RAILGUN CONDUCTOR HEATING FROM MULTIPLE CURRENT PULSES

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Abstract — A numerical technique for solving current- and thermal-diffusion problems in railgun conductors has been used to study joule heating in rails that are subject to multiple current pulses. Copper rails that are 25 mm high by 12.5 mm wide with a 20-mm-square bore and a current pulse with 1-MA peak current and 1-ms pulse width at half maximum were assumed. This combination of parameters is sufficient to accelerate an 80-g projectile to 2-3 km/s with each current pulse. Three parameters were varied in the analysis: the repetition rate or current pulse frequency (3.3 to 100 Hz), the coolant heat-transfer coefficient (5 x 10^4 and 5 x 10^5 W/m²K), and the coolant-channel distribution in the rail. Detailed results are used to illustrate the acceptability or unacceptability of particular combinations of these parameters for operation at steady state. An uncooled rail was not acceptable for steady-state operation. Repetition rates of about 30 Hz were acceptable with the higher coolant heat-transfer coefficient and the best coolant-channel distribution; this included cooling the rail exterior surface.

INTRODUCTION

Railguns or electromagnetic accelerators are devices that accelerate projectiles by the interaction of an electric current and a magnetic field. Recent interest in railguns has lead to tests in which a projectile of a few grams was accelerated to 10 km/s and a projectile of a few hundred grams was accelerated to 3 km/s [1,2]. One-shot railguns use the heat capacity of the rails to absorb joule heating from current flow. However, multiple-shot railguns require cooling if they operate for more than a few pulses.

Two-dimensional current- and thermal-diffusion calculations were done for a rail cross section near the breech using a technique that had previously been developed for this type of analysis [3]. At this location, the rail is subject to the entire current pulse. The finite difference calculation used a 10 x 10 rectangular mesh with variable node spacing; only half of one rail was modeled because of symmetry. Figure 1 shows a schematic of the node layout with some of the node numbers identified. Node numbers run from 101 to 110 from left to right along the top row, from 201 to 210 along the second row, etc. With the variable node spacing, the edge and corner nodes, where current-density and thermal gradients are steep, could be made smaller. The rails were assumed to be copper. In this analysis, calculations were continued even though node temperatures above the melting point were indicated. In reality, some material redistribution would probably occur because of magnetic forces and mechanical forces acting on the parts of the rail that were near melting. This phenomenon was not considered in the analysis. Thus, node temperatures above the melting point of copper should not be

copper, 25 mm high by 12.5 mm wide, with a 20-mm-square bore. The high-frequency inductance gradient for this geometry is 0.44 μH/m [5]. With the current pulse employed, this rail configuration should be sufficient to accelerate 80-g projectiles to approximately 3 km/s [6].

The current pulse was approximated as

\[ I = I_m \sin^2\left(\frac{\pi t}{2 t_m}\right) \]  

for \( 0 < t < 2 t_m \) and zero otherwise. In (1), \( I \) is current, \( t \) is time, \( I_m \) is the maximum current, and \( t_m \) is the time at which the maximum occurs. For all the calculations, \( I_m \) was taken as 1 MA and \( t_m \) as 1 ms. This same current pulse was repeated at frequencies of 3.3-100 Hz to achieve the various repetition rates. The calculations were all done for 10 pulses; it was apparent whether a steady state had been attained in which temperatures were below the melting point of the rails by this time.

This paper presents the results of calculations of rail heating for a railgun that would achieve a specific objective, acceleration of an 80-g projectile to a velocity of 2-3 km/s for each shot [4]. The repetition rate or current pulse frequency, the coolant-channel distribution, and the heat-transfer coefficient to the coolant were varied as parameters. Repetition rates of 3.3-100 Hz were considered.

DESCRIPTION OF THE CALCULATION

During a recent symposium on railguns, a description of a system that would accelerate 80-g projectiles to 2-3 km/s was given [4]. The description primarily focused on the design of a power supply that would provide multiple current pulses with approximately 1-MA peak current and 1-ms pulse width at half maximum. Those parameters were also chosen for this analysis. The rails were assumed to be

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Fig. 1. A 10 by 10 rectangular mesh on half of one rail; node sizes are not shown to scale.
considered as an attempt to predict actual rail temperatures; rather, rail temperatures at or above melting indicate that failure of parts of the rail is likely.

In this formulation of current-diffusion analysis, it is assumed that current density, or equivalently, electric field strength, enters or leaves the conductor only through an exterior surface [3]. To approximate this boundary condition, we made changes in total current flow by changing current density in the edge nodes only. The relative current-density distribution on the conductor surface (in terms of current per unit length) was obtained from a static calculation of the high-frequency inductance and current distribution for the rails [5]. It was converted into a current density in the edge nodes (in units of current per unit area) by distributing all the current on the surface of an edge node uniformly over the node area.

Calculations were done for a rail without cooling and for four distributions of coolant channels in the rail. The coolant channels were defined by replacing one or more nodes with coolant channels or by assuming that the coolant contacts specific surface nodes. The heat transfer area to the coolant was taken as the area of the node surfaces; that is, no enhancement of the surface area was assumed. Figure 2 shows the four node layouts that employ coolant channels; the node sizes are shown to scale on this figure and the coolant channels are shown shaded. The single-channel cooling layout has one central coolant channel that is 3.7 mm wide by 6.0 mm high; only half the channel is shown in Fig. 2. The two-channel cooling layout has one channel (3.7 mm wide by 3.0 mm high) in each half of the rail; this arrangement brings the coolant closer to the rail corners, which are the highest temperature areas in the rail. The distributed cooling layout has six channels (1.85 mm wide by 1.5 mm high) in each half of the rail; this arrangement brings the coolant closer yet to the rail corners and surfaces. The surface/distributed cooling layout has six interior channels that are similar to the distributed cooling layout in each half of the rail and cooling channels that contact the exterior rail surface along the interior corner, the top, and the right side of the rail. The surface coolant channels would be defined by the rail surface and slots in the insulators. The surface/distributed arrangement brings the coolant very close to the high joule-heating areas of the rail.

Two values of $h_c$, the heat-transfer coefficient between the rail and the coolant, were used: $5 \times 10^4$ and $5 \times 10^5 \text{ W/m}^2\text{K}$. If the coolant is water, $h_c = 5 \times 10^4 \text{ W/m}^2\text{K}$ is easily attained with moderate water velocities. A value of $h_c = 5 \times 10^5 \text{ W/m}^2\text{K}$ would require high water velocities and possibly high pressures. Coolant temperatures and surroundings temperatures were assumed to be 300 K throughout each calculation. Cryogenic cooling (liquid nitrogen or

Fig. 2. Node layout and coolant-channel locations (shaded).
hydrogen) could reduce joule heating by reducing rail resistivity, however, this variation was not pursued here. The rail surfaces were assumed to lose heat to their surroundings (insulator and bore) with a heat-transfer coefficient of 100 W/m²-K in the areas that were not cooled by surface coolant channels. No heat-source term was included for the rail surface that is open to the bore; such a source term might come from the plasma in an arc-driven railgun. Thus, the calculated rail temperatures, particularly near the bore, would be lower than expected for an arc-driven projectile. The initial rail temperature was taken as 300 K at the start of a sequence of current pulses.

RESULTS

Three parameters were varied in these calculations: the coolant distribution (see Fig. 2); the current pulse frequency or repetition rate (3.3, 10, 33, and 100 Hz); and \( h_c \), the coolant heat-transfer coefficient. Calculations were not done for all combinations of these parameters; an attempt was made, however, to define the conditions under which the various coolant distributions could operate for an extended time period, that is, at steady state.

As a prelude to examining the multipulse results, it is instructive to examine in detail the effects of a single pulse. Figure 3 shows a plot of the total current flow in the half of the rail that was modeled; it follows the form of (1). Figure 4 shows a plot of current density (current per unit area) in three of the nodes (101, 205, and 1005) over the same time period. These results are for the no-cooling option. Over the time period shown, results for the other coolant distributions are similar for the first pulse except where a coolant channel replaces one of the nodes. It is evident that the current-density distribution is nonuniform. Node 101, at the corner near the bore (see Fig. 1), has the highest current densities. Node 205, near the middle and close to the top edge, has lower current densities, with the peaks coming a little later than those in Node 101. The peak current densities in the nodes near the rail edges are a reflection of the surface current distribution [5]. Node 1005, an interior node, shows much lower current densities. Even though total current flow is zero after 2 ms (see Fig. 3), some current flow persists to about 5 ms in the form of eddy currents. Figure 5 shows a plot of the temperatures of the same nodes over the same time period. The temperature rises over the first few milliseconds are mostly a function of the joule heating in each node because little heat transfer occurs during this time period. Joule heating is proportional to the square of the current density; thus, the temperature rise of Node 101 is about 25 times the rise of Node 1005. The pause in the temperature rise of Node 101 at 1-1.5 ms is caused by the current density in that node going to zero in that time period (see Fig. 4). After the first few milliseconds, node temperature changes are from the redistribution of heat throughout the rail. Node 101 is hotter than the surrounding nodes and thus drops in temperature as it loses heat (see Fig. 5). Nodes 205 and 1005 slightly increase in temperature, from 3-10 ms, as they gain heat from surrounding nodes. This phase continues until the next current pulse, which occurs at 10 ms for a 100-Hz repetition rate or 300 ms for a 3.3-Hz repetition rate.

No-Cooling Option

Without cooling, the only path for heat loss from the rail is through the exterior surfaces to the insulator or to the bore. However, without cooling, the heat-transfer coefficient from the exterior surfaces is low enough (100 W/m²-K) so that, in the time scale of interest, little heat is lost by this path. It would be difficult to conceive of a technique to increase this coefficient to the level at which exterior-surface heat losses would be effective without some type of forced cooling on the surface. Thus, for the two cases calculated without
cooling, pulse rates of 3.3 and 10 Hz, the rail is essentially an adiabatic system.

The results from the no-cooling option show that heat losses from the rail exterior without surface cooling are not large enough to allow steady-state operation under the conditions examined here. Rail heat capacity is sufficient to absorb about six current pulses before the inside corner of the rail (Node 101 and the surrounding nodes) begins to melt.

Single-Channel Cooling

The simplest cooling configuration is probably a single, central cooling channel in the rail. This configuration was approximated here by replacing four rail nodes (905, 906, 1005, and 1006) with a cooling channel (see Fig. 2). The other half of the rail contains a similar cooling arrangement that results in one central channel. Calculations were done for three cases: 3.3 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\text{K} \), 10 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \), and 10 Hz with \( h_c = 5 \times 10^6 \text{ W/m}^2\text{K} \).

Figure 6 shows a plot of temperatures of three nodes (101, 405, and 1001) for the 10-Hz calculation with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \). All three node temperatures show a steady rise during the 10 cycles; Node 101 shows significant melting during the last three cycles, and thus, not as large a temperature rise per cycle. At this repetition rate, heat flow to the central coolant channel. Thus, a decrease (by a factor of 10) in the coolant temperature (near Node 101) approached melting. The single central channel does not provide adequate cooling at this repetition rate. A similar calculation at 10 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\text{K} \) also showed over-heating. At 3.3 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\text{K} \), the rail reaches a steady state after about seven current pulses, and on subsequent pulses, peak temperatures do not increase. The lower repetition rate has allowed time for heat flow to the central coolant channel. Thus, this configuration could be considered as an acceptable steady-state design.

The results of the single-channel-cooling configuration show a restricted range of steady-state operation. Repetition rates of about 3 Hz and lower are acceptable; however, the heat-transfer coefficient to the coolant must be very high.

Two-Channel Cooling

A slightly more complicated coolant channel arrangement would be to put one channel in the center of each half of a rail. In Fig. 2, where Nodes 605, 606, 705, and 706 have been replaced by a coolant channel, this is called two-channel cooling. The total cross-sectional area of the coolant channels is the same as with single-channel cooling. Calculations were done for three cases: 3.3 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\text{K} \), 3.3 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \), and 10 Hz with \( h_c = 5 \times 10^6 \text{ W/m}^2\text{K} \).

As with the single-channel configuration for the same conditions, the two-channel cooling shows acceptable steady-state behavior at 3.3-Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \). Rail temperatures are lower with two-channel cooling because heat-flow paths from the high joule-heating areas to the coolant channel are shorter. A similar calculation at 3.3 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\text{K} \) showed a steady rise in rail temperatures during the 10 cycles; inside corner temperatures (near Node 101) approached melting. Thus, a decrease (by a factor of 10) in the coolant heat-transfer coefficient led to overheating at 3.3 Hz. Another calculation at 10 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \) also resulted in overheating.

The results from the two-channel cooling configuration did not show much improvement over single-channel cooling. Repetition rates of 3-4 Hz and lower are acceptable. Coolant heat-transfer coefficients must still be very high.

Distributed Cooling

The third cooling configuration examined is called distributed cooling (see Fig. 2). Six nodes (404, 405, 704, 707, 1004, and 1007) that are distributed around the half-rail cross section are replaced by coolant channels. The total cross-sectional area of the coolant channels is 50% greater than with single-channel or two-channel cooling, and the coolant channels are closer to the high joule-heating areas of the rail. Calculations were done for four cases: 3.3 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\text{K} \), 10 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\text{K} \), 10 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \), and 33 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \).

Figure 7 shows a plot of temperatures of three nodes (101, 405, and 1001) for the 10-Hz calculation with \( h_c = 5 \times 10^5 \text{ W/m}^2\text{K} \). The rail reaches a steady state after about five cycles. Although
Node-101 peak temperatures are over 1000 K, this configuration is probably an acceptable design. This is the first case that was acceptable at 10 Hz; the distributed cooling configuration has moved coolant channels close enough to the high joule-heating areas of the rail to cool it with only 100 ms between current pulses. At 10-Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\cdot\text{K} \), (a reduction by a factor of 10 of the coolant heat-transfer coefficient) rail melting occurs. These two calculations show the importance of the coolant heat-transfer coefficient when heat-flow paths through the rail are short. A calculation at 3.3 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\cdot\text{K} \) led to acceptable steady-state behavior. This is the first case in which the lower value of \( h_c \) was acceptable. Acceptable behavior occurs because the heat-flow paths in the solid are short enough that the overall resistance to heat flow from the high joule-heating areas to the coolant is low. At a higher repetition rate, (33-Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\cdot\text{K} \)) the inside corner of Node 101 and the surrounding nodes melts. Other parts of the rail are still increasing in temperature after 10 cycles. Thus, this higher repetition rate leads to unacceptable behavior.

The distributed cooling configuration shows the best behavior so far. Repetition rates up to about 10 Hz are acceptable if the coolant heat-transfer coefficient is kept very high. If \( h_c = 5 \times 10^4 \text{ W/m}^2\cdot\text{K} \), pulse frequencies up to about 3 Hz are acceptable.

**Surface/Distributed Cooling**

The final cooling configuration examined is called surface/distributed cooling in Fig. 2. Six nodes (404, 407, 704, 707, 1004, and 1007) were replaced by coolant channels; this is the same as with the distributed cooling configuration. In addition, coolant channels are assumed to contact the rail exterior surface along the interior corner, the top, and the right side (see Fig. 2). The total cross-sectional area of the coolant channels is greater than in any of the previous configurations, and the coolant is placed adjacent to the surfaces where joule heating is high. Calculations were done for four cases: 33 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\cdot\text{K} \), 100 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\cdot\text{K} \), 10 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\cdot\text{K} \), and 3.3 Hz with \( h_c = 5 \times 10^4 \text{ W/m}^2\cdot\text{K} \).

At 33-Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\cdot\text{K} \), the rail reaches a steady state in which all temperatures are below 1100 K after about 8 cycles. Because of the surface cooling around the interior corner and the high coolant heat-transfer coefficient, Node 101 is no longer the hottest node; Node 301 has the highest peak temperatures. The use of surface cooling has allowed an acceptable design at a repetition rate of 33 Hz. A calculation at 100 Hz with \( h_c = 5 \times 10^5 \text{ W/m}^2\cdot\text{K} \) led to rail melting along the rail surface adjacent to the bore where no surface cooling was available. Two other calculations were done for this coolant channel distribution with \( h_c = 5 \times 10^4 \text{ W/m}^2\cdot\text{K} \). At 10 Hz, rail temperatures approaching melting occurred, whereas a steady state was attained at 3.3 Hz, with peak temperatures of less than 1000 K near the interior corner.

The surface/distributed cooling configuration shows the best capabilities of all the cooling configurations tested. Placing coolant at the rail surface near the high joule-heating regions allowed repetition rates up to nearly 30 Hz if the coolant heat-transfer coefficient was kept very high. A rail design that incorporated surface cooling would be more difficult than using only interior cooling channels. The primary benefit of the more complex design would be about a factor of 3 higher allowable repetition rate.

**CORRELATION OF THE RESULTS**

The results that were presented in detail in the previous section can be summarized in terms of their acceptability or unacceptability for steady-state operation. Table I presents these judgments. Three parameters were varied in the calculations: the repetition rate or current pulse frequency, the heat-transfer coefficient to the coolant (\( h_c \)), and the coolant-channel distribution. The repetition rate and heat-transfer coefficient have numerical values and can thus be discussed quantitatively. So far, the various coolant-channel distributions have only been discussed qualitatively. In an attempt to correlate the acceptability criterion with these parameters, the various coolant-channel distributions were ordered by an estimate of the heat-flow path length from the regions of high joule heating to the coolant channels; Table I lists the values assigned. For single-channel cooling, the coolant channel is about 8 mm from the rail corners, where joule heating is highest (see Fig. 2). For two-channel cooling, this path length is reduced to about 5 mm. With distributed cooling, heat-flow path lengths from the corners to the coolant are 2.5 mm. With distributed/surface cooling, the corners are cooled by the surface coolant channels, but the rail edge in the bore is still about 1.5 mm from the coolant. Although these assignments are approximate, they have proved useful in correlating the results of this analysis.

<table>
<thead>
<tr>
<th>Repetition Rate (Hz)</th>
<th>No cooling</th>
<th>Single-channel cooling</th>
<th>Two-channel cooling</th>
<th>Distributed cooling</th>
<th>Surface/distributed cooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>U</td>
<td>U</td>
<td>A</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>10</td>
<td>U</td>
<td>U</td>
<td>A</td>
<td>A</td>
<td>U</td>
</tr>
<tr>
<td>33</td>
<td>U</td>
<td>U</td>
<td>A</td>
<td>A</td>
<td>U</td>
</tr>
<tr>
<td>100</td>
<td>U</td>
<td>U</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

(a) denotes unacceptable design; A denotes acceptable design; — denotes no results. Units of \( h_c \) are \( \text{W/m}^2\cdot\text{K} \).

**TABLE II**

**HEAT-FLOW PATH LENGTHS**

<table>
<thead>
<tr>
<th>Coolant-Channel Distribution</th>
<th>Heat-Flow Path Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-channel cooling</td>
<td>8</td>
</tr>
<tr>
<td>Two-channel cooling</td>
<td>5</td>
</tr>
<tr>
<td>Distributed cooling</td>
<td>2.5</td>
</tr>
<tr>
<td>Surface/distributed cooling</td>
<td>1.5</td>
</tr>
</tbody>
</table>
The slab (T) decays exponentially, before significant heat transfer occurs. Between current pulses, the temperature increase from the current pulse must decay back to the initial temperature before the next current pulse occurs. In the limit of long times, the temperature at the center of the slab (T) decays exponentially,

\[ T = \exp(-\alpha^2 t), \]

where \( \alpha \) is the first positive root of

\[ \tan(2\alpha) = 2\alpha(h_c/k)/[(\alpha^2 - (h_c/k)^2)]. \]

In these equations, \( \kappa \) is the thermal diffusivity of the slab, \( k \) is the thermal conductivity, \( h_c \) is the surface heat-transfer coefficient, and \( t \) is the time. The units of \( \alpha \) are (length\(^{-1}\)), the quantity \((1/\alpha)\) is a characteristic length that is a combination of the actual heat-flow path length \( L \), \( h_c \), and \( k \).

Temperature decay is controlled by the dimensionless number, \( \kappa a^2 t_d \), where \( t_d \) is the time available for the temperature to decay. If \( t_d \) is the time between current pulses, \( t_d = 1/f \), where \( f \) is the repetition rate or current pulse frequency, and the

**Fig. 8.** Correlation of acceptable and unacceptable designs as a function of pulse frequency and heat-flow path length for \( h_c = 5 \times 10^6 \text{ W/m}^2\text{K}. \)

Figure 8 shows a plot with pulse frequency and heat-flow path length as axes. The locations of the various calculations done with \( h_c = 5 \times 10^6 \text{ W/m}^2\text{K} \) are shown on the plot; squares are plotted for unacceptable design conditions and triangles for acceptable design conditions (see Table I). Thus, for two-channel cooling (path length = 5 mm), the result at 5.3 Hz repetition rate was acceptable, but the result of 10 Hz was unacceptable (see Table I). Figure 9 shows a similar plot for calculations done with \( h_c = 5 \times 10^6 \text{ W/m}^2\text{K} \). The general trend in both figures is the same; the lower-left region of each plot (lower repetition rates and shorter path lengths) contains acceptable designs, and the upper-right region contains unacceptable designs.

The solid lines drawn in Figs. 8 and 9 were obtained from an analysis of a one-dimensional heat-transfer problem: time-dependent heat flow in a slab of thickness \( 2\ell \) that is initially at a uniform temperature, \( T_0 \), and loses heat through its surfaces to a medium at zero temperature \([7]\). This is a one-dimensional analog of heat transfer between the high joule-heating regions of a rail and the coolant channels. The fast current pulse heats the rail before significant heat transfer occurs. Between current pulses, the temperature increase from the current pulse must decay back to the initial temperature before the next current pulse occurs. In the limit of long times, the temperature at the center of the slab (T) decays exponentially,

\[ T = \exp(-\alpha^2 t), \]

where \( \alpha \) is the first positive root of

\[ \tan(2\alpha) = 2\alpha(h_c/k)/[(\alpha^2 - (h_c/k)^2)]. \]

In these equations, \( \kappa \) is the thermal diffusivity of the slab, \( k \) is the thermal conductivity, \( h_c \) is the surface heat-transfer coefficient, and \( t \) is the time. The units of \( \alpha \) are (length\(^{-1}\)), the quantity \((1/\alpha)\) is a characteristic length that is a combination of the actual heat-flow path length \( L \), \( h_c \), and \( k \).

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**Fig. 9.** Correlation of acceptable and unacceptable designs as a function of pulse frequency and heat-flow path length for \( h_c = 5 \times 10^6 \text{ W/m}^2\text{K}. \)

characteristic number can be written as \( \kappa a^2/f \). The solid lines in Figs. 8 and 9 are lines for which \( \kappa a^2/f = 1 \); the slab was assumed to be copper with \( k = 400 \text{ W/mK} \) and \( \kappa = 1 \times 10^{-4} \text{ m}^2/\text{s} \).

The lines differ in the two figures because the values of \( \alpha \) depend on \( h_c \). It is an empirical observation that, for this particular value of \( \kappa a^2/f \), these lines tend to separate the acceptable and unacceptable regions of the plots. Thus, the parameter \( \kappa a^2/f \) represents a useful scaling parameter for estimating effects of changes in repetition rate, rail thermal conductivity, thermal diffusivity, or surface heat-transfer coefficient.

**REFERENCES**


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