Nonminimum Phase Channel Deconvolution Using the Complex Cepstrum of the Cyclic Autocorrelation

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Abstract—New discrete time blind deconvolution methods are proposed for nonminimum phase linear channels driven by cyclostationary inputs. The methods rely exclusively on second-order statistics and do not impose any constraints on the distribution of the channel input as in the case of methods based on higher-order statistics. The output of the channel is fractionally sampled and then the complex cepstrum of the cyclic autocorrelation is obtained. It is shown that this complex cepstrum preserves nonminimum phase information and thus the identification of nonminimum phase channels is possible. Practical constraints in the implementation of the methods and channel identifiability conditions are discussed. The applicability of the methods to both channel identification and fractionally spaced linear and DFE equalization is described and verified by means of computer simulations.

1. INTRODUCTION

The blind deconvolution problem of separating two unknown or partially known signals that have been convolved arises in speech/image processing [1], [2], seismic signal processing [2], [3], channel identification/equalization in digital communications [4]-[9], and many other applications. The situation considered in this paper is similar to the digital communications systems, where a linear channel (system) is driven by a cyclostationary random process signal. It is assumed that the channel characteristics are not known and that only partial information about the input signal is available, i.e., the cyclic period and its distribution.

Nonstochastic, time domain based approaches to this problem have been known for some time [1], [2]. One of the earliest approaches is that of homomorphic filtering that is based on separating the complex cepstrum of the input signal from that of the channel impulse response. However, most of these techniques require that the signals are easily distinguishable in the time or cepstrum domains. Also, they are very sensitive to the observation noise and delays introduced by the channel [2]. Recently, sequential methods have been proposed as an alternative but the research towards this direction is far from being complete.

On the other hand, there is a large number of discrete time stochastic approaches proposed in the literature. In most of them the input cyclostationary signal has been treated as a stationary random process. This has been the result of synchronously sampling (i.e., once per cyclic period) the observed output of the communication channel. However, synchronous sampling does not preserve the inherent cyclostationary properties of the input and observed signals. Then, by assuming that the channel is minimum phase or maximum phase, exclusively, the traditional linear prediction methods which are based solely on the second-order statistics (SOS) of the channel output can be applied to identify the channel characteristics. The input sequence is recovered through inverse filtering. By assuming that the channel is nonminimum phase, i.e., it has zeros inside and outside the unit circle, then the higher-order statistics (HOS) of the observed data need to be employed because of their ability to preserve nonminimum phase information. However, direct or indirect utilization of HOS requires either the minimization of nonlinear cost functions (Bussgang approaches), [4]-[7], or high computational complexity (Polyspectra approaches) [3], [8], [9]. Also, it is required that the input and observed sequences are not Gaussian processes. From the above we conclude that another approach to the blind deconvolution is to utilize the cyclostationary properties of digital communication signals. In few related papers that have appeared in the literature [10]-[14], it has been shown that the cyclic second-order statistics (cyclic autocorrelation and cyclic spectrum) do not share the limitations of the SOS (autocorrelation and power spectrum) and the blind identification of nonminimum phase channels is possible. This is true for most channels that are not strictly bandlimited to a bandwidth less than 1/T where T is the period of the cyclostationary signal [15]. Lately there is an increased interest towards this direction.

In this paper, we propose new discrete time blind deconvolution methods for linear channels driven by cyclostationary inputs. In particular, homomorphic approaches are applied to the cyclic autocorrelation of the fractionally-spaced sampled output of the channel. It is shown that the corresponding complex cepstrum preserves nonminimum phase information and thus the true channel characteristics can be recovered. The methods first identify the differential cepstrum coefficients and then the impulse responses of either the forward channel, a linear equalizer, or a decision feedback equalizer. Batch type solutions as well as an adaptive LMS realization are proposed and evaluated by means of computer simulations.

Compared with the approaches that utilize HOS under stationary signal assumptions, the methods that utilize
cyclic second-order statistics possess the following attractive characteristics:

i) Fewer data samples are required, and less complexity is encountered in the estimation of the cyclic second-order statistics.

ii) No restrictions on the distribution of the input data are imposed.

iii) They are insensitive (in theory) to stationary additive noise.

iv) Fractionally spaced sampling is less sensitive than synchronous sampling to timing errors.

Consequently, it is expected that the proposed methods could be the appropriate choice in many applications.

In Section II, the problem formulation is presented. In Section III, the complex cepstrum of the cyclic autocorrelation is derived. Practical constraints in the recovery of the complex cepstrum parameters are discussed. The applicability of the methods to blind identification and fractionally spaced equalization is presented in Section IV. Special considerations in the implementation of the examined methods are given in Sections V and VI. Performance evaluation and comparisons are given in Section VII and VIII. Conclusions are drawn in Section IX.

II. PROBLEM DEFINITION

Let us consider the situation depicted in Fig. 1. The discrete time sequence \( \{y(k)\} \) is written as

\[
y(k) = f(k) \ast x(k) + w(k) = \sum_{n=-\infty}^{\infty} a_n f(k-nL) + w(k) \tag{1}
\]

where \( \ast \) denotes linear convolution. The input data sequence \( x(k) = \sum_n a_n \delta(k-nL) \) where \( \{a_n\} \) is in general a complex, i.i.d., process and \( b(k) \) is the discrete delta function. For the moment we will assume that \( L \) is an integer. Thus, \( \{x(k)\} \) is a wide sense cyclostationary process, i.e., its mean \( m(k) = E\{x(k)\} \) and autocorrelation \( R(k, m) = E\{x(k+m)x^*(m)\} \) are periodic in \( k \) with period \( L \). Hence, \( \{x(k)\} \) is periodic and \( \{y(k)\} \) is a wide sense cyclostationary process, statistically independent from \( \{x(k)\} \). We assume that the system function \( F(z) \) of \( \{f(k)\} \) admits the factorization \( \prod_{i=1}^{r} \left( 1 - a_i z^{-1} \right) \), where \( A \) is a constant gain, \( r \) is constant delay in time, the \( I(z^{-1}) = \prod_{i=1}^{r} \left( 1 - a_i z^{-1} \right) \) is a minimum phase polynomial (i.e., \(|a_i| < 1 \), \(|b_i| < 1 \) and \( O(z) = \prod_{i=1}^{r} (1 - b_i z) \) is a maximum phase polynomial (i.e., \(|b_i| < 1 \)). Thus, the \( F(z = e^{j2\pi f}) \) exists and it is non-zero for all frequencies. The case of zeros on the unit circle will be considered later as a special case.

For example, (1) arises in digital communication links with data transmission rate \( f \) when the received signal after being demodulated is fractionally sampled at a rate \( \frac{1}{T} \). In other words, if \( y(t) = \sum_{n=-\infty}^{\infty} a_n \cdot f(t-nT) + w(t) \) is the continuous time counterpart of (1), then, \( y(t) = \sum_{n=-\infty}^{\infty} a_n \cdot f(t-nL) \) is the discrete time counterpart of (1), then, \( y(t) = \sum_{n=-\infty}^{\infty} a_n \cdot f(t-nL) + w(t) \) represents the total equivalent linear channel between transmitter and receiver corresponding to the sampling instances. Similar situations arise in digital recording [1], digital monitoring [7] and other data transmission applications.

The objective of blind deconvolution is to identify the unknown impulse response \( \{f(k)\} \), within a constant gain factor and constant time delay, using a finite number of samples from \( \{y(k)\} \) and only partial information about \( \{x(k)\} \). Then, recover the input sequence \( \{x(k)\} \) or \( \{a_n\} \) from \( \{y(k)\} \) by means of inverse filtering.

III. THE COMPLEX CEPSTRUM OF THE CYCLIC SPECTRUM

A. Cepstrum Relations—Definitions

Let us define the cyclic autocorrelation, \( R_y^o(m) \), and the cyclic spectrum, \( S_y^o(z) \), of the sequence \( \{y(k)\} \) as follows [16], [18]

\[
R_y^o(m) = \frac{1}{L} \sum_{k=-L}^{L} E\{y(k+m)\} y^*(k) z^m \tag{3}
\]

\[
S_y^o(z) = \sum_{m} R_y^o(m) z^{-m} \tag{4}
\]

where \( z = e^{j2\pi f}, z_n = e^{-j2\pi n}, \) and \( \{L\} \) denotes a period of \( L \) samples. Note, that \( R_y^o(k, m) = E\{y(k+m)y^*(k)\} \) is the classical autocorrelation function which is periodic in \( k \) with period \( L \). Thus, \( R_y^o(m) \) are the Fourier series coefficients of the \( R_y(k, m) \) with \( \alpha = \frac{m}{L}, \) \( l \in Z \) where \( Z \) is the set of integer numbers. Then, based on the problem assumptions it is shown in Appendix A that

\[
S_y^o(z) = q_x \cdot F(z) \cdot F^*(z \cdot z_{\alpha}) + S_w(z), \quad \alpha \in Z \tag{5}
\]

\[
S_y^o(z) = q_x \cdot F(z) \cdot F^*(z \cdot z_n), \quad \alpha = \frac{l}{L}, \quad l \neq nL \tag{6}
\]

where \( l, n \in Z, q_x = \frac{1}{L} E\{|a|^2\} \) and \( S_w(z) \) is the power spectral density of the wide sense stationary noise. By replacing (2) into (5) we obtain

\[
S_y^o(z) = q_x \cdot |A|^2 \cdot z_{\alpha} \cdot I(z^{-1}) \cdot O(z \cdot I^*(z^{-1} z_{\alpha})) \cdot O^*(z z_{\alpha}), \quad \alpha = \frac{l}{L}, \quad l \neq nL, \quad l, n \in Z. \tag{7}
\]

Based on the problem assumptions, we conclude that for a given value of \( \alpha \) in the range above, the \( S_y^o(z) \) (i) is not equal to zero or infinity at any frequency and (ii) contains
no linear phase components. Thus, \( R^0_p(m) \) has a well defined complex cepstrum \([1],[2]\), defined as
\[
c^0_p(m) = Z^{-1}[C^0_p(z)]
\]
(8)

where \( Z^{-1} \) denotes inverse Fourier transform, and
\[
C^0_p(z) = \ln \left\{ S^0_p(z) \right\}
\]
\[
= \ln(q_x \cdot |A|^2) - r \ln z_o + \sum_{i=1}^{L_1} \ln(1 - a_i z^{-1})
- \sum_{i=1}^{L_2} \ln(1 - b_i z^{-1}) + \sum_{i=1}^{L_2} \ln(1 - b_i z)
+ \sum_{i=1}^{L_1} \ln \left( 1 - a_i^* z^{-1} \right) - \sum_{i=1}^{L_3} \ln \left( 1 - c_i z^* z^{-1} \right)
+ \sum_{i=1}^{L_4} \ln \left( 1 - b_i^* z^{-1} z^{-1} \right)
\]
(9)

using the series expansions \( \ln(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k}, \quad |x| < 1 \)
and by defining
\[
A(k) = \sum_{i=1}^{L_1} a_i^k - \sum_{i=1}^{L_3} c_i^k, \quad B(k) = \sum_{i=1}^{L_2} b_i^k, \quad k = 1, 2, \ldots
\]
then
\[
C^0_p(z) = \ln(q_x \cdot |A|^2) - r \ln z_o + \sum_{k=1}^{\infty} \frac{-A(k)z^{-k}}{k}
- \sum_{k=1}^{\infty} \frac{B(k)z^{-k}}{k}
\]
(10)

and thus
\[
c^0_p(m) = \begin{cases} 
\ln(q_x \cdot |A|^2) - r \ln z_o, & m = 0 \\
\frac{-A(m)}{m} - \frac{B(m)}{m}, & m > 0
\end{cases}
\]
(12)

The \( \{A(k)\} \) and \( \{B(k)\} \) are the differential cepstrum parameters and contain the minimum phase and maximum phase information of the channel \( \{f(k)\} \), respectively \([3],[19]\).

From the problem formulation it is straightforward that the \( C^0_p(z) \) is analytic in its region of convergence. Thus, by taking the derivative with respect to \( z \), we obtain
\[
z \cdot \frac{dC^0_p(z)}{dz} = z \left[ \frac{d \ln \left\{ S^0_p(z) \right\}}{dz} \right] = \frac{d}{dz} \ln \left\{ S^0_p(z) \right\} \left[ \frac{dS^0_p(z)}{dz} \right]
\]
(13)

or
\[
m \cdot c^0_p(m) = Z^{-1} \left[ \frac{Z[m \cdot R^0_p(m)]}{S^0_p(z)} \right].
\]
(14)

Alternatively, from (13)
\[
S^0_p(z) \left[ \frac{dC^0_p(z)}{dz} \right] = z \cdot \frac{dS^0_p(z)}{dz}
\Rightarrow R^0_p(m) \ast \left[ -m c^0_p(m) \right] = -m R^0_p(m)
\]
(15)

or
\[
\sum_{k=-\infty}^{\infty} \left[ -m c^0_p(k) \right] R^0_p(m-k) = -m R^0_p(m).
\]
(16)

Then, by replacing (12) into (16), we obtain
\[
\sum_{k=1}^{\infty} \left[ (A(k) + B^*(k)z^{-k})R^0_p(m-k) \right] - \left( A^*(k)z^k + B(k) \right) R^0_p(m+k) = -m R^0_p(m).
\]
(17)

Remark: If in (3), due to the sampling operation, we replace \( y(k) \) by \( y(k + t_0) \) where \( t_0 \) is a constant delay, then \( R^0_p(m) \) should be replaced by \( R^0_p(m) e^{-j2\pi m t_0} \), everywhere. The term \( e^{-j2\pi m t_0} \) is independent of \( m \) and thus it is absorbed into \( c^0_p(0) \). However, the zero cepstral lag is not taken into account in the methods described next and thus we can assume \( t_0 = 0 \).

B. Recovery of the Complex Cepstrum

From (10), the differential cepstrum parameters decay exponentially at least as fast as
\[
|A(I)| < C_1 \cdot a^I, \quad |B(J)| < C_2 \cdot b^J, \quad 0 < I, J < \infty
\]
(18)

where \( \max(|a_k|,|c_k|) < a < 1, \max(|b_k|) < b < 1 \) and \( C_1, C_2 \) are constants. Thus, even though they are of infinite length, we can always choose two positive integers \( p \) and \( q \) to be sufficiently large, so that \( \{A(I)\} < C \) and \( \{B(J)\} < C \) for \( I > p \) and \( J > q \) \([3],[8]\). For example \( C = 10^{-4} \). Then, we can place (int denotes integer part)
\[
A(I) \simeq 0, \quad I > p = \text{int} \left[ \frac{\ln(C)}{\ln(a)} \right]
\]
\[
B(J) \simeq 0, \quad J > q = \text{int} \left[ \frac{\ln(C)}{\ln(b)} \right].
\]
(19)

Then, given \( \{R^0_p(m)\}, m = 0 \pm 1, \pm 2, \ldots \pm M, \) we proceed as follows.

1) \FFT Based Approach: Let \( Q > \max[2M+1,2 \max(p,q)] \). Then, from (14) by applying \( Q \)-FFT operations
\[
S_1(k) = \sum_{m=-M}^{M} [m R^0_p(m)] e^{-j2\pi \frac{k}{Q} m}
\]
\[
S_2(k) = \sum_{m=-M}^{M} R^0_p(m) e^{-j2\pi \frac{k}{Q} m}
\]
\[
S_3(m) = mc^0_p(m) = \sum_{k=-\frac{Q}{2}+1}^{\frac{Q}{2}} S_1(k) e^{j2\pi \frac{km}{Q}}
\]
(20)
Then, the differential cepstrum parameters of the cyclic autocorrelation are
\[
\begin{align*}
\{ C_\alpha(m) \} = A(m) + B^*(m)z_\alpha^{-m} = -S_\alpha(m), \\
\{ D_\alpha(m) \} = A^*(m)z_\alpha^{m} + B(m) = S_\alpha(-m).
\end{align*}
\]
\[ m = 1, 2, \ldots, Q - 1. \tag{21} \]

2) Least-Squares Approach: From (17) we obtain the approximate equation
\[
\sum_{k=1}^{\max(p, q)} \left[ (A(k) + B^*(k)z_\alpha^{-k})R_y^\alpha(m - k) - (A^*(k)z_\alpha^{k} + B^2(k))R_y^\alpha(m + k) \right] = -mR_y^\alpha(m).
\]

Then, we can define \( w \geq \max(p, q) \) and repeat (22) for \( m = -w, \ldots, -1, 1, \ldots, w \) to form the linear over-determined system of equations
\[
R_\alpha \cdot \mathbf{a} = r_\alpha
\]
where \( R_\alpha \) is \( 2w \times 2\max(p, q) \) matrix, and \( r_\alpha \) is \( 2w \times 1 \) vector with entries from the cyclic auto-correlation lags, and \( \mathbf{a} = \{ C_\alpha(1), \ldots, C_\alpha(\max(p, q)), D_\alpha(1), \ldots, D_\alpha(\max(p, q)) \} \) is \( 2\max(p, q) \times 1 \) vector with entries the unknown differential cepstrum parameters
\[
\begin{align*}
\{ C_\alpha(k) = A(k) + B^*(k)z_\alpha^{-k} \}, \\
\{ D_\alpha(k) = A^*(k)z_\alpha^{k} + B(k) \},
\end{align*}
\]
\[ k = 1, 2, \ldots, \max(p, q). \tag{24} \]

The least squares solution to (23) is
\[
\mathbf{a} = \left[ R_\alpha^H R_\alpha \right]^{-1} R_\alpha^H r_\alpha
\]
where \( H \) denotes conjugate transpose. The above solution exists and is unique provided that the over-determined matrix \( R_\alpha \) has linearly independent columns [21].

In practice, the cyclic autocorrelation lags \( \{ R_y^\alpha(m) \} \) in \( S_1(k), S_2(k)R_\alpha, \) and \( r_\alpha \) are replaced by their sample estimates \( \hat{R}_y^\alpha(m) \). Assuming that \( \{ y(k) \}, k = 1, \ldots, N \) are samples from an autocorrelation cycloergodic random process, then [18]
\[
\hat{R}_y^\alpha(m) = \frac{1}{N} \sum_{k=S_1}^{S_2} y(k + m)y^*(k)e^{-j2\pi nk}
\]
where \( S_1 = \max(1, -m) \) and \( S_2 = \min(N, N - m) \).

In many applications one could apply either frequency smoothing in the estimation of \( S_1(k) \) and \( S_2(k) \) or segmentation of the data and averaging over the estimates of \( \hat{R}_y^\alpha(m) \) for each segment to reduce the variance of estimation [3]. Also, methods other than transforming the cyclic autocorrelation for estimating the cyclic spectrum can be found in [16], [17].

C. Recovery of \( \{ A(k) \} \) and \( \{ B(k) \} \)

The objective is to investigate under what conditions the cepstrum parameters \( \{ A(k) \} \) and \( \{ B(k) \} \) can be recovered from (21) or (24). It is easy to see that recovery of the parameters is not possible when simultaneously \( B(k) = B^*(k)z_\alpha^{-k} \) and \( A(k) = A^*(k)z_\alpha^{k} \). Such a case occurs when all poles and zeros of the channel are spaced radially from 0 to \( 2\pi \) with an angle of \( \frac{\pi}{Q} \). Nevertheless, this situation rarely occurs and will not be met for different values of \( L \).

Assuming that the above condition is not met and that the channel impulse response \( \{ f(k) \} \) is complex, then, the \( \{ A(k) \} \) and \( \{ B(k) \} \) are complex as well. Thus, (21) or (24) can be solved with respect to \( \{ A(k) \} \) and \( \{ B(k) \} \) provided that the determinant
\[
D = \left| \begin{array}{cc}
z_\alpha^{-k} & 1 \\
1 & z_\alpha^k
\end{array} \right| \neq 0
\]
\[
\Rightarrow z_\alpha^{2k} \neq 1 \Rightarrow e^{j2\pi nk} \neq 1
\]
\[
\Rightarrow 2nk \neq \nu \Rightarrow \frac{2nk}{L} \neq \nu, \quad k, \nu \in Z.
\]
\[ \tag{27} \]

On the other hand, assuming that \( \{ f(k) \} \) is real, \( \{ A(k) \} \) and \( \{ B(k) \} \) are real as well. Then, the \( C_s(k) \) and \( D_s(k) \) in (21) and (24) are linearly dependent and thus \( D = 0 \) for all \( k \).

In such case, we can obtain two sets of equations with a different value of \( \alpha \) (same \( L \) but different \( l \)). For example, given \( \{ R_y^{s/L_1}(m) \} \), and \( \{ R_y^{s/L_4}(m) \} \) we can apply either (20) or (22) twice and obtain the equations
\[
\begin{align*}
\{ C_{s_1}(k) = A(k) + B(k)z_\alpha^{-k} \}, \\
\{ C_{s_4}(k) = A(k) + B(k)z_\alpha^{k} \},
\end{align*}
\]
\[ k = 1, 2, \ldots, \max(p, q). \tag{28} \]

We easily find that (28) has a nonzero determinant provided that
\[
eq e^{j2\pi \nu k} = \frac{(l_1 - l_2)k}{L} \neq \nu, \quad l_1, l_2, k, \nu \in Z
\]
\[ \tag{29} \]

Let us consider a few special cases for (27) and (29):
1) \( L = 1 \) (case of stationarity). In this case neither (27) or (29) is satisfied for any \( k \) and thus \( \{ A(k) \} \) and \( \{ B(k) \} \) can not be recovered.
2) \( l = 0, \pm L, \pm 2L, \ldots, \) or \( l_1 - l_2 = 0, \pm L, \pm 2L, \ldots \) for a complex and a real channel, respectively. The \( \{ A(k) \} \) and \( \{ B(k) \} \) can only be recovered for odd values of \( k \).
3) \( l = 1, L = 4, \) or \( l_1 - l_2 = 1, L = 2. \) Then, the \( \{ A(k) \} \) and \( \{ B(k) \} \) can only be recovered for odd values of \( k \).
4) \( 0 < \| l_1 - l_2 \| \leq L, L \) is odd number, \( L \neq 1. \) Then, the \( \{ A(k) \} \) and \( \{ B(k) \} \) can be recovered for all values of \( k \) except \( k = L, 2L, 3L, \ldots \).

It can be easily shown that case 4 represents the best case scenario and for this reason it is considered in the sequel.

Based on the choice for \( p, q \) and \( L \) the following cases might occur:
1) Let \( L \geq \max(p, q) \). This condition may be satisfied if the zeros and poles of the channel are far from the unit circle. Then, the \( A(k), B(k), k = 1, 2, \cdots, \max(p, q) \) can be obtained directly by solving the linear system (21) or (24).

2) Let \( q \leq L < p \). This condition could be satisfied for channels with dominant minimum phase information. Then, the \( A(k), B(k), k = 1, 2, \cdots, L \) can be obtained by solving the linear system (21) or (24), while the \( A(k) = C_{\alpha}(k), B(k) = 0, k = L + 1, \cdots, p \).

3) Let \( p \leq L < q \). This condition could be satisfied for channels with dominant maximum phase information. Then, the \( A(k), B(k), k = 1, 2, \cdots, L \) can be obtained by solving the linear system (21) or (24), while the \( A(k) = 0, B(k) = D_{\alpha}(k), k = L + 1, \cdots, q \).

4) Let \( L \leq \min(p, q) \). Then, the \( A(k), B(k) \) can be recovered from (21) or (24) for all \( k \in [1, \cdots, \max(p, q)] \) except for those values of \( k \) that are multiples of \( L \). Assuming that \( L \) is sufficiently high, a variety of interpolation methods could be employed to provide estimates of the missing values. For example, the Newton–Aitken iteration method with divided differences is described in Appendix B and utilized in the simulations.

We conclude that except from some pathological cases non-minimum phase channel information is preserved and can be recovered in the cyclic power spectrum (or cyclic autocorrelation) domain. It is the lack of conjugate symmetry in this domain (see (6) for \( \alpha \neq 0, \pm 1, \pm 2, \cdots \)) that allows the separation of the maximum phase from the minimum phase components of the channel. For example, in Fig. 2 the channels with system functions \( F_1(z) \neq F_2(z) \) have \( S^{0,2}_1(z) = S^{0,2}_2(z) \) but \( S^{2,4}_1(z) \neq S^{2,4}_2(z) \). The cyclo-cepstrum approach, by means of the logarithmic transformation, provides efficient linear solutions for separating the two components.

IV. BLIND CHANNEL IDENTIFICATION AND EQUALIZATION

A. Channel Reconstruction

Given the \( \{ A(k) \} \) and \( \{ B(k) \} \) for \( k = 1, \cdots, \max(p, q) \), we can recover the minimum and maximum phase characteristics of the channel impulse response as follows. Let \( i(k) = Z^{-1} [I(z^{-1})] \), \( i(0) = 1 \) and \( o(k) = Z^{-1} [O(z)] \), \( o(0) = 1 \). Then, [2], [3], [8]

\[
i(k) = -\frac{1}{k} \sum_{n=2}^{k-1} A(n-1) \cdot i(k-n+1), \quad k = 1, \cdots, N_1
\]

\[
o(k) = \frac{1}{k} \sum_{n=k+1}^{q} B(1-n) \cdot o(k-n+1), \quad k = -1, \cdots, -N_2
\]

where \( N_1 \) and \( N_2 \) are chosen arbitrarily. Finally, ("norm" refers to gain normalization)

\[
f_{\text{norm}}(k) = i(k) \ast o(k), \quad k = -N_2, \cdots, 0, \cdots, N_1.
\]
The gain as well as the phase of the equalizers output must be properly adjusted prior to threshold decoding by means of blind gain and phase tracking algorithms such as those described in Appendix C [9]. Since the rate of the desired data \( \{a_n\} \) is \( \frac{1}{2} \) the output of the linear equalizer and the all-pass filter in the DFE are sampled at a rate \( \frac{1}{2} \) (i.e., every \( L \) samples) before entering the decision device, as indicated in Fig. 3.

C. Adaptive Realization

Let \( \hat{R}_a^\alpha(m) \) be an estimate of \( R_a^\alpha(m) \) at iteration \( n \). Then, from (26) we get

\[
\hat{R}_a^\alpha(n+1) = \frac{n-1}{n} \hat{R}_a^\alpha(n-1)(m) + \frac{1}{n} y(S_\alpha^2 + m) y^*(S_\alpha^2) e^{-j2\pi \alpha S_\alpha^2},
\]

where \( S_\alpha^2 = \min(n, n - m) \) is as before. Alternatively [20]

\[
\hat{R}_a^\alpha(n)(m) = \rho \cdot \hat{R}_a^\alpha(n-1)(m) + y(S_\alpha^2 + m)y^*(S_\alpha^2)e^{-j2\pi \alpha S_\alpha^2},
\]

where \( 0 < \rho \leq 1 \), for example \( \rho = 0.99 \). Then, at iteration \( n = 0, 1, 2, \cdots \) the LMS adaptation rule is [8]

\[
\tilde{a}(n+1) = \tilde{a}(n) + \mu(n) \cdot \tilde{R}_a(n) \cdot e(n), \quad \tilde{a}(0) = 0 \tag{42}
\]

where \( e(n) = \tilde{r}_a(n) - \hat{R}_a(n) \cdot \tilde{a}(n) \) is the error vector, \( \tilde{R}_a(n) \), \( \hat{R}_a(n) \), and \( \tilde{a}(n) \) are the estimates of \( R_a(n) \), \( R_a(n) \), and \( a \) at iteration \( n \), respectively, and the step size \( \mu(n) \) is such that \( 0 < \mu(n) < 2/\text{trace}(R_a^2(n)R_a(n)) \) to satisfy stability requirements. The underlying cost function being minimized above is quadratic and thus, convergence to a global solution is guaranteed.

V. TREATING ZEROS ON THE UNIT CIRCLE

From (10) we observe that the closer the zeros and poles of \( F(z) \) get to the unit circle the larger \( p \) and \( q \) become. Assuming that the \( F(z) \) has zeros on the unit circle then, \( p \to \infty \) or \( q \to \infty \) and then the estimation of \( \{A(k)\} \) and \( \{B(k)\} \) by (20) or (22) becomes practically impossible. To overcome this, let us consider the weighted cyclic autocorrelation defined as

\[
R_{y,d}^a(m) = d^m \cdot R_y^a(m), \quad m = 0, \pm 1, \pm 2, \cdots, M \tag{43}
\]

where \( d \) is a real and positive number different than zero. Then, from (4) we obtain [2], [22]

\[
S_{y,d}^a(z) = S_y^a(z) \cdot d^{-1}. \tag{44}
\]

Thus, all zeros and poles of \( S_y^a(z) \) are moved inwards if \( d < 1 \) or outwards if \( d > 1 \) and provided that \( d \) is properly chosen, then the \( S_{y,d}^a(z) \) does not have any zeros or poles on the unit circle. Next we assume that \( d < 1 \) with no loss of generality.

Note that zeros on the unit circle become minimum phase when they are moved inside the unit circle. Assuming that \( F(z) \) has \( P \) zeros on the unit circle, then, it is easy to show that the (20) and (22) should be modified as follows

\[
S_{y,d}^a(m) = m S_{y,d}^a(m) - P \delta(0)
\]

\[
= \sum_{k=-Q}^{Q} \frac{S_{x,d}(k)}{S_{y,d}(k)} e^{2\pi \frac{k}{Q} m},
\]

\[
k, m = -Q + 1, \cdots, 0, \cdots, Q
\]

and (note that \( P = S_{y,d}(0) \))

\[
\sum_{k=1}^{\infty} [C_{a,d}(k)R_{y,d}^a(m-k) - D_{a,d}(k)R_{y,d}^a(m+k)]
\]

\[
= -(P + m) R_{y,d}^a(m) \tag{46}
\]
where \( S_{1,d}(k) \) and \( S_{2,d}(k) \) are defined in terms of \( R_{pd}(m) \), and

\[
\begin{align*}
S_{3,d}(z^{-k}) &= C_{o,d}(k)
\end{align*}
\]

\[
\begin{align*}
&= d^{k}A_{in}(k) + \sum_{i=1}^{P} e^{j\theta_{i}}[1 + z_{o}^{-k}]
\end{align*}
\]

\[
\begin{align*}
&+ B_{out}(k)z_{o}^{-k}
\end{align*}
\]

\[
\begin{align*}
S_{4,d}(k) &= D_{o,d}(k) = d^{-k}A_{m}(k) + B_{out}(k)
\end{align*}
\]

The \( \{A_{in}(k)\} \), \( \{B_{out}(k)\} \) and \( \sum_{i=1}^{P} e^{j\theta_{i}} \) denote the differential cepstrum coefficients formed from zeros and poles inside the unit circle, the zeros outside the unit circle, and the zeros on the unit circle, respectively. Thus, \( d \) controls the degree of exponential decay for the differential cepstrum parameters and the values of \( p_{d}, q_{d} \). There should be caution so that zeros that are outside the unit circle do not move inside the unit circle. Then, the following procedure could be applied to estimate the \( \{A_{in}(k)\} \) and \( \{B_{out}(k)\} \).

ii) Choose \( d \) and two different values of \( \alpha_{1} \), i.e., \( \alpha_{1} = \alpha_{2} = 1/2 \), and estimate the weighted differential cepstrum coefficients \( C_{1,1^2}(k), D_{1,1^2}(k), C_{2,1^2}(k), D_{2,1^2}(k), k = 1,2,\ldots,max(p_{d},q_{d}) \) via the FFT or least-squares method.

iii) From the equations for \( D_{1,1^2}(k) \) and \( D_{2,1^2}(k) \) obtain the \( A_{in}(k) \) and \( B_{out}(k) \), \( k = 1,2,\ldots,max(p_{d},q_{d}) \).

iv) Finally, \( A(k) = A_{in}(k) + \sum_{i=1}^{P} e^{j\theta_{i}}, k = 1,2,\ldots,p_{d} \) and \( B(k) = B_{out}, k = 1,\ldots,q_{d} \).

The following remarks are in order:

1) As an alternative to step (iv) above, we can consider placing \( A(k) = A_{in}(k), k = 1,\ldots,p_{d} \) and \( B(k) = B_{out}(k) + \sum_{i=1}^{P} e^{j\theta_{i}}, k = 1,\ldots,q_{d} \). The corresponding impulse responses differ only by a constant delay and gain factors.

2) In relations (45) and (46) the value \( P \) appears which indicates the number of zeros on the unit circle. Since \( P = S_{3,0}(0) \) it does not have any affect on the implementation of the FFT method. However, \( P \) must be incorporated in the least-squares method. In many cases the channel \( h(k) = h_{1}(k) + h_{2}(k) \) where the \( h_{1}(k) \) is not known but the \( h_{2}(k) \) is a known low-pass filter in the transmitter or receiver. While it is unlikely that \( h_{1}(k) \) has zeros on the unit circle, the \( h_{2}(k) \) is usually designed to have zeros on the unit circle. In this case \( P \) may be known a priori and used with the least-squares method. However, when \( P \) is unknown the FFT method should be utilized.

3) In (43), the function \( d^{m} \) that weights \( R_{pd}(m) \) affects the magnitude of estimation errors as well. For large negative values of \( m \) the magnification of estimation errors may become significant and affect seriously the performance of the methods. Thus, the length \( M \) of estimated cyclic autocorrelation lags in (43) must be carefully chosen not to be larger than the “effective length” of the cyclic correlation. Similar considerations should be made in choosing the differential cepstrum truncating parameters \( p_{d} \) and \( q_{d} \).

4) In addition to moving zeros away from the unit circle the above procedure can be applied in order to transform the channel to either dominantly minimum phase or dominantly maximum phase to satisfy cases 2) or 3) of Section III-C, respectively.

VI. IDENTIFICATION USING RATIONAL FRACTIONAL SAMPLING

Consider (1) with \( L = \frac{L}{\nu} \), be not an integer but a rational number. Then, the quantity \( E\{y(k+m)y^{*}(k)\} \) is periodic in \( k \) with period \( K = n \cdot L \) where, \( n \) and \( K \) are co-prime integers. In that case we define

\[
R_{pd}(m) = \frac{1}{K} \sum_{k=(K)} E\{y(k+m)y^{*}(k)\} z_{o}^{-k}.
\]

Following a procedure similar to that in Appendix A, it can be easily shown that relations (5) to (7) hold as well. Thus, the same algorithmic procedures in the identification of the channel apply. However, a potential advantage by considering noninteger \( L \) lies in the recovery of \( \{A(k)\} \) and \( \{B(k)\} \) from \( \{A_{n}(k)\} \) and \( \{D_{n}(k)\} \). This becomes clear, by observing that in the solution of (21) or (24) the corresponding determinant is nonzero for \( \frac{2k}{L} \neq \nu, l, k, \nu \in Z \) instead of \( \frac{2k}{L} \neq \nu, l, k, \nu \in Z \). Thus, if \( L \) is chosen properly, \( K \) becomes arbitrarily large and interpolation of nonrecoverable differential cepstrum parameters might not be necessary. For example, by choosing \( L = 3.3 \) (or \( L = 2.82 \)), which corresponds to a relatively small change in sampling rate, then \( K = 33 \) (or \( K = 141 \)) and only those \( \{A(k)\} \) and \( \{B(k)\} \) where \( k \) is multiple of 33 (or 141) need to be interpolated.

VII. PERFORMANCE EVALUATION

In this section we evaluate the performance of the methods by means of Monte Carlo computer simulations. Computational complexity issues and the potential advantages and limitations of the proposed approaches compare to existing ones are discussed.
A. Computer Experiment 1

We have assumed that $L = 3$, $\alpha = \frac{1}{3}$. The data symbols $\{a_n = a_n, + j a_n,\}$ take values from a 16-QAM signal constellation, i.e., the $\{a_n,\}$ and $\{a_n,\}$ are i.i.d. sequences independent from each other and each taking the values $\pm 1, \pm 3$ with equal probability. The channel is modeled as either of the nonminimum phase impulse responses shown in Figs. 4 and 6 with system function

$$F(z) = \frac{(1 - a_1 z^{-1})(1 - b_1 z)(1 - b_2 z)}{(1 - c_1 z^{-1})(1 - c_2 z^{-1})}$$

with $a_1 = -0.2$, $b_1 = 0.7 (\sqrt{2} + j \frac{\sqrt{2}}{2})$, $b_2 = 0.5 (\sqrt{2} + j \frac{\sqrt{2}}{2})$, $c_1 = 0.6 (\sqrt{2} - j \frac{\sqrt{2}}{2})$, $c_2 = 0.6 (0.809 - j 0.5878)$, or $a_1 = -0.5$, $b_1 = 0.7 (\sqrt{2} - j \frac{\sqrt{2}}{2})$, $b_2 = 0.5 (\sqrt{2} + j \frac{\sqrt{2}}{2})$, $c_1 = 0.8 (\sqrt{2} + j \frac{\sqrt{2}}{2})$, and $c_2 = 0.8 (0.809 - j 0.5878)$, respectively. The second channel causes more severe distortion than the first channel since its zeros and poles are closer to the unit circle. To obtain $x(k)$ (see Fig. 1) two zeros were introduced between any two of the data symbols. The resulting sequence was then convolved with the channel impulse response. Additive white Gaussian noise was added with signal-to-noise ratio $\text{SNR} = 30 \text{dB}$. The SNR is defined as

$$10 \log_{10} \sum_{k=0}^{K-1} \frac{|s(k)|^2}{|e(k)|^2}$$

where $s(k) = f(k) + x(k)$. The estimated frequency characteristics of the channels obtained with the proposed least-squares approach are illustrated in Figs. 5 and 6.

Fig. 4. Impulse response and zero-pole location of channel 1.

Fig. 5. True and estimated frequency response of channel 1. Least squares approach with $L = 3$. $\alpha = 1/3$. $p = q = 8$. $w = 16$. (a) 128 symbols. (b) 512 symbols.

7. In all cases, the estimated magnitude and phase response from 50 independent realizations of the experiment, the mean of the estimated characteristics, and the true frequency characteristics of the channel have been drawn. Also, the mean and variance of the estimated differential cepstrum parameters are given in Tables I and II. We conclude that the proposed least squares approach was able to identify correctly both the magnitude and nonminimum phase characteristics of the channels. Notice that the performance was satisfactory with as few as 128 data symbols. As expected the variance of the estimation decreases as the number of data symbols employed increases. In all cases the differential cepstrum parameters $A(k)$ and $B(k)$ for $k = 3, 6, 9, \cdots$ were interpolated using the Newton-Aitken method with four estimated values symmetrically located around the interpolated one. Using more values in the interpolation would further improve the estimated characteristics.
B. Computer Experiment 2

In this experiment we have applied the proposed adaptive realization of the least-squares approach to carry out fractionally spaced linear and DFE equalization of channel 1. Two cases were considered. In the first case \( \{a_n\} \) takes values from a 16-QAM constellation and the SNR = 20 dB. In the second case \( \{a_n\} \) takes values from a 64-QAM signal constellation (where both \( \{a_{n,r}\} \) and \( \{a_{n,i}\} \) take the values \( \pm 1, \pm 3, \pm 5, \pm 7 \) with equal probability) and the SNR= 30 dB. In both cases \( L = 3 \) and \( \alpha = 1/3 \). In total 31 taps were utilized in both equalization schemes. The performance metrics considered were: (i) the mean square error at iteration \( n \), defined as \( \text{MSE}(n) = E[|\hat{a}_n - a_n|^2] \) where \( \hat{a}_n \) and \( a_n \) are the output of the equalizer and threshold decoding device, respectively, and (ii) the discrete eye patterns in the output of the equalizer. The MSE\((n)\) was obtained by averaging over 10 independent realizations of the experiment. The eye pattern at iteration \( n \) at the output of the equalizer was obtained by taking a number of points symmetrically located around \( n \) for all 10 realizations. The obtained results for cases 1 and 2 are illustrated in Figs. 8 and 9, respectively. The eye patterns in the input of the equalizers are completely closed due to the distortion introduced by the channel and noise. However, the performance metrics at the output of the equalizer clearly demonstrate that the proposed adaptive equalization algorithms were able to compensate for the channel distortion.

C. Computer Experiment 3

In this case, we assume that \( y(k) = h_L(k) * [h_1(k) * x(k) + w(k)] \). The \( h_1(k) \) is the impulse response of a nonminimum phase channel described by the linear difference equation

\[
h_1(k) = 0.6h_1(k - 1) - 0.36h_1(k - 2) + 0.49h_1(k + 2) - 1.0878h_1(k + 1) + 1.1986h_1(k) - 0.29h_1(k - 1).
\]
The $h_2(k)$ is the nonminimum phase impulse response of a low-pass filter, that is

$$h_2(k) = 0.9^k \cdot \frac{\sin(\pi k/L)}{\pi k/L} \cdot \frac{\cos(\beta \pi k/L)}{1 - (4\beta^2 k^2/L^2)^2},$$

for $k = -8, \ldots, 0, \ldots, 8$

with $\beta = 0.1$. The two impulse responses and the corresponding zero-pole diagrams are illustrated in Fig. 10. The $w(k)$ is white Gaussian noise, and $x(k) = \sum_n a_n h(k - nL)$ with $\{a_n\}$ taking values from a 16-QAM constellation and $L = 3$. Thus, according to (1) the total impulse response is $f(k) = h_1(k) * h_2(k)$ and the noise is Gaussian and colored by the spectral characteristics of $h_2(k)$. The noise power was adjusted so that SNR = 20 dB. DFE equalization of the $f(k)$ was carried out successfully with the adaptive least-squares approach as it is illustrated in Fig. 11. The eye diagram after 3600 iterations (1200 symbols) is open. The convergence speed is slower than that observed in experiment two, however, this is due to the significantly more severe distortion introduced by the channel $f(k)$ in this case. The FFT method was also applied and the estimated magnitude and phase responses of the $f(k)$ obtained from 50 independent realizations of the experiment are illustrated in Fig. 12. The mean and variance of the corresponding estimated differential cepstrum parameters are given in Table III. We observe that the characteristics of the magnitude and phase characteristics were estimated correctly. As expected higher bias and variance of estimation were exhibited in the stopband of the channel transfer function.

### Table I

**Mean and Variance of the First Six Estimated Differential Cepstrum Parameters of Channel 1 with the Least Squares Approach**

<table>
<thead>
<tr>
<th></th>
<th>True parameters</th>
<th>Estimated parameters of channel 1 (LS method, 50 realizations)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N=128$, $L=3$</td>
<td>$N=512$, $L=3$</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td>Real</td>
<td>Imag</td>
<td>Real</td>
</tr>
<tr>
<td>A(1)</td>
<td>-0.8564</td>
<td>-0.1669</td>
</tr>
<tr>
<td>A(2)</td>
<td>0.1087</td>
<td>0.0306</td>
</tr>
<tr>
<td>A(3)</td>
<td>0.2747</td>
<td>0.2054</td>
</tr>
<tr>
<td>A(4)</td>
<td>0.1712</td>
<td>0.1884</td>
</tr>
<tr>
<td>A(5)</td>
<td>0.0355</td>
<td>0.0673</td>
</tr>
<tr>
<td>A(6)</td>
<td>0.0088</td>
<td>0.0274</td>
</tr>
</tbody>
</table>

### Table II

**Mean and Variance of the First Six Estimated Differential Cepstrum Parameters of Channel 2 with the Least Squares Approach**

<table>
<thead>
<tr>
<th></th>
<th>True parameters</th>
<th>Estimated parameters of channel 2 (LS method, 50 realizations)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N=256$, $L=3$</td>
<td>$N=512$, $L=3$</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td>Real</td>
<td>Imag</td>
<td>Real</td>
</tr>
<tr>
<td>A(1)</td>
<td>-1.5472</td>
<td>-0.2225</td>
</tr>
<tr>
<td>A(2)</td>
<td>0.3722</td>
<td>0.0544</td>
</tr>
<tr>
<td>A(3)</td>
<td>0.5452</td>
<td>0.4869</td>
</tr>
<tr>
<td>A(4)</td>
<td>0.5806</td>
<td>0.5054</td>
</tr>
<tr>
<td>A(5)</td>
<td>0.1325</td>
<td>0.2837</td>
</tr>
<tr>
<td>A(6)</td>
<td>-0.0384</td>
<td>-0.1580</td>
</tr>
</tbody>
</table>

### D. Computer Experiment 4

In this experiment we investigate the performance of the FFT method with integer and noninteger values of $L$, i.e., $L = 3$ and $L = 3.3$. First the impulse response

$$h_2(k) = 0.96^k \cdot \frac{\sin(\pi k/L_1)}{\pi k/L_1} \cdot \frac{\cos(\beta \pi k/L_1)}{1 - (4\beta^2 k^2/L_1^2)^2},$$

for $k = -98, \ldots, 0, \ldots, 98$
with $p = 0.1$ and $L_1 = 33$, and the sequence $x(k) = \sum_{n} a_n e^{j(k - nL_1)}$ with $a_n$ taking values from a 16-QAM constellation were generated. The $h_1(k)$ and $w(k)$ were as in experiment 3. The observed data sequence was generated as in experiment 3 and then down-sampled by taking either every 11th sample (for $L = 3$), or every 10th sample (for $L = 3.3$). The equivalent channel impulse responses after down-sampling are indicated in Fig. 13 by $x$ and $a$, respectively.

The estimated magnitude and phase frequency responses of the two impulse responses are illustrated in Figs. 14 and 15, respectively. The mean and variance of the cepstrum parameters are given in Table IV. In both cases we have taken $\alpha = \pm 1/L$ (real channel), $N = 500$ symbols, and $Q = 64$. For $L = 3$ interpolation with the Newton-Aitken method was carried out. For $L = 3.3$ no interpolation was carried out since theoretically the first unrecoverable cepstrum parameter appears at $k = 33$. From the results we conclude that in both cases the characteristics of the channels were identified. However, the performance with $L = 3$ was sufficiently better than that with $L = 3.3$.

E. Computer Experiment 5

We examine the feasibility of identifying channels with zeros on the unit circle by employing the procedures of Section V. The simulation setup is similar to that of experiment 3, i.e., $y(k) = h_2(k) * [h_1(k) * x(k) + w(k)]$. We consider two cases:
Case 1: The $h_1(k)$ is as in experiment 3, and

$$h_2(k) = \frac{\sin(\pi k/L)}{\pi k/L} \cdot \frac{\cos(\beta\pi k/L)}{1 - (4\beta^2 k^2/L^2)},$$

$k = -10, \cdots, 0, \cdots, 10$

with $L = 3$ and $\beta = 0.1$. The combined channel $f(k) = h_1(k) * h_2(k)$ is infinite length and nonminimum phase with 14 zeros on the unit circle.

Case 2: The $h_3(k) = \delta(k) + 0.8\delta(k-1) - 0.4\delta(k-3)$, and

$$h_2(k) = \frac{\sin(\pi k/L)}{\pi k/L} \cdot \frac{\cos(\beta\pi k/L)}{1 - (4\beta^2 k^2/L^2)},$$

$k = -8, \cdots, 0, \cdots, 8$

with $L = 3$ and $\beta = 0.11$. The combined channel $f(k) = h_3(k) * h_2(k)$ is as that of Tong et al. [11]. It is finite length and nonminimum phase with 12 zeros on the unit circle. The assumptions about the noise are as in experiment 3. The results obtained from 25 independent realizations of applying the procedure of Section V with the FFT method are depicted in Fig. 16 and Table V for case 1 and in Fig. 17 and Table VI

16-QAM, SNR = 20 dB, Ch. 3 plus lowpass

MSE (dB)

0.00 10.00 20.00

16-QAM, SNR = 20 dB, Ch. 3 plus lowpass, DFE

Eye at 1200 symbols

16-QAM, SNR = 20 dB, Ch. 3 plus lowpass, DFE

Eye at 1400 symbols

Fig. 11. As in Fig. 8, but with channel 3 and low-pass filter 1,

$L = 3, \alpha = \pm 1/3, p = 23, q = 11, w = 69$, 16-QAM signal,

and $SNR = 20$ dB.

Fig. 12. True and estimated frequency response of channel 3 with low-pass filter 1, FFT method with $L = 3, \alpha = \pm 1/3$, $Q = 80$, and $N = 1000$ symbols.
TABLE III
MEAN AND VARIANCE OF THE FIRST 23 MINIMUM PHASE AND THE FIRST 11 MAXIMUM PHASE ESTIMATED DIFFERENTIAL CEPSLUM PARAMETERS OF CHANNEL 3 WITH LOW-PASS FILTER 1 AND THE FFT METHOD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Estimate</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truncated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(case 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N=1000, L=3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A(1)</td>
<td>-4.3283</td>
<td>0.4099</td>
</tr>
<tr>
<td>A(2)</td>
<td>-1.1247</td>
<td>0.4744</td>
</tr>
<tr>
<td>A(3)</td>
<td>0.8389</td>
<td>1.0531</td>
</tr>
<tr>
<td>A(4)</td>
<td>1.2199</td>
<td>0.2378</td>
</tr>
<tr>
<td>A(5)</td>
<td>0.4579</td>
<td>0.5576</td>
</tr>
<tr>
<td>A(6)</td>
<td>-0.4007</td>
<td>0.1974</td>
</tr>
<tr>
<td>A(7)</td>
<td>-0.6310</td>
<td>0.1028</td>
</tr>
<tr>
<td>A(8)</td>
<td>-0.2462</td>
<td>0.0667</td>
</tr>
<tr>
<td>A(9)</td>
<td>0.2372</td>
<td>0.1111</td>
</tr>
<tr>
<td>A(10)</td>
<td>0.3307</td>
<td>0.0896</td>
</tr>
<tr>
<td>A(11)</td>
<td>0.0814</td>
<td>0.0580</td>
</tr>
<tr>
<td>A(12)</td>
<td>-0.2992</td>
<td>0.2357</td>
</tr>
<tr>
<td>A(13)</td>
<td>-0.2521</td>
<td>0.0869</td>
</tr>
<tr>
<td>A(14)</td>
<td>-0.6513</td>
<td>0.0862</td>
</tr>
<tr>
<td>A(15)</td>
<td>0.1893</td>
<td>0.2978</td>
</tr>
<tr>
<td>A(16)</td>
<td>0.3160</td>
<td>0.1245</td>
</tr>
<tr>
<td>A(17)</td>
<td>0.2844</td>
<td>0.1478</td>
</tr>
<tr>
<td>A(18)</td>
<td>-1.6417</td>
<td>0.3420</td>
</tr>
<tr>
<td>A(19)</td>
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<td>0.2114</td>
</tr>
<tr>
<td>A(20)</td>
<td>0.2020</td>
<td>0.2414</td>
</tr>
<tr>
<td>A(21)</td>
<td>-0.2718</td>
<td>0.4932</td>
</tr>
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<td>-0.2001</td>
<td>0.5567</td>
</tr>
<tr>
<td>A(23)</td>
<td>0.0328</td>
<td>0.2077</td>
</tr>
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<td>B(2)</td>
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<tr>
<td>B(2)</td>
<td>0.8304</td>
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</tr>
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<td>-0.1115</td>
<td>0.2931</td>
</tr>
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<td>B(4)</td>
<td>-0.6344</td>
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</tr>
<tr>
<td>B(7)</td>
<td>-0.1311</td>
<td>0.0798</td>
</tr>
<tr>
<td>B(8)</td>
<td>-0.0831</td>
<td>0.0573</td>
</tr>
<tr>
<td>B(9)</td>
<td>-0.0911</td>
<td>0.1269</td>
</tr>
<tr>
<td>B(10)</td>
<td>-0.0056</td>
<td>0.0038</td>
</tr>
<tr>
<td>B(11)</td>
<td>-0.0782</td>
<td>0.1226</td>
</tr>
</tbody>
</table>

for case 2. In case 1, the parameters \( N = 1200 \) symbols, \( Q = 64 \), \( \alpha = \pm 1/3 \) and \( d = 0.83 \) were chosen. To reduce estimation errors the cyclic autocorrelation was truncated to \( M = 20 \) lags and the differential cepstrum parameters were truncated to \( p_d = 13 \) and \( q_d = 8 \). For case 2, the corresponding parameters were \( N = 1200 \), \( \alpha = \pm 1/3 \), \( Q = 80 \), \( d = 0.9 \), \( M = 22 \), \( p_d = 16 \) and \( q_d = 5 \). From the results obtained, we conclude that in both cases the identified characteristics approximate well the characteristics of the true channels. However, higher estimation bias and variance is observed compared to the results of experiments 3 and 4.

F. Computational Complexity

The major contribution to the computational complexity of the proposed methods comes from the solution of the overdetermined system of equations and the size of FFT algorithms utilized. Thus, for large values of \( p \) and \( q \) both the number of complex multiplications and additions required is of the order \( \sim [2 \max(p, q)]^2 \).

Compared to the existing gradient type blind deconvolution methods (linear predictors, Bussgang approaches), which have a complexity of the order of number of coefficients to be iden-
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blind deconvolution techniques, the proposed methods are in higher complexity. Nevertheless, compared to the polyspectra identified [4]–[6], the proposed algorithms exhibit considerably computational savings are obtained from the lower complexity in the estimation of the cyclic autocorrelation compared to the estimation of higher-order moments and cumulants. For example, given that the channel is of length \( M \) and \( N \) data samples are made for the other lags. In addition, assuming there are nonzero different fourth-order cumulant lags \( \rho_{4}(0,0,0) \) requires \( 2N+1 \) complex multiplications while the estimation of the fourth-order cumulant lag \( C_{4}(0,0,0) \) requires \( 4N+4 \) complex multiplications. Similar observations are made for the other lags. In addition, assuming that the channel is of length \( M \) (no loss of generality) there are \( 2M+1 \) nonzero different cyclic autocorrelation lags while there are \( \binom{2M+1}{2} \) nonzero different fourth-order cumulant lags (taking into account symmetry properties). Thus, assuming \( M = 5 \) \((M = 10)\), the total number of multiplications required for the estimation of the cyclic autocorrelation is approximately 10 times (35 times) less than that for the estimation of fourth-order cumulants. Also, note that the implementation of the proposed FFT approach requires three 1-D FFT transformations, however, a similar approach in the fourth-order cumulant domain requires three 3-D FFT transformations [3], i.e., two orders of magnitude more multiplications. More computational savings are obtained by recognizing that for the same estimation variance less data samples are needed to estimate the second-order statistics of a process compared to the estimation of its higher-order statistics.

VIII. DISCUSSION

The methods proposed in this paper possess the following strong points/limitations compared to other techniques proposed in the literature [10]–[14]. In [10], training by means of a cyclostationary pilot superimposed on the data is provided in order to identify channels longer than the data cyclic period. Thus, the procedure is not truly blind. The methods in [11] and [12] are based on multichannel modeling of the sampled signal and make some assumptions for the rank of a channel coefficient matrix. Also, they are unable to deal with

<table>
<thead>
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<th>TABLE IV</th>
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<tbody>
<tr>
<td>MEAN AND VARIANCE OF THE FIRST 10 MINIMUM PHASE AND THE FIRST FIVE MAXIMUM PHASE ESTIMATED DIFFERENTIAL CEPSRUM PARAMETERS OF TWO IMPULSE RESPONSES (WITH ( L = 3 ) AND ( L = 3.3 )) WITH THE FFT METHOD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( N=500, L=3 )</th>
<th>( N=500, L=3.3 )</th>
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<tbody>
<tr>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td>A(1)</td>
<td>0.8477</td>
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<tr>
<td>A(2)</td>
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</tr>
<tr>
<td>A(3)</td>
<td>0.9438</td>
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<tr>
<td>A(4)</td>
<td>0.5200</td>
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<tr>
<td>A(5)</td>
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<tr>
<td>A(6)</td>
<td>-0.7803</td>
</tr>
<tr>
<td>A(7)</td>
<td>-1.0599</td>
</tr>
<tr>
<td>A(8)</td>
<td>-1.1983</td>
</tr>
<tr>
<td>A(9)</td>
<td>1.1407</td>
</tr>
<tr>
<td>A(10)</td>
<td>0.9264</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>TABLE V</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN AND VARIANCE OF THE FIRST 13 MINIMUM PHASE AND THE FIRST EIGHT MAXIMUM PHASE ESTIMATED DIFFERENTIAL CEPSRUM PARAMETERS OF CHANNEL 6 WITH LOW-PASS FILTER 2 AND THE FFT METHOD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( N=1200, L=3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>A(1)</td>
</tr>
<tr>
<td>A(2)</td>
</tr>
<tr>
<td>A(3)</td>
</tr>
<tr>
<td>A(4)</td>
</tr>
<tr>
<td>A(5)</td>
</tr>
<tr>
<td>A(6)</td>
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<td>A(7)</td>
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<tr>
<td>A(8)</td>
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<td>A(9)</td>
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<tr>
<td>A(10)</td>
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<tr>
<td>A(11)</td>
</tr>
<tr>
<td>A(12)</td>
</tr>
<tr>
<td>A(13)</td>
</tr>
<tr>
<td>R(1)</td>
</tr>
<tr>
<td>R(2)</td>
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<tr>
<td>R(4)</td>
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<tr>
<td>R(6)</td>
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<tr>
<td>R(7)</td>
</tr>
<tr>
<td>R(8)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>True &amp; Mean Estimated Magnitude</th>
<th>True &amp; Mean Estimated Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized Frequency</td>
<td>Normalized Frequency</td>
<td>Normalized Frequency</td>
</tr>
<tr>
<td>Fig. 16. True and estimated frequency response of channel 6 with low-pass filter 2. FFT method with ( d = 0.83 ). ( L = 3 ). ( \alpha = \pm 1/3 ). ( Q = 64 ), and ( N = 1200 ) symbols.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
infinite impulse response (IIR) channels. Their implementation is based on singular value decomposition and thus depends on accurate separation of the signal from noise subspaces. In [13] and [14], methods for the identification of the zeros and poles of the ARMA modeled transfer function of the channel are proposed that require some a-priori knowledge or determination of the ARMA model order. Among the restrictions imposed are those of no zero-pole cancelation in the cyclic spectrum domain and no zeros on the unit circle. All of the above methods are basically channel identification methods and thus different equalization methods need to be devised based on the estimated channel.

The methods proposed in this paper are able to identify separately the minimum and maximum phase information of the channel. By using simple linear transformations in the cepstrum domain and efficient recursive formulas they can recover directly the coefficients of either the channel impulse response, a linear equalizer, or a decision feedback equalizer. They can assume either batch type or adaptive realizations. The methods work equally well with finite impulse response (FIR) and IIR (AR or ARMA) channels, without the need for model order selection. They are only sensitive to the severe underestimation of the differential cepstrum truncating parameters $p$ and $q$. The methods are not directly applicable to channels with zeros on the unit circle because in this case the complex cepstrum is not well defined. By properly weighting the cyclic autocorrelation, as it is explained in Section V, the zeros can be moved away from the unit circle and then the proposed methods can be applied. However, this procedure is sensitive to estimation errors and requires careful implementation.

A limitation shared by all methods proposed in [11]–[14] and in this paper is the inability to identify channels with all their zeros and poles evenly spaced radially with angle $\frac{2\pi}{L}$. However, this problem is rare and will occur for specific values of $L$ only. Finally, we should mention that cyclic second-order statistics cannot be utilized for the identification of nonminimum phase discrete channels that are strictly bandlimited to less than $\frac{2\pi}{L}$ rads/sample. In this case the cyclic spectrum reduces to the classical power spectrum because it is nonzero only for integer values of $\alpha$ as it can be easily seen by (5). A formal proof of this has been provided in [15].

**IX. Conclusion**

New discrete time blind deconvolution methods for nonminimum phase linear channels driven by cyclostationary inputs of period $L$ were introduced. They utilize the complex cepstrum of the cyclic autocorrelation of the channel output and they can identify separately the minimum and maximum phase information of the channel. They are applicable to all channels except those

i) with zeros on the unit circle

ii) with zeros and poles evenly spaced radially from 0 to $2\pi$ with angle $\frac{2\pi}{L}$

iii) strictly bandlimited to less than $\frac{2\pi}{L}$ rad/sample.

However, a procedure for treating channels with zeros on the unit circle was proposed and successfully applied. Computer simulations have demonstrated the effectiveness of the proposed batch type and adaptive methods for channel identification as well as linear and decision feedback equalization.
APPENDIX A

From (4), and the problem assumptions

\[ S^p_v(z) = \frac{1}{L} \sum_{k=(L)}^{\infty} \sum_{m=-\infty}^{\infty} a_n f(k + m - nL) + w^p(k + m) \]

\[ \times \sum_{k=(L)}^{\infty} \left( \sum_{n=-\infty}^{\infty} a_n f(k - (L) + w^*(k)) \right) z_k \]

\[ = \frac{1}{L} \sum_{k} \sum_{m} \sum_{n} \sum_{l} E\{a_m a_n^* f(k + m - nL) + w^*(k) \} z_k z_m z_l \]

\[ \times f^*(k - (L)) + E\{w(k + m)n^*(k) \} \]

\[ \times f^*(k - (L)) + E\{w(k + m)n^*(k) \} \]

\[ = \frac{1}{L} E\{a_n^2\} \sum_{k} \sum_{m} \sum_{n} f(k + m - nL) \]

\[ \times f^*(k - nL)z_k^2 z_m^2 \]

\[ + \frac{1}{L} \sum_{k} R_w(m)z^m \sum_{k} z_k \]

\[ = \frac{1}{L} E\{a_n^2\} \sum_{k} \sum_{m} \sum_{n} f(k + m - nL) z_m \]

\[ \times f^*(k - nL)z_k^2 + \frac{1}{L} \sum_{k} R_w(m)z^m \sum_{k} z_k \]

\[ = \frac{1}{L} E\{a_n^2\} F(z) \]

\[ \times \sum_{k=(L)}^{\infty} \sum_{n=-\infty}^{\infty} f^*(k - nL)(z - z_0)^{(k-nL)} \]

\[ \times z_k n + \frac{1}{L} S_w(z) \sum_{k=(L)}^{\infty} z_k \]

\[ = \frac{1}{L} E\{a_n^2\} F(z) \sum_{n=-\infty}^{\infty} f^*(z - z_0)^{-x} \]

\[ \times z_k n + \frac{1}{L} S_w(z) \sum_{k=(L)}^{\infty} z_k. \]

Since \( \alpha = \frac{1}{L} \), we find \( z_n^L = e^{-j2\pi \frac{n}{L}} = 1 \). Thus

\[ S^p_v(z) = \frac{1}{L} E\{|a_n|^2\} F(z) F^*(z - z_0) + S_w(z) \sum_{k=(L)}^{\infty} e^{-j2\pi \alpha k}, \]

\[ \alpha = \frac{1}{L}, \quad L \in \mathbb{Z}. \quad (50) \]

APPENDIX B

Let us assume that \( I(\cdot) \) stands for \( A(\cdot) \) or \( B(\cdot) \) and that the \( I(k), k = k_0, k_1, \ldots, k_i \) where \( \{k_j\} \) are integers greater than zero are known. Also, let \( I(k/k_0, k_1, \ldots, k_i) \) denote the unique polynomial of \( i \)th degree so that \( I(k) = I(k/k_0, k_1, \ldots, k_i), k = k_0, k_1, \ldots, k_i \). Then, an \( i \)th-order approximation of the value \( I(k) = I(k/k_0, k_1, \ldots, k_i), k \neq k_0, \ldots, k_i \) by using the Newton-Aitken iteration method with divided differences is obtained as follows [23] (see (51) at the bottom of this page). For example, assuming that we want to interpolate the value of \( I(k) \) given the values \( I(k_0 = k - 1), I(k_1 = k + 1), I(k_2 = k - 2), I(k_3 = k + 2), \ldots \), then we can obtain a

First degree approximation:

\[ I(k/k_0, k_1) = \frac{1}{(k_1 - k_0)} I(k_0) k_0 - k \]

Second degree approximation:

\[ I(k/k_0, k_1, k_2) = \frac{1}{(k_2 - k_0)} I(k_0) k_2 - k \]

Third degree approximation:

\[ I(k/k_0, k_1, k_2, k_3) = \frac{1}{(k_3 - k_0)} I(k_0) k_3 - k \]

and so on. Thus, by incorporating more and more given values, one can successively apply the above iteration until the desired precision (according to some constraint) is achieved. In our case, the following constraint can be employed at the \( i \)th-order approximation. Let \( C_\alpha(k/k_0, k_1, \ldots, k_i) = A(k/k_0, k_1, \ldots, k_i) + B^*(k/k_0, k_1, \ldots, k_i)z_0^{-n} \) as suggested by (24). Then, the approximation is satisfactory provided that

\[ |C_\alpha(k) - C_\alpha(k/k_0, k_1, \ldots, k_i)| \leq \epsilon \quad (52) \]

where \( \epsilon \) is a small positive real number, for example \( \epsilon = 10^{-2} \). Otherwise, proceed to a higher-order approximation until all given samples are employed. Note, that the larger the variation of the functions \( A(k) \) and \( B(k) \) the higher the approximation order necessary to achieve the desired precision becomes. From the definition of \( A(k) \) and \( B(k) \) in (10) we observe that high variation between neighboring samples would occur when the zeros and poles of the channel are concentrated near the negative axis in the \( Z \)-transform domain. For such cases, a rule
of thumb is to choose odd (even) values for the $k_0, k_1, \ldots, k_i$ if the missing value is at $k$ odd (even) because the underlying interpolation function exhibits less variation, but this assumes some a priori knowledge of the zeros and poles locations.

APPENDIX C

For the estimation of the complex gain $\frac{1}{A(n)} = G \cdot e^{j\phi(n)}$ needed in equalization applications one can apply the following blind recursive procedures (see Fig. 3) [9]

$$\hat{\phi}(n + 1) = \hat{\phi}(n) + \mu \hat{a}_n [\hat{a}_n - \hat{a}_n],$$
$$\hat{G}(n + 1) = \sqrt{\frac{E[\hat{a}_n^2]}{Q(n + 1)}},$$
$$Q(n + 1) = (1 - \nu)Q(n) + \nu \left| \frac{\hat{a}_m}{G(n)} \right|^2,$$
$$\hat{G}(0) = 1, Q(0) = 0,$$
$$\frac{1}{A(n)} = \hat{G}(n) \cdot e^{j\phi(n)}, \quad n = 0, 1, \ldots \quad (53)$$

REFERENCES


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Dr. Hatzinakos is a member of the IEEE Statistical Signal and Array Processing (SSAP) Technical Committee, EURASP, and Technical Chamber of Greece.
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