]^T \). We note that the Lyapunov function (20) is partly motivated by [35] that designs adaptive consensus protocols for second-order multi-agent systems with unknown disturbances and uncertainties. Because of the term \( \text{sign}(s_i) \), the Lyapunov function is not locally Lipschitz but absolutely continuous. Therefore, we cannot use the classical Lyapunov theorem, which requires the Lyapunov function to be continuously differentiable or at least locally Lipschitz. Thus, using the Zubov theorem, which only requires a Lyapunov function is continuous [35], the derivative of \( \zeta_i \) can be derived as
\[
\dot{\zeta}_i = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s_i \ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \text{sgn}(s_i) & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \text{sign}(s_i) \dot{s}_i \\ 0 \end{bmatrix} \\
-k_2 \text{sgn}(s_i) + \dot{w}_i
\]
\[
= \frac{1}{2} |s_i| - \frac{1}{2} \begin{bmatrix} -k_1 \text{sign}(s_i) \ 0 \end{bmatrix} \left[ -2[k_2 - \text{sgn}(s_i) \dot{w}_i] \text{sign}(s_i) \right] = |s_i|^{-\frac{1}{2}} A_i \zeta_i,
\]
where
\[
A_i = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix}.
\]
(21)

Then, the derivative of (20) along its trajectory is
\[
\dot{V} = \sum_{i=1}^{N} \dot{V}_i = \sum_{i=1}^{N} |s_i|^{-\frac{1}{2}} \zeta_i^T P_i + P_i^T A_i \zeta_i.
\]
(22)

The matrix \( A_i \) is Hurwitz if and only if \( k_1 > 0 \) and \( k_2 > \eta \) [35]; since \( P_i \) is positive definite, then
\[
\dot{V} = \sum_{i=1}^{N} \dot{V}_i = -\sum_{i=1}^{N} |s_i|^{-\frac{1}{2}} \zeta_i^T Q_i \zeta_i,
\]
where
\[
Q_i = P_i + A_i^T P_i A_i = -Q_i, \quad i = 1, 2, ..., N.
\]
(24)

We note that for every \( Q_i > 0 \) exists a unique solution \( P_i > 0 \). Because \( |s_i|^{-\frac{1}{2}} = |\text{sign}(s_i)| \leq |\zeta_i| \leq \frac{1}{2} \lambda_{\text{max}}(P_i) V_i^{-\frac{1}{2}} \), the equality (23) can therefore be converted to
\[
\dot{V} \leq -\beta \sum_{i=1}^{N} V_i^{-\frac{1}{2}} \frac{\lambda_{\text{min}}(Q_i)}{\lambda_{\text{max}}(P_i)} V_i
\]
\[
\leq -\beta \sum_{i=1}^{N} V_i^{-\frac{1}{2}},
\]
(25)

where \( \beta = \min_{i=1,2,...,N} \left\{ \frac{\lambda_{\text{min}}(Q_i)}{\lambda_{\text{max}}(P_i)} \right\} \). We use the following inequality [35]:
\[
\sum_{i=1}^{N} V_i^{-\frac{1}{2}} \geq \left( \sum_{i=1}^{N} V_i \right)^{-\frac{1}{2}}.
\]
(26)

Therefore from inequality (25), we have
\[
\dot{V} \leq -\beta \left( \sum_{i=1}^{N} V_i \right)^{-\frac{1}{2}}.
\]
(27)

As a result, the continuous protocol
\[
V_{ni} = u_{ni} = \frac{1}{g_i(x_i)} (v_i - f_i(x_i))
\]
\[
v_i = v_i^{nom} + v_i^{ct}
\]
\[
v_i^{nom} = -k_1 \text{sign}(x_i) + k_2 \text{sign}(x_i)\alpha_2
\]
\[
v_i^{ct} = -k_1 \text{sign}(s_i) + \rho_i
\]
\[
\rho_i = k_2 \text{sgn}(s_i), \quad i = 1, 2, ..., N
\]
(28)
can guarantee that the direct term of DG output voltage, \( v_{odi} \), synchronizes with \( v_{ref} \) in a finite time despite the uncertainties and disturbances. The stable finite-time closed-loop systems demonstrate better robustness and disturbance rejection properties [34]. Therefore, the control inputs \( (V_{ni}) \) solve the voltage synchronization problem in a finite time, that is:
\[
\lim_{t \to T} [v_{odi} - v_{adj}] = 0, \quad \forall i, j
\]
\[
v_{odi} = v_{adj}, \quad \forall t \geq T, \quad \forall i, j.
\]
(29)

Owing to the line impedance effect, the simultaneous control of voltage and accurate reactive power sharing can only be achieved under a symmetric configuration [8], [9], [36]. Therefore, accurate voltage regulation results in large errors in reactive power sharing. Conversely, the precise reactive power sharing leads to the poor voltage regulation. Therefore, we should establish the trade-off between the voltage regulation and the reactive power sharing. In this section, we have focused on the voltage control.

![Block diagram of the distributed robust finite-time secondary voltage control](image-url)

**Fig. 2.** Block diagram of the distributed cooperative robust finite-time secondary voltage control. The control parameters \( k_1, k_2, \alpha_1, \) and \( \alpha_2 \) can tune the speed rate of the DG voltage.
V. DISTRIBUTED ROBUST FINITE-TIME SECONDARY FREQUENCY CONTROL

In this section, a distributed robust finite-time cooperative control selects the proper control inputs \( \omega_{ni} \) in (3) to synchronize the frequency of DGs, \( \omega_i \), to the reference value \( \omega_{ref} \). We note that the primary control provides accurate real power sharing among DGs according to (30); therefore, after applying the secondary control, the controller should also guarantee real power sharing accuracy:

\[
\frac{P_j}{P_i} = \frac{m_p}{m_p j}, \quad \forall i, j \in N. \tag{30}
\]

We assume that DGs can communicate with one another through the prescribed communication undirected graph \( G_r \). Therefore, the secondary frequency control selects \( u_{i2} \) in (4) so that \( y_{i2} \rightarrow \omega_{ref} \) and \( m_p k P_k \rightarrow m_p P_i, \forall i \). As in the previous section, we can obtain the input-output partial feedback linearization for the system with uncertainties and disturbances (4) as

\[
\dot{y}_{i2} = L_{y_2} y_{i2} + L_{y_1} y_{i2} u_{i2} + d_{i2} u_{i2} \\
\dot{\mu}_{i2} = W_{i2} (\dot{y}_{i2}, \mu_{i2}), \tag{31}
\]

where \( \mu_{i2} \in \mathbb{R}^{n_{i2}} \) represents the set of internal dynamics. We assume that internal dynamics are asymptotically stable [12], [33]. Equation (31) can be written as the following first-order system:

\[
\dot{y}_{i2} = \dot{\omega}_i = \dot{\omega}_{ni} - m_p P_i. \tag{32}
\]

Now we define the auxiliary control \( u_{\omega_i} \) as

\[
u_{\omega_i} = \dot{\omega}_i + m_p P_i, \tag{33}\]

where (33) determines the input-output feedback linearization that leads to the first-order linear systems:

\[
\omega_{ni} = u_{\omega_i}, \quad i = 1, 2, \ldots, N. \tag{34}
\]

Inspired from [37], we have designed the following auxiliary distributed robust finite-frequency control \( u_{\omega_i} \) based on its own information and the information of its neighbors on the communication graph, so that the angular frequency of each DG, \( \omega_i \), synchronizes to its nominal value, \( \omega_{ref} \), while guaranteeing the real power sharing accuracy:

\[
u_{\omega_i} = -k_\omega \sum_{j \in N_{ci}} \text{sign}(\omega_i - \omega_j) \alpha_\omega + g_i \text{sign}(\omega_i - \omega_{ref}) \alpha_\omega \\
- k_\omega \sum_{j \in N_{ci}} \text{sign}(m_p P_i - m_p P_j) \alpha_\omega, \tag{35}\]

where \( k_\omega \) and \( \alpha_\omega \) are the control parameters. It is worth mentioning that the second term of (35) guarantees that real power sharing will be maintained after applying the secondary control even in the event of communication failures. According to (33), (34), and (35), we can write \( \omega_{ni} \) as follows:

\[
\omega_{ni} = \int u_{\omega_i}, \quad i = 1, 2, \ldots, N. \tag{36}
\]

Therefore, the control inputs \( \omega_{ni} \) solve the frequency synchronization problem in a finite time:

\[
\lim_{t \to T} [\omega_i - \omega_j] = 0, \quad \forall i, j \\
\omega_i = \omega_j \quad \forall t \geq T \quad \forall i, j
\]

\[
\lim_{t \to T} [m_p P_i - m_p P_j] = 0, \quad \forall i, j \\
m_p P_i = m_p P_j, \quad \forall t \geq T, \quad \forall i, j. \tag{37}
\]

Compared with the conventional distributed control, the stable finite-time closed-loop systems have better robustness and disturbance rejection properties [34] because \( \alpha_\omega \) adds a further degree of freedom to the controller. As (35) demonstrates, the secondary control \( u_{\omega_i} \) contains two parts. The first part of the controller leads to the steady-state track of the reference frequency (i.e., \( \omega_i \rightarrow \omega_{ref} \)), despite the uncertainties and disturbances, and the second part ensures real power sharing accuracy (i.e., \( m_p P_j \rightarrow m_p P_i \)). Fig. 3 shows the block diagram of the distributed robust finite-time secondary frequency control.

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its sensitivity to failures. Moreover, the use of the sparse communication network makes them an viable option. From a computational point of view, the proposed method does not use any particularly problematic operation or heavy computations (suitable for using in a DSP or a microcontroller). We have also considered the intrinsic communication link delays in the simulation section; however, because of the large time scale of the secondary control, this delay does not significantly affect system performance. As mentioned in Assumption 4.1, the communication requirements to implement the proposed method can be easily achieved. Moreover, the communication topology among DGs should be a graph containing a spanning tree (i.e., the leader is globally reachable) in which the consensus protocols only require its own information and local neighbors in the communication graph. Therefore, according to the microgrid’s physical structure, it is easy to select a graph with a spanning tree that connects all the DGs in an optimal manner. There are many ways to design graphs including minimal lengths of the communication links, maximal use of existing communication links, and minimal number of links, and so on.

VII. SIMULATION RESULTS

To verify the proposed secondary controller’s effectiveness, we simulate the 380 V, 50 Hz islanded microgrid, which Fig. 4 shows, under various scenarios in the MATLAB/SimPowerSystems software environment. The islanded microgrid test consists of four inverter-based DGs. The loads and lines are modeled as series RL branches. Table I provides the DGs, lines, and loads parameters. In this table, \( K_{PV}, K_{IV}, K_{PC}, \) and \( K_{IC} \) are the voltage and current controllers parameters.

![Fig. 4. Block diagram of the islanded microgrid test system.](image)

We assume that DGs communicate with one another as shown through the undirected graph in Fig. 5. For both frequency and voltage restoration problems, we consider the voltage and frequency references as the leader node outputs. Moreover, DG_1 is the only DG connected to the leader node with the pinning gain \( g = 1 \). Note that to verify the robustness of the proposed method with respect to the parameters, the simulations are carried out with a 50% additive uncertainty (i.e., 50% additive deviation from the nominal value) in the parameters, as shown in Table I. Moreover, the parameters of the secondary controllers are \( k_1 = 100, k_2 = 100, \alpha_1 = 0.79, \alpha_2 = 0.79, k_{\omega} = 100, \) and \( \alpha_{\omega} = 0.79 \). This section consists of four parts, beginning with a robust performance evaluation of the proposed protocols, and then verification of robustness of the proposed method under communication network changes and plug-and-play capability, and finally a comparison of the proposed method with the methods in [11] and [12].

A. Study 1: Controller Performance

In this part, we have performed a number of simulations, including structure reconfiguration and load changes, to verify the robust performance of the proposed method. The simulation scenario of this part is defined as follows:

1) at \( t = 0.0 \) s (simulation initialization period), only the primary control is activated.
2) at \( t = 1.5 \) s, the proposed secondary control is applied.
3) at \( t = 2 \) s, Load #1 is increased.
4) at \( t = 3 \) s, Load #3 is disconnected.
5) at \( t = 3.5 \) s, switch S in Fig. 4 is opened.

Fig. 6 shows the real and reactive power of microgrid loads.

![Fig. 6. Study 1- the real and reactive power components of microgrid loads.](image)
their reference values by their droop controller. Therefore, the microgrid’s voltages and frequency must be restored in the secondary control layer of each inverter-based DG. In engineering applications, the voltage and frequency control should be applied instantly after occurring the disturbance.

Therefore, when we apply the secondary control, at \( t = 1.5 \) s, both voltage and frequency are rapidly restored to their reference values (\( v_{ref} = 380 \) V, \( f_{ref} = 50 \) Hz, respectively). Fig. 8(c) shows the real powers multiplied by the real power droop coefficients (i.e., \( m_P P_i \)) for DGs. As is seen, the frequency control meets (30). In other words, after applying the secondary control, the real power sharing is satisfied according to the rated power of DGs.

In the following, we investigate the performance of the islanded microgrid with respect to the inclusion of a three-phase load. Subsequent to the previous case study, at \( t = 2 \) s, we connect an RL load with \( R = 30 \) \( \Omega \) and \( L = 477 \) mH to Load #1. Fig. 7(a) and Fig. 8(a) show that the proposed secondary controller demonstrates good tracking and robust performance against the connection of the RL load and accurately restores the microgrid’s voltages and frequency. Despite connection to the RL load, Fig. 8(c) shows that the real power sharing is satisfied.

To highlight the controller’s robust performance, at \( t = 3 \) s, we disconnect load #3 from the islanded microgrid. The results show that the distributed secondary controller can restore the voltages and frequency of the microgrid after the small transient time, as Fig. 7(a) and Fig. 8(a) show. As in the previous case studies, the real power sharing is still satisfied.

To further evaluate the performance of the distributed secondary controller under microgrid structure reconfiguration, at \( t = 3.5 \) s, we open switch S and change the configuration of the microgrid system to a radial distribution network.

### Table 1

<table>
<thead>
<tr>
<th>Parameters of the Microgrid System</th>
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<tbody>
<tr>
<td><strong>DG #1 &amp; 2</strong></td>
</tr>
<tr>
<td>( m_P )</td>
</tr>
<tr>
<td>( n_Q )</td>
</tr>
<tr>
<td>( Z_c )</td>
</tr>
<tr>
<td>( L_{f1}, L_{f2} )</td>
</tr>
<tr>
<td>( R_{f1}, R_{f2} )</td>
</tr>
<tr>
<td>( C_f )</td>
</tr>
<tr>
<td>( K_{PV} )</td>
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<td>( K_{IV} )</td>
</tr>
<tr>
<td>( K_{PC} )</td>
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<td>( K_{IC} )</td>
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### Table 8

<table>
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<tr>
<th>Study 1- DGs (a) frequency, (b) output real power, and (c) reactive power ratio.</th>
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<tr>
<td><strong>Fig. 7.</strong> Study 1- DGs output (a) voltage magnitude, (b) reactive power with secondary control, and (c) reactive power without secondary control.</td>
</tr>
<tr>
<td><strong>Fig. 8.</strong> Study 1- DGs (a) frequency, (b) output real power, and (c) reactive power ratio.</td>
</tr>
<tr>
<td><strong>Fig. 8(e).</strong> Study 1- DGs (a) frequency, (b) output real power, and (c) reactive power ratio.</td>
</tr>
</tbody>
</table>
As Fig. 7(a) and Fig. 8(a) show, the distributed secondary controller robustly regulates the microgrid's voltages and frequency after the reconfiguration of the understudy microgrid.

Fig. 8(b) depicts the real power of DGs. As this figure shows, the real power sharing among the DGs is accurately performed according to the droop coefficients of DGs in various load changes and microgrid reconfiguration, while the proposed secondary controller restores the islanded microgrid's voltages and frequency. Fig. 7(b) shows the reactive power of DGs. Because of the microgrid lines impedance, small mismatches among the reactive power of DGs are observed. Moreover, Fig. 7(c) shows the DGs reactive power without applying the proposed secondary control. In general, after the arrival of the secondary control, we cannot predict the accuracy of reactive power sharing. As Figs. 7(b) and (c) show, when we apply the secondary control, the error of reactive power sharing for some DGs increases, while it decreases for others.

Moreover, Fig. 9 depicts the microgrid frequency and voltage magnitude of Bus 1. As this figure shows, the proposed voltage and frequency restoration can effectively compensate for the microgrid's frequency and voltage deviations (which the primary controller imposes) with uncertain parameters.

B. Study 2: Communication Network Changes

To verify the performance of the proposed protocols under communication network changes, the protocols are implemented through the communication network shown in Fig. 10. Control parameters are selected in the same manner as in Study 1. As Figs. 11, 12, and 13 show, the proposed secondary control returns all the voltage and frequency terms to their reference values in a finite time (less than 0.1 s). Compared to the simulation results of Study 1, Figs. 11, 12, and 13 show that the proposed secondary control works appropriately under communication network changes.

C. Study 3: Plug-and-Play Capability

In this study, the plug-and-play capability of the proposed method is tested by disconnecting DG unit 4 at $t = 2$ s, and reconnecting it at $t = 2.5$ s. Note that DGs have already reached their steady state before the plug out of DG4. Control parameters are selected in the same manner as in Study 1, and the results are shown in Figs. 14, 15, and 16. As Fig. 15(b) shows, once DG4 is disconnected from the microgrid, the remaining DGs produce more power to compensate for the amount of power previously generated by DG4. As a result,
Fig. 13. Study 2- (a) and (b) the microgrid frequency and voltage magnitude of Bus 1.

there are increases in the power ratios of the remaining DGs, whereas those of DG4 drop to zero during the plug out, as shown in Fig. 15(c). Moreover, at $t = 2.5$ s, the synchronization process is activated and the seamless plug in of DG4 into the microgrid is achieved. Moreover, as seen in Figs. 14, 15(a) and (c), despite plug out and plug in operations of DG4, aside from some transients, the proposed controllers maintain accurate power sharing and frequency and voltage regulation before, during, and after the plug-and-play procedure. Therefore, the ability of the proposed control method to meet the requirement of plug-and-play operation is verified. In addition, the microgrid frequency and voltage magnitude of Bus 1 remain well regulated despite the connection/disconnection of DG4.

Fig. 14. Study 3- DGs output voltage magnitude.


As previously stated in the introduction, a few studies solve both voltage and frequency restoration problems in a distributed manner, especially in a finite time [10], [29], [30]. Therefore, we compare the proposed voltage controller (28) with the distributed voltage controller in [12], and the proposed frequency controller (36) with the distributed frequency controller in [11]. We note that both controllers in [11] and [12] are asymptotic controllers that guarantee the global asymptotic stability of the system, whereas our proposed method reaches consensus in a finite time. Figs. 17 and 18 show the simulation results of Study 1 when the distributed voltage control in [12] and the distributed frequency controller in [11] are used to restore the microgrid’s output voltage magnitudes and frequency. The control gains are set the same in both our proposed controllers and the controllers in [11] and [12]. For the sake of simplicity, only frequency and voltage responses of DG1 are depicted. Figs. 17 and 18 verify the following results: 1) despite our proposed robust and adaptive protocols
being fully independent of the DG parameter, they do not deteriorate the synchronization of DGs voltage magnitude and frequency; 2) unlike distributed control methods in [11] and [12], our proposed protocols rapidly reach consensus and show more accurate robust performance for controller activation, structure reconfiguration, and load changes; 3) by increasing protocols gains, conventional methods may obtain the same performance as our method. However, the proposed method yields better disturbance rejection properties in the structure reconfiguration and load changes than conventional methods; and 4) unlike existing methods, we model DGs unmodeled dynamics, including unknown disturbances and uncertainties \( (u_i) \) and show how the proposed method effectively rejects such disturbances in the understudy system. However, the existing distributed controllers may excite the high-frequency dynamics neglected in modeling.

Fig. 17. Study 4- performance comparison of the proposed secondary voltage control and the one presented in [12].

VIII. CONCLUSION

This paper proposes a fully distributed and robust finite-time secondary control for both voltage and frequency restoration of an islanded microgrid while demonstrating accurate real power sharing among DGs. The microgrid is designated as a multi-agent system, in which DGs communicate with one another through an undirected sparse communication network. Unlike a central control structure, the proposed controller uses only the information of some neighboring DGs that can communicate with one another through a sparse communication undirected network. Therefore, the controller increases system reliability and eliminates the need for a central computing and communication unit. Not only does the controller consider the coupling between the frequency and voltage control loop, but it is also robust with respect to the parametric uncertainties, unmodeled dynamics, and disturbances in the microgrid system. We remark that the use of a new robust finite-time secondary control for both voltage and frequency restoration of an islanded microgrid system has not been proposed or investigated before. To verify the effectiveness of the proposed method, we carried out various simulation studies in MATLAB/SimPowerSystems software environment. The simulation results verify that the proposed strategy

is robust to the reconfiguration of the microgrid test system;

- enables the system with plug-and-play capability (i.e., the possibility of adding/removing a DG, line, and/or load without distorting the stability of the microgrid); and
- robustly restores the voltage and frequency of the overall system to their reference values within finite time in the presence of the parametric uncertainties, unmodeled dynamics, and disturbances in the microgrid system, while maintaining good real power sharing accuracy.

REFERENCES


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