Abstract—This paper presents an analysis by which the stability of a multiple-input–multiple-output system of simultaneous speed and stator resistance estimators for sensorless induction motor drives can be successfully predicted. The instability problem of an adaptive flux observer (AFO) is deeply investigated. In order to achieve stability over a wide range of operation, a design of the observer feedback gain is proposed. Furthermore, closed-loop control systems of the independent use of the two estimators are developed. Therefore, all gains of the adaptive proportional–integral controllers are selected and generalized to provide good tracking performance as well as fast dynamic response. The performance of the AFO using the proposed gains, with a sensorless field-oriented-controlled induction motor drive, is verified by simulation and experimental results. The results show a good improvement in both convergence and stability, particularly in the regenerative mode at low speeds, which confirm the validity of the proposed analysis.

Index Terms—Adaptive proportional–integral (PI) controllers, observer feedback gain, sensorless control, stability analysis.

I. INTRODUCTION

SPEED-SENSORELESS control of induction motor drives has many advantages such as reduced hardware complexity, low cost, reduced size, elimination of direct sensor wiring, better noise immunity, increased reliability, and less maintenance requirements. Nevertheless, the sensitivity to variations of machine parameters and stability at low and zero speeds can be considered as the main limitations of sensorless drives [1].

In the past two decades, numerous endeavors have been devoted to achieve better speed estimation of high-performance sensorless induction motor drives using an adaptive flux observer (AFO). However, there is a well-known unstable region of the AFO, particularly in the low-speed regenerative mode, as reported in [2]–[5]. This unstable region can be reduced by proper design of both observer feedback gain and adaptive law using several techniques. Remedies to cope with this instability problem of the AFO are suggested in [6]. Another technique utilizes the Routh–Hurwitz criterion [7]. Others are based on the linearized model of the speed-adaptive full-order flux observer [8] or use Lyapunov theory [9]. Alternatively, improving the stability of the AFO by modifying the adaptive law is based on extensive numerical calculations of the current loci, and its stability is analyzed using a two-time-scale approach [10]. The ramp response characteristics of the speed estimator are used as design guidelines for adaptation gains [9]. Stabilizing the model-reference-adaptive-system-based estimator for combined speed and stator resistance is achieved by adjusting the adaptive laws [11]. In [12], a modified speed-adaptation law has been proposed to stabilize the full-order flux observer in the regenerative mode at low speeds by changing the projection of the current estimation error in this region of operation. The instability problem for simultaneous estimation of speed and stator resistance using an average technique has been studied in [13]. In this paper, the derived stability conditions reveal that the coupling between the speed and the stator resistance estimation loops is the main cause of instability. Stability analysis of the speed estimation of the AFO and simplification of the structure of a sensorless control system by means of the decoupling control have been proposed in [14]. Reduced-order observers for flux and speed estimation of sensorless induction motor drives have been analyzed, and proper gains to give stability for all operating conditions have been selected [15]. The reduction of the unstable region in the regenerative mode and, consequently, the reduction of the inobservability phenomenon by an appropriate design of feedback gains and speed adaptation law have been proposed in [16].

Recently, many attempts have been made to improve the performance of speed-sensorless induction motor drives during a wide speed range [17]–[22]. In this subject, different techniques have been proposed for extending the operating region of sensorless drives near zero stator frequency by identifying stator resistance together with speed [23], [24]. Several online stator resistance adaptation schemes have been proposed for this purpose [10], [11], [13], [23], [24]. However, the sensitivity to variations of other machine parameters such as rotor resistance, mutual inductance, and leakage inductance has been studied. It has been concluded that, among all the effects of parameter variations, the stator resistance mismatch has the most serious influence on the speed estimation. However, variations of rotor resistance, mutual inductance, and leakage inductance do not cause the instability problem in the speed estimation [9], [13].

Basically, much recent research efforts have been focused on deriving general stability conditions in the case of speed estimation [5], [16], [25], [26], and this problem seems to be solved. However, simultaneous speed and stator resistance estimation is a more complicated problem which is not solved at this time and more research work should be carried out [10], [11], [13], [27]. Moreover, the combined speed and stator resistance estimation is not stable for all the operating points...
if feedback gains are not chosen properly. In addition, the adaptation proportional–integral (PI) gains for simultaneous estimators, which are considered an important parameter for specifying the estimation process of the AFO, need to be designed to achieve fast transient response as well as good tracking performance.

The main objectives of this paper are as follows.

2. Develop the $2 \times 2$ multiple-input–multiple-output (MIMO) closed-loop control systems of speed and stator resistance estimators for both independent and simultaneous uses.
3. Study the instability problem of the proposed drive system, particularly in the regenerative mode at low speeds.
4. Design the observer feedback gain to guarantee stability in the regenerative mode at low speeds for both independent and simultaneous uses.
5. Design the adaptation PI gains of speed and stator resistance estimators for a wide speed range to achieve fast transient response as well as good tracking performance.
6. Confirm the validity of the proposed analysis with both simulation and experimental results under different operating conditions.

II. MATHEMATICAL MODELS

A. Dynamic Model of an Induction Motor

The dynamic model of an induction motor can be expressed in the stationary reference frame in terms of stator current and rotor flux as follows [3]:

$$\begin{align*}
\dot{p}\vec{i}_s &= A_{11}\vec{i}_s + A_{12}\vec{\lambda}_r + b_1\vec{v}_s^s \\
\dot{p}\vec{\lambda}_r &= A_{21}\vec{i}_s + A_{22}\vec{\lambda}_r
\end{align*}$$

where

$$\begin{align*}
\vec{i}_s &= \begin{bmatrix} \vec{i}_{ds}^s \\ \vec{i}_{qs}^s \end{bmatrix}, \\
\vec{\lambda}_r &= \begin{bmatrix} \lambda_{dr}^r \\ \lambda_{qr}^r \end{bmatrix}, \\
\vec{v}_s^s &= \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix}, \\
A_{11} &= \alpha I, \\
A_{12} &= cI + dJ, \\
A_{21} &= eI, \\
A_{22} &= -\varepsilon A_{12},
\end{align*}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

$$a = -\left(\frac{R_r}{\sigma L_s} + \frac{L_m}{\sigma L_s T_s T_r} \right), \quad c = \frac{1}{\varepsilon T_r},$$

$$d = -\frac{\omega_r}{\varepsilon}, \quad \varepsilon = \frac{\sigma L_s L_r}{L_m},$$

$$b = \frac{1}{\sigma L_s}, \quad \sigma = 1 - \frac{L_m^2}{L_s T_r}, \quad T_r = \frac{L_r}{R_r}.$$

B. AFO

The AFO can be constructed using (1) and (2) as follows [4]:

$$\begin{align*}
\dot{p}\vec{i}_s &= \hat{A}_{11}\vec{i}_s + \hat{A}_{12}\vec{\lambda}_r + b_1\vec{v}_s^s - K (\vec{i}_s - \vec{i}_s) \\
\dot{p}\vec{\lambda}_r &= A_{21}\vec{i}_s + \hat{A}_{22}\vec{\lambda}_r.
\end{align*}$$

The rotor speed and stator resistance are estimated using (4) and (5) similar to [3], [4]

$$\begin{align*}
\dot{\omega}_r &= \left( K_{P\omega} + K_{I\omega} \right) \int dt \vec{e}_r^T \vec{\lambda}_r \quad (5) \\
\dot{R}_s &= \left( K_{PR} + K_{IR} \right) \int dt \vec{e}_r^T \vec{i}_s \quad (6)
\end{align*}$$

where

$$\vec{e}_r = \begin{bmatrix} \vec{i}_{dr}^r \\ \vec{i}_{qr}^r \end{bmatrix}, \quad \vec{\lambda}_r = \begin{bmatrix} \lambda_{dr}^r \\ \lambda_{qr}^r \end{bmatrix}, \quad \vec{e}_r^i = \vec{i}_s - \vec{i}_s.$$

$K_{P\omega}, K_{I\omega}, K_{PR},$ and $K_{IR}$ are the adaptive gains of speed and stator resistance estimators, respectively. The symbol "∧" signifies the estimated values and variables. $K$ is the feedback gain matrix defined as $K = K_1 I + K_2 J, A_{11}, A_{12},$ and $A_{22}$ are matrices of coefficients in which $\omega_r$ and $R_r$ are replaced by estimated speed $\dot{\omega}_r$ and stator resistance $\hat{R}_s$ set in the observer.

The block diagram of the proposed sensorless indirect-field-oriented (IFO)-controlled induction motor drive is shown in Fig. 1. In this scheme, the unit vector $\theta_1$, required for axis transformation, can be calculated from the estimated speed and slip speed as $\theta_1 = \int (\dot{\omega}_r + \omega_{slip}) dt.$ Equations related to IFO control of induction motors have been described in [28].

III. STABILITY ANALYSIS

A. Error Equations of Stator Current and Rotor Flux

By subtracting (1) and (2) from (3) and (4), the state error of (MIMO) closed-loop control systems of speed and stator resistance can be calculated from the estimated speed and variables.

$$p\vec{e}_r = (A_{11} - K)\vec{e}_r + A_{12}\vec{e}_\lambda + \Delta A_{11}\vec{i}_s + \Delta A_{12}\vec{\lambda}_r$$

$$p\vec{e}_\lambda = A_{21}\vec{e}_r + A_{22}\vec{e}_\lambda + \Delta A_{22}\vec{\lambda}_r$$

where

$$\vec{e}_\lambda = \vec{\lambda}_r - \vec{\lambda}_r$$

$$\Delta A = \dot{A} - A = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix}.$$

If the rotor speed and stator resistance are only considered as variable parameters, the error matrix $\Delta A$ can be expressed as follows:

$$\Delta A_{11} = \frac{\Delta R_s I}{\sigma L_s}, \quad \Delta A_{12} = \frac{\Delta \omega_r J}{\varepsilon},$$

$$\Delta A_{21} = 0, \quad \Delta A_{22} = -\Delta \omega_r J$$

where

$$\Delta R_s = R_s - \hat{R}_s, \quad \Delta \omega_r = \omega_r - \dot{\omega}_r.$$

The relationship between the speed error and stator resistance error can be derived by taking the Laplace transform of (7) and (8). Hence

$$\begin{align*}
[sI - (A_{11} - K)] \vec{e}_r &= A_{12}\vec{e}_\lambda + \Delta A_{12}\vec{\lambda}_r + \Delta A_{11}\vec{i}_s \\
[sI - A_{22}]\vec{e}_\lambda &= A_{21}\vec{e}_r + \Delta A_{22}\vec{\lambda}_r
\end{align*}$$
from which the stator current error equation in terms of both speed and stator resistance errors can be expressed as follows [9], [13]:

\[
e_1 = e_1 + e_2
\]  

(11)

where

\[
e_1 = G_\omega(s) \cdot \frac{\lambda_s^*}{R_s} \Delta \omega_r
\]

(12)

\[
e_2 = G_R(s) \cdot \frac{\sigma L_s}{\sigma L_s} \Delta R_s
\]

(13)

and

\[
G_\omega(s) = \frac{s^2 I + s(K - A_{11} - A_{22})}{s^2 I + s(K - A_{11} - A_{22})}
\]

(14)

\[
G_R(s) = \frac{(s - A_{22})}{\frac{s}{\sigma L_s} + s(K - A_{11} - A_{22})}
\]

(15)

The rotor speed and stator resistance estimators, given by (5), (6), (12), and (13), can be represented by the 2 × 2 MIMO closed-loop control system, as shown in Fig. 2. The speed and stator resistance estimators may be stable when designed independently. There is an effect from each estimator that appears as a disturbance on the other one. Therefore, stability cannot be guaranteed for simultaneous use of the two estimators in all operating conditions and needs to be investigated. In the next section, stability for simultaneous estimators will be investigated, taking the effect of each estimator on the other into consideration. Afterward, the transfer functions of the speed and stator resistance estimators treated as single-input—single-output (SISO) control system are derived to select their gains of adaptive PI controllers.

**B. Transfer Function of Speed Estimator**

Simplifying (14), one obtains

\[
G_\omega(s) = \frac{s^2 I + s(a_1 I + a_2 J) + (a_3 I + a_4 J)}{s^2 I + s(a_1 I + a_2 J) + (a_3 I + a_4 J)}
\]

(16)

where

\[
a_1 = \left( \frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s \sigma L_s T_r L_r} + \frac{1}{T_r} + K_1 \right)
\]

(17)

\[
a_2 = (K_2 - \omega_r)
\]

\[
a_3 = \left[ \frac{1}{T_r} \left( \frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s \sigma L_s T_r L_r} - \frac{L_m}{\sigma L_s T_r L_r} + K_1 + K_2 \omega_r \right) \right]
\]

(18)

\[
a_4 = \left( \frac{K_2}{T_r} - \frac{\omega_r}{T_r} \left( \frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s \sigma L_s T_r L_r} - \frac{L_m}{\sigma L_s T_r L_r} + K_1 + K_2 \omega_r \right) \right)
\]

Then, the error equation \( e_1 \) can be expressed as

\[
\begin{bmatrix}
e_{1d} \\
e_{1q}
end{bmatrix} = \begin{bmatrix}
G_{\omega_{11}}(s) & G_{\omega_{12}}(s) \\
G_{\omega_{21}}(s) & G_{\omega_{22}}(s)
end{bmatrix} \begin{bmatrix}
0 \\
\lambda^*_r
end{bmatrix} \Delta \omega_r.
\]

(19)

As shown in (17), the term \( J^* \lambda^*_r \) has its component only in the \( q \)-axis in the rotor flux reference frame. Therefore, the transfer functions related to this input and that are relevant to the closed-loop stability are \( G_{\omega_{12}}(s) \) and \( G_{\omega_{22}}(s) \). Considering only the error component in the \( q \)-axis, \( e_{1q} \), it is concluded that only the signals in the \( q \)-axis form the closed-loop system. This means that only \( G_{\omega_{22}}(s) \) is involved in stability analysis of the speed estimation system. Therefore, the error equation \( e_{1q} \) can be written from (17) as follows:

\[
e_{1q} = \left| \lambda^*_r \right|^2 G_{\omega_{22}}(s) \Delta \omega_r.
\]
The transfer function $G_{\omega_{22}}(s)$ is calculated onto the rotor reference frame as given in (19), shown at the bottom of the page. Therefore, the SISO block diagram of the speed estimator can be obtained as shown in Fig. 3. Stability analysis of the speed estimator can be investigated using $G_{\omega_{22}}(s)$ based on the root-locus plot. The dominant-pole/zero allocation of $G_{\omega_{22}}(s)$ in a regenerative mode with zero feedback gain ($K_1 = K_2 = 0$) is shown in Fig. 4. It is obvious that $G_{\omega_{22}}(s)$ has one zero on the right-hand side of the s-plane. As a consequence, one of the dominant poles will move toward the unstable zero, and the speed estimation process will be unstable regardless of the adaptation PI gains.

C. Transfer Function of Stator Resistance Estimator

In order to obtain the transfer function of the stator resistance estimator, the estimated-current reference frame is used in the transformation to simplify the derivation of the SISO control system from the MIMO block diagram.

The error equation $\vec{e}_2$ can be expressed as

$$
\begin{bmatrix}
\vec{e}_{2d}' \\
\vec{e}_{2q}'
\end{bmatrix} =
\begin{bmatrix}
G_{R11}(s) & G_{R12}(s) \\
G_{R21}(s) & G_{R22}(s)
\end{bmatrix}
\begin{bmatrix}
\vec{i}_s' \\
0
\end{bmatrix} \Delta R_s.
$$

(20)

Considering only the $d$-axis components, we have

$$
\vec{e}_{2d}' = \frac{\vec{i}_s'}{\sigma L_s} G_{R11}(s) \Delta R_s.
$$

(21)

Therefore, $G_{R11}(s)$ is the only transfer function used in stability analysis of the SISO of the stator resistance estimator. Thus, $G_{R11}(s)$ can be obtained from (15) as given in (22), shown at the bottom of the page. It should be noted that $G_{R11}(s)$ is derived in the estimated stator current reference frame in order to obtain the SISO block diagram of the stator resistance estimator, as shown in Fig. 5. The dominant-pole/zero allocation of $G_{R11}(s)$ in a regenerative mode with zero feedback gain ($K_1 = K_2 = 0$) is shown in the root-locus plot of Fig. 6. It is observed that all dominant poles and zeros are located on the left-hand side of the s-plane. Therefore, the
stator resistance estimator is stable under zero feedback gain. This agrees with the physical interpretation that during a short transient period, if a slow motor resistance is considered to track only the thermal variations of the stator resistance, the cross-coupling effect from stator resistance estimation can be neglected, and the sensorless drive behaves as if the stator resistance is fixed. However, there is only a speed estimation error between the actual and estimated speeds. The stability property during this short transient period should be focused, and the cross-coupling effect from the stator resistance should be considered.

D. MIMO Stability Investigation

The mismatch of the stator resistance estimator heavily affects the speed estimation process and, consequently, the stability of the sensorless drive. Therefore, the observer feedback gains should be designed such that it guarantees the stability of the AFO in the existence of the stator resistance estimator. The simultaneous speed and stator resistance estimators, which are shown in Fig. 2, have been considered for stability analysis. The simultaneous use of the two estimators may lead to instability, particularly at low speeds in a regenerative mode. Therefore, the stability of the complete 2 × 2 MIMO system must be investigated. The stability analysis is performed by finding the locations of the poles and zeros of the open-loop transfer function for the MIMO system. From Fig. 2, the relationship between the system outputs \( \dot{\omega}_r \) and \( \dot{R}_s \) and the errors \( \Delta \omega_r \) and \( \Delta R_s \), which represents the forward transfer function of the 2 × 2 MIMO system, is derived as follows:

\[
\begin{align*}
\dot{\omega}_r & = G_{c\omega}(s) \left( \hat{e}^T_1 \hat{J} \hat{\lambda}^\omega_r + \hat{e}^T_2 \hat{J} \hat{\lambda}^\omega_r \right) \\
& = G_{c\omega}(s) \left( \hat{\lambda}^\omega_r \hat{G}_{c\omega22}(s) \Delta \omega_r + \hat{e}^T_2 \hat{J} \hat{\lambda}^\omega_r \right) \tag{23} \\
\dot{R}_s & = G_{cR}(s) \left( \hat{e}^T_1 \hat{r}_s + \hat{e}^T_2 \hat{r}_s \right) \\
& = G_{cR}(s) \left( \hat{e}^T_1 \hat{r}_s + \hat{e}^T_2 \hat{r}_s \right)^2 G_{R11}(s) \Delta R_s \tag{24}
\end{align*}
\]

Simplifying (23) and (24), the transfer function of the 2 × 2 MIMO system can be given in matrix form as follows:

\[
\begin{bmatrix}
\dot{\omega}_r \\
\dot{R}_s
\end{bmatrix} =
\begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega_r \\
\Delta R_s
\end{bmatrix}
\tag{25}
\]

where

\[
\begin{align*}
m_{11} & = |\hat{\lambda}^\omega_r|^2 G_{c\omega} G_{c\omega22} \\
m_{12} & = |\hat{\lambda}^\omega_r|^2 G_{c\omega}(s) \left( G'_{R21}(s) \hat{i}_r + G'_{R22}(s) \hat{i}_q \right) \\
m_{21} & = |\hat{\lambda}^\omega_r|^2 G_{cR}(s) \left( G'_{R12}(s) \hat{i}_r + G'_{R22}(s) \hat{i}_q \right) \\
m_{22} & = |\hat{\lambda}^\omega_r|^2 G_{cR}(s) G'_{R11}(s)
\end{align*}
\]

and \( G_{c\omega} = K_{P\omega} + (K_{I\omega}/s) \) and \( G_{cR} = K_{P\omega} + (K_{I\omega}/s) \) are the transfer functions of the adaptive PI controllers for rotor speed and stator resistance, respectively. It should be noted that the estimated flux value \( |\hat{\lambda}^\omega_r| \) as well as the estimated stator current value \( |\hat{r}_s| \) are time-varying variables. Thus, in the derivation of (18), (21), and (23)–(25), it is assumed that the values of \( |\hat{\lambda}^\omega_r| \) and \( |\hat{r}_s| \) are approximately constant.

The transfer functions \( G'_{R12}(s), G'_{R11}(s), \) and \( G'_{R21}(s) \) on the rotor flux reference frame can be derived as given in (26), (27), and (28), respectively, shown at the bottom of the page, where \( a_5 = \omega_o^2 + \omega_o a_2 - a_3 \) and \( a_6 = \omega_o^2 + \omega_o a_2 - a_3; \omega_o \) is the rotor flux frequency and \( \omega_o \) is the stator current frequency.

The terms \( \omega_o, a_2, a_3, \) and \( a_4 \) are dependent on \( \omega \), which is a time-varying variable, so \( \omega_o a_2, a_3, \) and \( a_4 \) are time-varying variables as well. Hence, the operator \( s \omega_o \) can be rewritten as \( s \omega_o = s \omega_o s \). The term \( (s \omega_o) = d \omega_o / d t \) is the rate of change of operating frequency. The operators \( s a_2, s a_3, \) and \( s a_4 \) can be treated with the same approach as that with \( s \omega_o \). The terms \( (s \omega_o), (s a_2), (s a_3), \) and \( (s a_4) \) denote the derivative terms \( (d \omega_o / d t), (d a_2 / d t), (d a_3 / d t), \) and \( (d a_4 / d t) \), respectively. Therefore, the transfer functions \( G'_{R12}(s), G'_{R11}(s), G'_{R21}(s), \) and \( G'_{R22}(s) \) are approximately calculated by assuming that these derivative terms can be usually neglected [14].

It should be noted that the derived transfer functions do not depend on the linearization process. Therefore, its validity is not limited locally as in the linearization method.

\[
G'_{R11}(s) = \frac{s^3 + (a_1 + 1/T_r)s^2 + (a_1/T_r + \omega_o)(2\omega_o + a_2) - a_3)s + \omega_o(\omega_o a_1 + a_4) - a_5/T_r}{\sigma L_s \left( s^2 + a_1 s - \omega_o^2 - \omega_o a_2 + a_3 \right)^2 + (2\omega_o a_2 + a_3)^2 + \sigma L_s \left( a_1 s - \omega_o^2 - \omega_o a_2 + a_3 \right)^2}
\tag{26}
\]

\[
G'_{R21}(s) = \frac{(\omega_o - 2\omega_o - a_2)s^2 + (a_1(\omega_o - a_2)) - 1/T_r(2\omega_o + a_2) - a_4)s - a_5\omega_o - 1/T_r(\omega_o a_1 + a_4)}{\sigma L_s \left( s^2 + a_1 s - \omega_o^2 - \omega_o a_2 + a_3 \right)^2 + (2\omega_o a_2 + a_3)^2}
\tag{27}
\]
The characteristic equation to calculate the eigenvalues of the closed-loop system can be obtained from (25) as

\[ \left| I + \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \right| = 0. \]  

(29)

The determinant of (29) has been calculated to obtain (30) as

\[ 1 + m_{11} + m_{22} + m_{11}m_{22} - m_{12}m_{21} = 0. \]  

(30)

The root-locus plot of the simultaneous use of the two estimators at 1 rad/s with zero feedback gain \( K = 0 \) is shown in Fig. 7. It is observed that the system has one zero on the right-hand side of the \( s \)-plane. As a consequence, one of the dominant poles will move toward the unstable zero, and the speed estimation process will be unstable regardless of the adaptation PI gains. As a result, the simultaneous use of the two estimators becomes unstable in the regenerative mode at low speeds. It is also observed that the unstable zero of the simultaneous use of the two estimators is diverged from the left-hand side of the \( s \)-plane more than the corresponding one in the speed estimator only. This confirms that the stator resistance estimator, which is stable when designed independently, affects the simultaneous use of the two estimators.

IV. DESIGN OF THE OBSERVER FEEDBACK GAIN

The presented analytical results using the root-locus method reveal that there is an unstable region in the regenerative mode at low speeds for both the speed estimator only and the simultaneous use of the speed and stator resistance estimators with zero feedback gain. This is clearly observed as a result of the existence of an unstable zero on the right-hand side of the \( s \)-plane. Thus, it is very important to eliminate this instability problem and guarantee the stability of speed estimation in all operating conditions, specifically in the regenerative mode at low speeds by assigning the proper feedback gain value.

A. Stability Conditions of Speed Estimation

The stability conditions of speed estimation can be derived using the transfer function \( G_{\omega,22}'(s) \). To guarantee the stability conditions of speed estimation, the location of the zeros of \( G_{\omega,22}'(s) \) must be restricted to the left-hand side of the \( s \)-plane; otherwise, the dominant closed-loop poles of Fig. 4 will cause instability. The stable-zero conditions can be determined by applying the Routh–Hurwitz criterion. It is found that the following necessary and sufficient coefficients for stable speed estimation are obtained [9]:

\[
\begin{align*}
  a_1 & > 0 \\
  a_1a_3 - \omega_0a_4 & > 0 \\
  \omega_0a_1 + a_4 & > 0
\end{align*}
\]  

(31)

from which the observer feedback gains shown in (32) are proposed to satisfy the stable-zero conditions and, consequently, the stability of speed estimation

\[ K_1 > - \left( \frac{R_s}{\pi T_r} + \frac{1}{\sigma_T} \right) \]  

(32)

The dominant-pole/zero allocation of \( G_{\omega,22}'(s) \) in the regenerative mode of operation with feedback gain \( K \) (32) is shown in the root-locus plot of Fig. 8. It is clear that the unstable zero is moved into the left-hand side of the \( s \)-plane and that the speed estimator becomes stable with the proposed feedback gain (32).

B. Stability Conditions of Simultaneous Use of Speed and Stator Resistance Estimators

Although the stator resistance estimator is stable with zero feedback gain when used independently and the instability comes only from the speed estimation in the regenerative mode at low speeds, the cross coupling between the two estimators has an effect on system stability. Therefore, stability analysis of the simultaneous use of the two estimators becomes mandatory to design the observer feedback gain in order to stabilize not only the speed estimator but also the simultaneous use of speed and stator resistance estimators. The coupling between the
speed and stator resistance estimation loops is the main cause of instability, and stabilization of each individual estimator loop is necessary but insufficient to guarantee stability.

The feedback gain should be such that all zeros of the numerator of the transfer function (30) lie on the left-hand side of the $s$-plane. The sufficient conditions of feedback gain $K$ to make the system zeros lie on the left-hand side of the $s$-plane and to guarantee the stability conditions of (30) are given in (33), shown at the bottom of the page, where

$$l_1 = \left( \frac{R_s}{\sigma L_s} + \frac{L_m}{\sigma L_s T_r L_r} + \frac{1}{T_r} \right)$$

$$l_2 = -\frac{1}{T_r} \left( -\frac{R_s}{\sigma L_s} - \frac{L_m}{\sigma L_s T_r L_r} + \frac{L_m}{\varepsilon T_r} \right)$$

$$l_3 = l_2 + \frac{1}{T_r} - \omega_{r_o}^2$$

$$l_4 = \frac{l_2}{T_r} - l_2 \omega_{r_o}^2$$

$$l_5 = \frac{\sigma L_s}{2} \left| \hat{x}_r \right|^2$$

The conditions in (33) are validated by numerically investigating the locations of the closed-loop poles for various operating modes of the drive for a range of speeds starting at ±0.1 up to ±150 rad/s for the motor parameters given in Appendix B.

Fig. 9 shows the dominant-pole/zero allocation of the MIMO control system at 1 rad/s during regenerative mode with the feedback gain (32) which is designed for the speed estimator. As shown, this gain minimizes the unstable region of the simultaneous use. However, it is still an unstable region which must be eliminated. On the other hand, Fig. 10 shows the dominant-pole/zero allocation of the MIMO control system at 1 rad/s during regenerative mode with the proposed feedback gain (33) which is designed for the simultaneous use of the two estimators. It is highly observed that the MIMO control system is stable and that the unstable region is now completely eliminated since the unstable zero is moved to the left-hand side of the $s$-plane and the observer stability is guaranteed. The effectiveness of the proposed feedback gain is proved by analytical results, as shown in Fig. 10.

V. DESIGN OF ADAPTIVE PI GAINS FOR SPEED AND STATOR RESISTANCE ESTIMATORS

The adaptive PI gains of the speed and stator resistance estimators are responsible for ensuring the good convergence of the estimated parameters, and consequently, the stability of the identification system for the two estimators is investigated. Therefore, the aim of this section is to design the adaptive PI gains for these estimators, based on stability analysis, using the root-locus method. From Fig. 2, the block diagrams of the closed-loop speed and stator resistance estimators treated as SISO systems, by ignoring the effect of each estimator on the other one, are shown in Figs. 3 and 5, respectively. The speed estimator is designed by assuming that the effect of the error $\bar{\omega}_2$ is a bounded external disturbance, which is ignored for the purpose of stability analysis. This can be justified since the poles of the speed estimator using the proposed feedback gain are located on the left-hand side of the $s$-plane. This ensures the convergence of $\Delta \omega_r$ to zero, whatever the value of the disturbance signal. This agrees with the physical interpretation that, at steady state, the speed error is exactly equal to zero, since $\omega_r$ is set equal to $\omega_r$. Similar to the speed estimator, the same is done for the stator resistance estimator by assuming that the effect of the error $\bar{\omega}_1$ is a bounded external disturbance, which is ignored for the purpose of stability analysis.

$$K_1 = 0$$

$$K_2 > \left\{ \frac{n_1(l_2 l_6 K_P R + l_4 l_6 K_P R + l_4 l_6 K_I R) + n_2(2 l_5 l_6 K_P R + l_4 l_6 K_P R + l_4 l_6 K_I R + l_1 l_6 K_I R + \frac{l_5 K_I R}{\varepsilon l_1 R})}{n_1(l_2 l_5 K_P R + 2 l_4 l_6 K_P R + 2 l_6 K_I R \lambda_{r o} - n_2(l_5 K_P R + 2 l_6 K_P R + 2 l_6 K_I R \lambda_{r o}) - n_2(l_5 K_P R + 2 l_6 K_P R + 2 l_6 K_I R \lambda_{r o})} \right\}$$

(33)
Fig. 11. (a) Locations of dominant poles for speed range extending from 0.1 up to 150 rad/s. (b) PI controller zero against rotor speed.

The root-locus plot of $G_{\omega 22}'(s)$ at 1 rad/s is shown in Fig. 8; the zoomed-in view of the dominant poles and zeros is shown. The transfer function $G_{\omega 22}'(s)$ has four poles and three zeros. All poles and three zeros are located on the left-hand side of the $s$-plane. The location of the closed-loop transfer function poles characterizes the control system dynamics. The location of the PI controller zero should be on the real axis at $-5.8$ to make sure that the fast dynamic closed-loop poles are always located to the left of the dominant conjugate poles, achieving fast transient response and good tracking performance. Therefore, $K_{P\omega}$ is taken as a large value, and $K_{I\omega}/K_{P\omega}$ should equal the negative real part of the dominant poles (the values $K_{P\omega} = 500$ and $K_{I\omega}/K_{P\omega} = 5.8$ are used). This allows for fast exponentially speed error decaying with time without overshoot or undershoot.

The root-locus plot of $G_{R11}(s)$ at 1 rad/s using the proposed feedback gain is shown in Fig. 6; the zoomed-in view of the dominant poles and zeros is shown. Similar to the speed estimator, the adaptive PI controller gains are designed. The transfer function $G_{R11}(s)$ has four poles and three zeros. All are located on the left-hand side of the $s$-plane. To investigate both fast dynamic response and good convergence between the actual and estimated values of stator resistance, the PI controller zero is selected. This can be achieved by assigning a large value to $K_{PR}$, and $K_{IR}/K_{PR}$ equals the negative real part of the dominant poles. Therefore, $K_{PR}$ is chosen as 300 and $K_{IR}/K_{PR} = 5.8$.

To generalize the design of adaptive PI gains for speed and stator resistance estimators to be suitable for practical implementation, the dominant poles were plotted for a range of speed starting at 0.1 up to 150 rad/s, as shown in Fig. 11(a). As a consequence, the value of the PI controller zero was plotted against the rotor speed, as shown in Fig. 11(b). This helps one to choose the adaptive PI gains at any rotor speed.

Fig. 12. Simulation results at low-speed regenerating operation at 5 rad/s and $T_L = -2$ N·m. (a) Instability problem without the feedback gain ($K = 0$). (b) Remedy of the instability problem with the proposed feedback gain $K$.

VI. SIMULATION RESULTS

A sensorless IFO-controlled induction motor drive, shown in Fig. 1, is used, where the actual speed feedback signal is replaced by the estimated one. The simulation results of Fig. 12(a) demonstrate the divergence of the estimated speed with zero feedback gain $K = 0$ in the regenerative mode at low speed (5 rad/s, $T_L = -2$ N·m) due to the unstable zero. This verifies the analytical results of Fig. 7.

The feedback gain was adjusted inside the observer according to (33), and the same simulation results were taken at the same conditions as that in Fig. 12(a) to demonstrate the effect of the proposed feedback gain on stability. The simulation results of Fig. 12(b) demonstrate a good convergence of the estimated speed in the regenerative mode at low speed (5 rad/s, $T_L = -2$ N·m) with the proposed feedback gain (33). This verifies the analytical results of Fig. 10.

Fig. 13 shows the performance at acceleration and deceleration in forward and reverse directions. It is observed that
VII. EXPERIMENTAL RESULTS

An experimental setup was built using an induction motor interfaced with a digital control board DS1102 based on a Texas Instruments TMS320C31 digital signal processor. The induction motor is coupled with a dc generator for loading. The rating and parameters of the induction motor, as well as the speed controller gains of the IFO control system, are given in the Appendix. The adaptive PI controller gains of the speed and stator resistance estimators are selected based on the relation of Fig. 11(b). This can be easily done by representing the PI controller zero as a function of rotor speed using quadratic curve fitting. It is found that this relation is

$$K_I / K_P = -0.0008 \omega_r^2 + 0.02 \omega_r + 5.6.$$  \hspace{1cm} (34)

This relation is incorporated in the software program, and then, $K_P$ and $K_I$ can be calculated at any speed to give the best performance.

Experimental results are presented to demonstrate the instability problem with zero feedback gain and its remedy using the proposed feedback gain in the regenerative mode at low speed. In addition, the effectiveness of the estimation process with the proposed gains is examined during a wide speed range in motoring and regenerative modes of operation.

A. Instability in Low-Speed Regenerative Mode and Its Remedy

Experimental results have been carried out to demonstrate the unstable behavior in the low-speed regenerative mode, as shown in Fig. 15, where the speed reference is set at 5 rad/s and the load torque is about $-2 \text{ N} \cdot \text{m}$. It is obvious that the estimated signals $\hat{\omega}_r$ and $i_{qs}^e$ diverge from the measured ones $\omega_r$ and $i_{qs}$. The same experimental results were taken with the proposed feedback gain $K$ (33) to illustrate the stability of
Fig. 15. Experimental results showing the instability problem at low-speed regenerative operation at 5 rad/s and $T_L = -2 \text{ N} \cdot \text{m}$ with $K = 0$.

The estimated variables in the low-speed regenerative mode at 5-rad/s speed reference and a load torque of about $-2 \text{ N} \cdot \text{m}$, as shown in Fig. 16. The obtained results show the effectiveness of the proposed feedback gain and confirm the validity of the analytical results of Figs. 7 and 10, respectively. In addition, these results verify the simulation results of Fig. 12 with and without feedback gains.

B. Performance during Starting and Speed Reversal

The experimental results at starting operation are shown in Fig. 17. A good agreement between the actual and estimated speeds is achieved. Moreover, the experimental results at a speed reversal from 10 to $-10$ rad/s under no-load condition are shown in Fig. 18. As shown, the estimated signals $\hat{\omega}_r$ and $\hat{i}_{qs}$ can successfully track the actual ones $\omega_r$ and $i_{qs}$ in transient and steady states. A speed reversal at very low speed from 2.5 to $-2.5$ rad/s is shown in Fig. 19.

C. Effect of Sudden Load Impact

The step response to the sudden load change from 0 to $2 \text{ N} \cdot \text{m}$ is shown in Fig. 20 at a speed of 16 rad/s. It is obvious that the estimated speed $\hat{\omega}_r$ tracks correctly the actual one $\omega_r$ and recovers quickly to the steady-state value. This confirms the robustness of the speed estimator to load disturbances and the effectiveness of the control system.

D. Effect of Stator Resistance Mismatch

It is important to investigate experimentally both stability and robustness of the proposed AFO when stator resistance varies. Practically, parameter variations are unavoidable due to temperature rise and skin effects. The influence of parameter variations on speed estimation is investigated by showing how parameter mismatch affects the speed estimation error. Fig. 21 shows the experimental results of the actual and estimated speeds as well as the estimated stator resistance during 50% stator resistance variations at very low speed (2 rad/s). The initial detuning in the stator resistance takes a value of 50% of the actual value. As shown, there exists a substantial speed estimation error between the actual and estimated speeds, and this degrades the performance of the sensorless drive, specifically at low speeds. In order to avoid this, the online stator resistance adaptation scheme has been applied at $t = 1$ s. It is obvious that the stator resistance estimator quickly removes the initial stator resistance...
error, and consequently, a considerable reduction of the speed error is observed.

E. Zero Speed Operation

A sensorless induction motor drive is tested at zero-speed operation with nominal load torque, which considered the main problem of this drive system. It is observed from Fig. 22 that a good estimation is achieved at zero-speed operation. However, the actual speed contains some ripples.

To confirm and validate the resultant drive performance, including the stability on the torque–speed plane, Fig. 23(a) and (b) shows the simulation and experimental results of the torque–speed characteristics for various speed commands at very slow torque variations. The stability boundary is represented as a straight line in the torque–speed plane. It is obvious that the sensorless system performs well for the entire speed range (3–150 rad/s) in both motoring and regenerative modes. These results indicate that with the proposed feedback gain, the instability problem in the regenerative mode is successfully removed. This verifies the validity of the analytical results. It can be said that the performance of the sensorless drive with the proposed gains is comparable to that of the field-oriented control with sensors.
VIII. CONCLUSION

This paper has presented stability analysis of the simultaneous use of speed and stator resistance estimators for sensorless induction motor drives. The instability problem in the regenerative mode at low speeds has been clearly demonstrated for the two estimators. It has been concluded that stabilization of each of the independent speed and stator resistance estimators is insufficient to ensure stability when used simultaneously. To achieve good stability over a wide range of operation, particularly in the regenerative mode at low speeds, the observer feedback gain has been designed. Based upon the root-locus method, the adaptive PI controller gains of the two estimators have been determined and generalized for a wide speed range. The obtained results showed that the dynamic performance of the proposed sensorless drive system has been highly improved. Moreover, a significant improvement in both convergence and stability over wide-speed-range estimation, specifically in the regenerative mode at low speeds, has been achieved. A close correlation between the simulated and experimental results was found, which verifies the validity of the proposed analysis.

APPENDIX

A. List of Symbols

- $L_m$: Magnetizing inductance.
- $L_s, L_r$: Stator and rotor self-inductances.
- $R_s$: Stator resistance.
- $T_r$: Rotor time constant.

B. Induction Motor Parameters

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<th>Induction Motor Parameters</th>
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<tr>
<td>Rated power (w)</td>
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<td>Rated current (Amp.)</td>
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<td>Integral gain</td>
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<th>Table III</th>
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REFERENCES


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