AN ANALYTICAL MODEL FOR RUNWAY SYSTEM CAPACITY ANALYSIS

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Abstract: Runway complexes are the principal bottlenecks of airports, and therefore knowledge of their capacity is of vital importance. We propose a new, analytical, model that estimates the capacity of a wide range of runway complexes. This model captures all the major parameters that affect the runway capacity, while, at the same time, it is fast and easy-to-use. Furthermore, it provides an estimate of the potential capacity benefits from sequencing arriving aircraft. This model can provide support to decision makers and planners for strategic decisions related to airport planning and operations.

Keywords: Aircraft operations, Air traffic control, Analytic approximations, Capacity, Controllers, Model, Sequences, Sequence estimation

I. INTRODUCTION AND PREVIOUS RELATED WORK

Runway complexes have been for the past thirty years, and continue to be today, the principal bottlenecks of airports and for that matter of the entire ATM system. The reason, quite simply, is that runways constitute the "interface" between the three-dimensional airspace and a "single-file-flow" traffic regime. Moreover, runways are expensive to build, "consume" a great amount of land area and, most important, have significant environmental and other external impacts that necessitate long and complicated review-and-approval processes with uncertain outcomes. Because of its importance, several analytical and simulation models have addressed issues related to the capacity of the runway system (Harris, 1972; Hockaday and Kanafani, 1972).

A number of simulation models like SIMMOD, TAAM and the Airport Machine for the airfield exist, some of them enjoy strong and continued support. Some significant concerns for the use of such models are: (1) they require a lot of time and effort (it can take a few months even for experienced users to set up the simulation of an airport) (2) they have a steep learning curve, and (3) in order to have confidence on the averages provided, several data points (runs) are required. This paper concentrates on analytical models for the estimation of capacity of runway systems.

The first important analytical model of airport operations, due to Blumstein (1959, 1960), deals with the issue of estimating the capacity of a single runway. The model takes into account safety rules that impose limits on the maximum "acceptance rate" of the runway, i.e., its "capacity". Such rules are the minimum allowed longitudinal separations among arriving aircraft during the final approach and...
the requirement for "single occupancy" of runways. Although superseded now by other models, the Blumstein Model is still noteworthy for developing the essential "physics" of the analytical capacity models that followed.

![Image](https://via.placeholder.com/150)

Fig. 1: The Runway Capacity Envelope

A generalized analytical and stochastic model for computing (single) runway capacity has been developed recently (Wingrove, et al., 1995) by the Logistics Management Institute for the NASA Terminal Area Productivity (TAP) program. A very significant contribution of the LMI model is that it takes explicitly into account the random nature of aircraft operations. Furthermore, the LMI model takes a "controller-based view" of aircraft operation and calculates the spacing between aircraft such that, with reasonable confidence, no violations of ATC separation requirements will occur.

Finally, the best-known analytical model that is available for approximating the capacity of complex runway configurations is the FAA Airfield Capacity Model (Swedish, 1981). The scope of the FAA Airfield Capacity Model includes fifteen runway geometry configurations ranging from one to four runways. The model approximates single runway capacity based on the fundamental concepts of the Blumstein model and its extensions and uses "base models" that extend the single-runway analysis to multiple-runway analysis.

2. THE PROPOSED APPROACH

The objective of the work presented in this paper is to develop a model that provides very good estimates of the capacity of runway complexes, is easy-to-use (has limited input requirements), is fast, and can analyze a wide range of runway complexes. The LMI Capacity Model for a single runway and the FAA Airfield Capacity Model for multiple runway operations represent the state of the art in low-level detail models, appropriate for policy analysis and strategic planning. Both, however, require improvements and modifications to address sets of deficiencies which are different in each case.

A new runway system capacity model that integrates an improved version of the LMI single-runway capacity model with the FAA Airfield Capacity model's approach for extending the single-runway analysis to more complex configurations is proposed. This model consists of: (1) a single-runway capacity model with some improvements over the LMI single-runway; (2) a generalized two-runway model that extends the single-runway capacity model by applying to it algorithms similar to those used by the FAA Airfield Capacity model for two runway configurations, and (3) a model for the evaluation of the potential capacity benefits from sequencing arriving aircraft.

3. COMPUTATION OF THE (SINGLE) RUNWAY CAPACITY ENVELOPE

The model computes the so-called "runway capacity envelope", i.e., the set of points that define the envelope of the maximum throughput capacities that can be achieved at the runway, under the entire range of possible arrival and departure mixes. Specifically, it identifies four points on the runway capacity curve. By interpolating between pairs of points with straight-line segments one can then obtain (approximately) the full runway capacity curve. The four points are the following:

Point 1: The "all arrivals" point, i.e., the capacity of the runway when it is used for arrivals only.

Point 2: The "freely inserted departures" point which has the same arrivals capacity as Point 1 and a departures capacity equal to the number of departures that can be inserted into the arrival stream "for free" by only exploiting large interarrival gaps.

Point 3: The "alternating arrivals and departures" point, i.e., the point at which an equal number of departures and arrivals is performed. This is achieved through an arrival-departure-arrival-departure-... sequencing, implemented by "stretching", when necessary, the interarrival gaps, so that a departure can always be inserted between two successive arrivals.

Point 4: The "all departures" point, i.e., the capacity of the runway when it is used for departures only.

An important feature of the model is that it takes a "controller-based", "look-ahead view" of runway operations. In this respect, it calculates the spacing between aircraft pairs ("arrival followed by arrival", "departure followed by departure", "arrival followed by departure", or "departure followed by arrival" as the case may be) such that, with reasonable
Another important feature is that the model takes explicitly into account the random nature of airport operations. So, for example, the approach speeds, the runway occupancy times and the delay in communication time between airport controllers and pilots are all incorporated into the model as normal random variables.

Key input parameters to the model include: the mix and number of aircraft types at the runway \((\rho)\); the length of the common approach path \((D)\); the mean and standard deviation of the approach speed of each aircraft type \((V_i, \sigma_v)\); the mean and standard deviation of the arrival and departure runway occupancy times \((R_{\text{A}}, \sigma_{RA}; R_{\text{D}}, \sigma_{RD})\); the miles-in-trail separation minima for all pairs of aircraft types \((S_{ij})\); and the mean and standard deviation of the communication time delay \((c, \sigma_c)\). All the input random variables are assumed to be normally distributed.

The following illustrates how the model combines its look-ahead and probabilistic features to compute airport capacity. Consider, first, a runway used for arrivals only. The model first calculates the time by which controllers must separate successive arrivals as they enter the common approach path, such that the controllers can be 95% confident that the aircraft will not violate the miles-in-trail airborne separation rule during final approach. Then, the model calculates the required time separation such that controllers can be 98.7% confident that the single-occupant rule on the runway are violated. This constraint does not allow a departure to start to roll unless the next arrival is further than a pre-specified, by the user, minimum distance from the runway threshold. This is a crucial constraint that often plays a major role in determining runway capacity at European airports, but is currently not included in the LMI model.

1. In the proposed single-runway capacity model there is an additional constraint that may prohibit a departure from being released. This constraint does not allow a departure to start to roll unless the next arrival is further than a pre-specified, by the user, minimum distance from the runway threshold. This is a crucial constraint that often plays a major role in determining runway capacity at European airports, but is currently not included in the LMI model.

2. The proposed single-runway capacity model calculates the expected average capacity of the runway under study as is the standard practice currently. By contrast, the LMI model computes the capacity that can be achieved 95% of the time, i.e., can be achieved with 95% confidence. Thus the capacity computed by the LMI model is always smaller than the capacity computed by the proposed model.

3. The LMI model uses a complex algorithm to calculate the all departures capacity (point 4), while the proposed single-runway capacity model uses an entirely different and much simpler algorithm which computes the weighted average of the inter-departure times (plus, if applicable, any penalty due to delays in communications between controllers and pilots).

4. The aircraft position uncertainty is treated by the LMI model in a way similar to the standard deviation of approach speeds and of winds. In contrast, the proposed single-runway capacity model considers the position uncertainty as the standard deviation, due to measurement inaccuracies, of the reported position of the aircraft. Then, it adds an appropriate length to the in-trail minimum separation requirements of approaching aircraft such that the controllers can be
95% confident that no violations will occur as a result of this uncertainty.

5. In the proposed single-runway capacity model, controllers will not allow a "free" departure to be inserted between arrivals if it is projected that such an operation will violate some separation constraint with probability greater than 0.5. The LMI model, by contrast, does allow for that possibility.

Mixed operations: In this case (Module 4), departures on one runway are typically independent of arrivals and departures on the other. However, there is still a dependence between arrivals on the two runways. Specifically, an arrival on one of the runways, taking place immediately after an arrival on the other runway, is subject to a diagonal separation from the preceding arrival on the other runway. (This is common practice when two parallel runways are intermediate-spaced and the airport operates under IFR.) The size of the diagonal separation is specified by the rules utilized by the local ATC authorities. As shown in Figure 2, the model examines quadruplets of arriving aircraft, with two aircraft arriving on each of the two runways. The first aircraft is arriving on one of the two runways (e.g., the "left" one), the second on the other ("right"), the third on the first ("left") runway again, and the fourth on the second ("right") runway. The time interval between the arrival of the first and the second aircraft is calculated based on the required diagonal separation between the two. The arrival time of the third aircraft is then constrained by its diagonal separation from the second aircraft, the in-trail separation from the first aircraft, and the runway occupancy time of the first aircraft. The same procedure is repeated for the last aircraft. As mentioned, departures are considered independent of operations on the other runway and are inserted between arrivals on either or both of the runways.

4.2 Two intersecting runways

In the case of two intersecting runways (Module 5), one used for departures and the other for arrivals, the inter-arrival times and the capacity of the runway used for arrivals is calculated with the single-runway model (Point 1), while the departures capacity is calculated on the basis of the probability of achieving one or two departures per interarrival gap. The constraints taken into account are (i) the time required for an arrival to clear the intersection of the two runways or to exit the arrival runway, (ii) the minimum separation of an approaching arrival from the runway such that a departure can be released, and (iii) the required separations between successive departures on the same runway.

4.3 More than two simultaneously active runways
The application of analytical models to configurations involving three or more simultaneously active runways is more problematic, because of the large number of possibilities concerning the interactions among operations taking place at different runways and the complex operation-sequencing strategies that can be used. The typical modeling approach taken in such cases is to approximate the airport's total capacity by "decomposing" the airport into more easily analyzable components, each consisting of single- or two-runway combinations, and then finding the capacity of each of these components. The type of decomposition most appropriate for each case will depend, of course, on the particular "geometry" of the airfield.

5. SEQUENCING ARRIVING AIRCRAFT

In the vast majority of airports around the world, a first-come, first-serve policy has been adopted for arriving aircraft. Still, due to increasing congestion of runway complexes that cause increasing delays with heavy economic and environmental cost, a lot of interest exists for the possibility of introducing other disciplines that might increase the capacity of the runway complex. It has been observed (Venkatakrishnan et al., 1993) that higher capacity levels for arriving aircraft can be achieved when longer streams of aircraft of the same type land consecutively.

The goal for developing the technique described here is to evaluate the potential (arrival) capacity benefits of the runway complex form sequencing arriving aircraft. A constraint for having very long streams of aircraft of the same type land consecutively is that, in practice, no aircraft type can be excluded any for a long period of time. The model developed takes as input an upper bound on the time any aircraft type can be excluded from landing due to sequencing and evaluates the length of sequences of aircraft of the same type land consecutively.

The runway capacity model uses in its analysis the fraction of aircraft pairs, when a single runway is used for landings, and the fraction of quadruplets of aircraft, in module 4 where two parallel dependent runways are used simultaneously for landings. It evaluates those fractions assuming a first-come, first-serve policy, and it multiplies the fraction of the aircraft types in the pair or quadruplet. A methodology for the evaluation of the fraction of pairs and quadruplets respectively, such that the capacity gains from sequencing arriving aircraft can be estimated follows.

The policy proposed is to have as many consecutive arrivals with aircraft of the same type as possible, without excluding any aircraft type for more than 30 minutes (upper bound), where x is an input parameter.

5.1 One runway for arrivals

Let n be the number of different aircraft types, and \( P_{ij} \) the fraction of sequences of aircraft of type j followed by type i. Also, let C be the arrival capacity of the runway complex. As a first order approximation for C, the capacity obtained with a first-come, first-serve discipline is used. A two step process is followed:

**Step 1:** Find the best transition sequence among aircraft types i.e., the sequence with which aircraft types alternate.

An exhaustive search of all the possible transition sequences is done. Then, the transition sequence with minimum sum of the transition inter-arrival times is chosen.

**Step 2:** Find the fraction of pairs of consecutive arrivals.

Let us divide the hour in 60/x equal time intervals. In order to make sure the policy requirement is met, we must have at least n transitions of aircraft types within each of those intervals. Then,

\[
P_{ij} = \frac{60}{x} / C \quad \text{for all } i, j, k, l\text{ of step 1}
\]

\[
P_{ij} = P_{i} - \frac{60}{x} / C \quad \text{when } i = j, \text{ and}
\]

\[
P_{ij} = 0 \quad \text{otherwise.}
\]

Note that, if the policy requirement is unreasonable (i.e. x is too small for the number of different aircraft types and mix), smaller capacity than the one under a first-come, first-serve discipline, and even negative \( P_{ij} \) will be observed.

5.2 Two runways used simultaneously for arrivals

The logic and the steps are very similar with the above case. Some modifications are required since in module 4 the fraction of each possible quadruplet instead of the fraction of each possible pair of aircraft types is needed.

**Step 1:** Find the best transition sequence among aircraft types.

As above, an exhaustive search of all the possible transition sequences is done. Now, the "cost" of the transition from aircraft type i to aircraft type j is the
sum of the inter-arrival times of the following quadruplets: \([i, i, i, j], [i, i, j, j],\) and \([i, j, j, j].\)

**Step 2:** Find the fraction of each landing quadruplet.

Again the hour is divided into \(60/x\) equal time intervals.

\[
P_{ijl} = \frac{(60/x)}{C} \quad \text{for all } i, j, k, l \text{ of step 1}
\]

\[
P_{ijl} = P_i - 3\left(\frac{60/x}{C}\right) \quad \text{when } i = j = k = l, \text{ and}
\]

\[
P_{ijl} = 0 \quad \text{otherwise.}
\]

6. CONCLUSIONS

The runway capacity model developed captures the important parameters that affect the capacity of airports, has limited input requirements (easy-to-use), and is very fast. Early test runs indicate that the model can provide, quickly and with little effort, good approximations of the capacity of runway complexes. This model, in conjunction with an analytical, fast and easy-to-use, model that approximates delays for a given demand profile (Kivestu, 1976; Malone, 1995) can become a tool for decision makers in the strategic level.

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