A Sequential Monte Carlo Probability Hypothesis Density Algorithm for Multitarget Track-Before-Detect

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ABSTRACT

In this paper, we present a recursive track-before-detect (TBD) algorithm based on the Probability Hypothesis Density (PHD) filter for multitarget tracking. TBD algorithms are better suited over standard target tracking methods for tracking dim targets in heavy clutter and noise. Classical target tracking, where the measurements are pre-processed at each time step before passing them to the tracking filter results in information loss, which is very damaging if the target signal-to-noise ratio is low. However, in TBD the tracking filter operates directly on the raw measurements at the expense of added computational burden. The development of a recursive TBD algorithm reduces the computational burden over conventional TBD methods, namely, Hough transform, dynamic programming, etc. The TBD is a hard nonlinear non-Gaussian problem even for single target scenarios. Recent advances in Sequential Monte Carlo (SMC) based nonlinear filtering make multitarget TBD feasible. However, the current implementations use a modeling setup to accommodate the varying number of targets where a multiple model SMC based TBD approach is used to solve the problem conditioned on the model, i.e., number of targets. The PHD filter, which propagates only the first-order statistical moment (or the PHD) of the full target posterior, has been shown to be a computationally efficient solution to multitarget tracking problems with varying number of targets. We propose a PHD filter based TBD so that there is no assumption to be made on the number of targets. Simulation results are presented to show the effectiveness of the proposed filter in tracking multiple weak targets.

Keywords: Probability Hypothesis Density Filter, Track-Before-Detect, Sequential Monte Carlo Methods, Multitarget Tracking, Nonlinear Filtering.

1. INTRODUCTION

Detecting and tracking the targets in low Signal-to-Noise Ratio (SNR) environments is a very difficult problem. The thresholding of the sensor data in classical target tracking throws away potentially useful information and results in poor performance. Track Before Detect (TBD) techniques, which allow simultaneous detection and tracking using unthresholded data, show better performance over classical target tracking methods. The conventional TBD algorithms such as Hough transform [1], dynamic programming [2, 3], maximum likelihood estimation [4] require batch processing of several scans of measurement data. The development of a recursive Bayesian TBD algorithm reduces the computational burden over conventional TBD methods. Particle filter based implementations of recursive TBD are given in [5, 6]. The particle filter based recursive TBD algorithms offer flexibility in the target and noise models. Moreover, the effects of unknown and fluctuating target intensity, point spread functions and extended objects can be accommodated. A detailed explanation of an implementation of the filter is given in [7] with derivations of Cramér-Rao bounds on system performance. An efficient particle filter implementation for a more realistic simulation scenario with Rayleigh noise to reflect signal processing of an operational sensor such as radar could be found in [8].

An extension of particle based TBD algorithms for multitarget tracking is given in [9]. In this approach a modeling setup is used to accommodate the varying number of targets where a multiple model SMC based TBD approach is used to solve the problem conditioned on the model, i.e., the number of targets. The algorithm can deal with a limited number of targets and it was assumed that the maximum possible number of targets is known.

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In this paper, we propose the Probability Hypothesis Density (PHD) filter [10, 11] for recursive TBD algorithm together with a sequential Monte Carlo approach to implement the algorithm [12]. The resulting algorithm does not require a modeling setup to accommodate varying number of targets and it is computationally efficient when the number of the targets is high. The algorithm requires suitably chosen models for target birth, spawning and disappearance. The number of estimated targets in the surveillance region is given by integral of the PHD or the total weight of the samples. The proposed filter does not have a restriction on the maximum number of possible targets. A simulation based on similar target and sensor models as in [7] for a multitarget setup is presented with results to show the effectiveness of the proposed filter.

This paper is organized as follows. Section 2 describes the track-before-detect problem in the multitarget context. Section 3 gives the implementation details of the proposed algorithm to the problem. The performance of the algorithm on simulated data is given in section 4. Finally, conclusions are given in section 5.

2. SYSTEM SETUP

2.1. Target Model

A general parameterized target dynamics of the t th target is given by

$$\mathbf{x}_{k+1}^t = \mathbf{f}_k(\mathbf{x}_k^t, \mathbf{v}_k) \qquad t = 1, \ldots, N_k \tag{1}$$

where $\mathbf{x}_k^t = [x_k^t \; \dot{x}_k^t \; y_k^t \; \dot{y}_k^t \; I_k^t]$ is the target state vector at time step k, N_k is the number of targets at time step k, $\mathbf{f}_k(\cdot)$ is a nonlinear function and $\mathbf{v}_k$ is the process noise of known statistics. Here (x_k^t, y_k^t), $(\dot{x}_k^t, \dot{y}_k^t)$ and I_k^t denote the position, velocity and the intensity of the target, respectively.

2.2. Sensor Model

The sensor provides two-dimensional images of the surveillance region at an interval of T and each image consists of $n \times m$ resolution cells. Each cell corresponds to a rectangular region of dimensions $\Delta_x \times \Delta_y$ and the center of each cell (i,j) is defined to be at $(i\Delta_x, j\Delta_y)$ for $i = 1, \ldots, n$ and $j = 1, \ldots, m$. The measured intensity $z_k^{(i,j)}$ at resolution cell (i,j) is given by

$$z_k^{(i,j)} = \begin{cases} \displaystyle\sum_{t=1}^{N_k} h_k^{(i,j)}(\mathbf{x}_k^t) + w_k^{(i,j)} & \mathcal{H}_1\text{: if there are } N_k \text{ targets} \\ w_k^{(i,j)} & \mathcal{H}_0\text{: if there are no targets} \end{cases} \tag{2}$$

where $w_k^{(i,j)}$ is the measurement noise of known statistics. The noise is assumed to be independent from cell to cell and from image to image. The target contribution intensity $h_k^{(i,j)}(\mathbf{x}_k^t)$ can be either due to target extent or to the sensor point spread function. If we assume a point target, the target intensity will be distributed over the surrounding cells according to the sensor's point spread function. Thus, for a point target of state $\mathbf{x}_k^t$, the contribution to cell (i,j) is approximated as

$$h_k^{(i,j)}(\mathbf{x}_k^t) \approx \frac{\Delta_x \Delta_y I_k^t}{2\pi\Sigma^2} \exp\left\{-\frac{(i\Delta_x - x_k^t)^2 + (j\Delta_y - y_k^t)^2}{2\Sigma^2}\right\} \tag{3}$$

where Σ represents the amount of blurring introduced by the sensor.

The complete set of measurements at time step k is denoted by

$$\mathbf{z}_k = \left\{z_k^{(i,j)} : \; i = 1, \ldots, n, \; j = 1, \ldots, m\right\} \tag{4}$$

and the set of complete measurements collected up to time step k is denoted by

$$Z_{1:k} = \{\mathbf{z}_i : i = 1, \ldots, k\} \tag{5}$$

The likelihood function for the above sensor model is given as

$$p(\mathbf{z}_k|\mathbf{x}_k^1, \mathbf{x}_k^2, \ldots, \mathbf{x}_k^{N_k}, \mathcal{H}_h) = \begin{cases} \prod_{i=1}^{n}\prod_{j=1}^{m} p_{S+N}(z_k^{(i,j)}|\mathbf{x}_k^1, \mathbf{x}_k^2, \ldots, \mathbf{x}_k^{N_k}) & \text{under } \mathcal{H}_1 \\ \prod_{i=1}^{n}\prod_{j=1}^{m} p_N(z_k^{(i,j)}) & \text{under } \mathcal{H}_0 \end{cases} \tag{6}$$

Here $p_N(z_k^{(i,j)})$ is the pdf of background noise in pixel (i,j), while $p_{S+N}(z_k^{(i,j)}|\mathbf{x}_k^1, \mathbf{x}_k^2, \ldots, \mathbf{x}_k^{N_k})$ is the likelihood of summation of individual target signal plus noise in pixel (i,j), given that the targets are in states $\mathbf{x}_k^1, \mathbf{x}_k^2, \ldots, \mathbf{x}_k^{N_k}$

The probability density functions can be expressed as

$$p_N(z_k^{(i,j)}|\mathcal{H}_0) = \mathcal{N}(z_k^{(i,j)}; 0, \sigma^2) \tag{7}$$

$$p_{S+N}(z_k^{(i,j)}|\mathcal{H}_1) = \mathcal{N}(z_k^{(i,j)}; \sum_{t=1}^{N_k} h_k^{(i,j)}(\mathbf{x}_k^t), \sigma^2) \tag{8}$$

Here, we assume a zero-mean Gaussian background noise $w_k^{(i,j)}$ with standard deviation σ. However, the algorithm presented in this paper is applicable to non-Gaussian measurement noise, as long as the statistics of $w_k^{(i,j)}$ are known for each cell (i,j).

Since the target will affect largely the cells in the vicinity of its location, we can approximate $p(\mathbf{z}_k|\mathbf{x}_k^t)$ as

$$p(\mathbf{z}_k|\mathbf{x}_k^t, \mathcal{H}_1) \approx \prod_{i\in C_i(\mathbf{x}_k^t)}\prod_{j\in C_j(\mathbf{x}_k^t)} p_{S+N}(z_k^{(i,j)}|\mathbf{x}_k^t) \prod_{i\notin C_i(\mathbf{x}_k^t)}\prod_{j\notin C_j(\mathbf{x}_k^t)} p_N(z_k^{(i,j)}) \tag{9}$$

where $C_i(\mathbf{x}_k^t)$ and $C_j(\mathbf{x}_k^t)$ are the sets of subscripts i and j, respectively, corresponding to pixels affected by the target. Here we use the approximation in order to reduce the computational load of the sequential Monte Carlo PHD filter. $C_i(\mathbf{x}_k^n)$ is usually selected as $C_i(\mathbf{x}_k^n) = \{i_0 - q, \ldots, i_0 - 1, i_0, i_0 + 1, \ldots, i_0 + q\}$, where i_0 is the nearest integer value of the particle state vector component $x_k^n = \mathbf{x}_k^n[1]$ and q is a design parameter. $C_j(\mathbf{x}_k^n)$ is also selected by a similar procedure. Let us define the cell that has the most influence by the target.

$$D_r(\mathbf{x}_k^t) = \max_r h_k^{(r,s)}(\mathbf{x}_k^t) \tag{10}$$

$$D_s(\mathbf{x}_k^t) = \max_s h_k^{(r,s)}(\mathbf{x}_k^t) \tag{11}$$

3. PHD FILTER IMPLEMENTATION

This section presents the implementation details of the proposed filter. The PHD filter has a property of estimating the number of targets and it is not necessary to assume the maximum possible number of targets within the surveillance region. However, the algorithm requires suitably chosen models for target birth, spawning and disappearance.

We use an SMC approach to implement the algorithm which provides a mechanism to represent the posterior density by a set of random samples or particles. Each particle consists of a target state with an associated weight. The SMC implementation considered here is structurally similar to Sampling Importance Resampling (SIR) type of particle filter [13].

Let the posterior $D_{k-1|k-1}(\mathbf{x}_{k-1}|Z_{1:k-1})$ be represented by a set of particles $\left\{w_{k-1}^{(p)}, \mathbf{x}_{k-1}^{(p)}\right\}_{p=1}^{L_{k-1}}$. That is,

$$D_{k-1|k-1}(\mathbf{x}_{k-1}, Z_{1:k-1}) = \sum_{p=1}^{L_{k-1}} w_{k-1}^{(p)}\delta(\mathbf{x}_{k-1} - \mathbf{x}_{k-1}^{(p)}) \tag{12}$$

3.1. Prediction

We apply importance sampling to generate samples that approximate the predicted density. Generate samples $\{\mathbf{x}_{k|k-1}^{(p)}\}_{p=1}^{L_{k-1}}$ from the proposal density $q_k(\cdot|\mathbf{x}_{k-1}, Z_k)$ and i.i.d. samples $\{\mathbf{x}_{k|k-1}^{(p)}\}_{p=L_{k-1}+1}^{L_{k-1}+J_k}$ corresponding to new spontaneously born targets from another proposal density $p_k(\cdot|Z_k)$.

$$\mathbf{x}_{k|k-1}^{(p)} \sim \begin{cases} q_k(\cdot|\mathbf{x}_{k-1}, Z_k) & p = 1, \ldots, L_{k-1} \\ p_k(\cdot|Z_k) & p = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases} \tag{13}$$

Then, the weighted approximation of the predicted density is given by

$$D_{k|k-1}(\mathbf{x}_{k|k-1}|Z_{1:k-1}) = \sum_{p=1}^{L_{k-1}+J_k} w_{k|k-1}^{(p)} \delta(\mathbf{x}_{k|k-1} - \mathbf{x}_{k|k-1}^{(p)}) \tag{14}$$

where

$$w_{k|k-1}^{(p)} = \begin{cases} \frac{e_{k|k-1}(\mathbf{x}_{k-1}^{(p)}) f_k(\mathbf{x}_{k|k-1}^{(p)}|\mathbf{x}_{k-1}^{(p)}) + b_{k|k-1}(\mathbf{x}_{k|k-1}^{(p)}|\mathbf{x}_{k-1}^{(p)})}{q_k(\mathbf{x}_{k|k-1}^{(p)}|\mathbf{x}_{k-1}^{(p)}, Z_k)} & p = 1, \ldots, L_{k-1} \\ \frac{\gamma_k(\mathbf{x}_{k|k-1}^{(p)})}{p_k(\mathbf{x}_{k|k-1}^{(p)}|Z_k)} & p = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases} \tag{15}$$

where $e_{k|k-1}(\mathbf{x}_{k-1})$ denotes the probability that a target with state $\mathbf{x}_{k-1}$ will survive at time step k and $b_{k|k-1}(\mathbf{x}_{k|k-1}^{(p)}|\mathbf{x}_{k-1}^{(p)})$ denote the PHD of spawned targets at time step k from a target with state $\mathbf{x}_{k-1}$. The PHD of the new born spontaneous targets at time step k is denoted by $\gamma_k(\mathbf{x}_{k|k-1}^{(p)})$. The samples for new born targets are drawn as follows. For target position components, the proposal density is uniform over the regions of the surveillance area for which $z_k^{(i,j)} > \gamma$, where γ is a predefined threshold value. For target velocity components, the density is uniform between $[-v_{max}, v_{max}]$ where v_{max} is the maximum target speed. For target intensity components, the density is uniform between $[I_{min}, I_{max}]$ where I_{min} and I_{max} are the minimum and maximum target intensity levels, respectively.

3.2. Update

With the availability of measurement image at time step k, the importance weights are computed using the likelihood ratio in cell (i,j) for a target in state $\mathbf{x}_k^n$ as follows.

$$l(z_k^{(i,j)}|\mathbf{x}_k^n) \triangleq \frac{p_{S+N}(z_k^{(i,j)}|\mathbf{x}_k^n)}{p_N(z_k^{(i,j)})} \tag{16}$$

$$= \exp\left\{-\frac{h_k^{(i,j)}(h_k^{(i,j)} - 2z_k^{(i,j)})}{2\sigma^2}\right\} \tag{17}$$

The updated particle weights can be calculated by

$$w_k^{*(p)} = \frac{\prod_{i \in C_i(\mathbf{x}_{k|k-1}^{(p)})} \prod_{j \in C_j(\mathbf{x}_{k|k-1}^{(p)})} l(\mathbf{z}_k^{(i,j)}|\mathbf{x}_{k|k-1}^{(p)})}{\lambda_k + \Psi_k(\mathbf{z}_k^{(r,s)})}$$

$$\text{for } r = D_r(\mathbf{x}_{k|k-1}^{(p)}),\ s = D_s(\mathbf{x}_{k|k-1}^{(p)}) \tag{18}$$

where λ_k is a normalization constant. $\Psi_k(z_k^{(r,s)})$ is given by

$$\Psi_k(z_k^{(r,s)}) = \sum_{p \in P^{r,s}} \prod_{i \in C_i(\mathbf{x}_{k|k-1}^{(p)})} \prod_{j \in C_j(\mathbf{x}_{k|k-1}^{(p)})} l(\mathbf{z}_k^{(i,j)}|\mathbf{x}_{k|k-1}^{(p)}) w_{k|k-1}^{(p)} \tag{19}$$

where the set of particles $P^{r,s}$ is given by

$$P^{r,s} = \left\{ p : p \in \{1, \ldots, L_{k-1} + J_k\};\ r = D_r(\mathbf{x}_{k|k-1}^{(p)}) \text{ and } s = D_s(\mathbf{x}_{k|k-1}^{(p)}) \right\} \tag{20}$$

3.3. Resample

To perform resampling, since the weights are not normalized to unity in PHD filters, the expected number of targets is calculated by summing up the total weights, i.e.,

$$\hat{n}_k^X = \sum_{p=1}^{L_{k-1}+J_k} w_k^{*(p)} \tag{21}$$

Then, the updated particle set $\left\{ w_k^{*(p)}/n_k^X, \mathbf{x}_{k|k-1}^{(p)} \right\}_{p=1}^{L_{k-1}+J_k}$ is resampled to get $\left\{ w_k^{(p)}/n_k^X, \mathbf{x}_k^{(p)} \right\}_{p=1}^{L_k}$ such that the total weight after resampling remains n_k^X. Now, the discrete approximation of the updated posterior density at time step k is given by

$$D_{k|k}(\mathbf{x}_k|Z_{1:k}) = \sum_{p=1}^{L_k} w_k^{(p)} \delta(\mathbf{x}_k - \mathbf{x}_k^{(p)}) \tag{22}$$

4. SIMULATIONS

In this section, a two dimensional tracking example is presented to show the efficacy of the proposed track-before-detect algorithm. The simulation generates random synthetic target trajectories with

- a nearly constant velocity model for target motion.
- a random walk model for target intensity.

Then the dynamic model equation for target dynamics is given by

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{v}_k \tag{23}$$

where

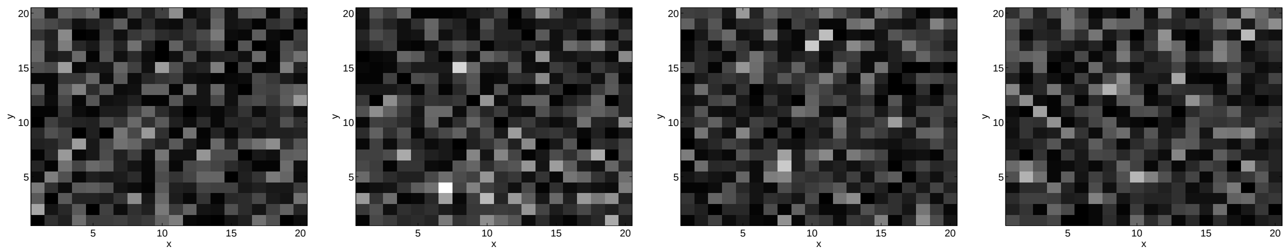
$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

and $\mathbf{v}_k$ is zero-mean white Gaussian noise with covariance $\mathbf{Q}$ given by

$$\mathbf{Q} = \begin{bmatrix} \frac{q_1}{3}T^3 & \frac{q_1}{2}T^2 & 0 & 0 & 0 \\ \frac{q_1}{2}T^2 & q_1 T & 0 & 0 & 0 \\ 0 & 0 & \frac{q_1}{3}T^3 & \frac{q_1}{2}T^2 & 0 \\ 0 & 0 & \frac{q_1}{2}T^2 & q_1 T & 0 \\ 0 & 0 & 0 & 0 & q_2 T \end{bmatrix} \tag{25}$$

where $q_1 = 0.001$ and $q_2 = 0.01$ denote the level of process noise in target motion and intensity, respectively. The adopted target model accommodates deviations from the straight-line motion as well as fluctuations in target intensity.

A sequence of 80 frames of sensor data has been generated with a time interval of $T = 1$ s. The other parameter values are $\Delta_x = \Delta_y = 1$, $n = m = 20$, $\sigma = 1$ and $\Sigma = 0.7$.



(a) Frame 1 (b) Frame 7 (c) Frame 15 (d) Frame 50

Figure 1. Simulated sensor data

Target 1 enters the measurement region at time $T = 7$ s with initial target state [4.2 0.15 7.2 0.10 30]. Target 2 and target 3 enter the region at times $T = 20$ s and $T = 40$ s with initial target states [8.2 0.10 0 0.10 20] and [0 0.10 8.2 0.10 20], respectively. The SNR is given by

$$\mathrm{SNR} = 10 \log \left[\frac{I \Delta_x \Delta_y / 2\pi\Sigma^2}{\sigma} \right]^2 \tag{26}$$

Then, the initial SNR values are 10.23 dB, 6.71 dB and 6.71 dB for target 1, target 2 and target 3, respectively. After entry, all the targets remain within the surveillance region until the last time step. This simulation does not include any spawning of new targets from existing ones.

The filter parameters are taken as follows: The number of particles representing one target $N = 3000$, number of particles representing new born targets $J_k = 40000$, the probability of target survival = 0.99, the probability of target spawning = 0, the probability of spontaneous target birth = 0.01, threshold $\gamma = 2$, $v_{max} = 1$ unit/s, initial intensity range from $I_{min} = 10$ to $I_{max} = 40$, $q = 2$ and $\lambda_k = 2 \times 10^{-5}$.

Figure 1 shows four synthesized measurement data sequence of Frames 1, 7, 15 and 50. The cell intensity is shown in grey linear scale with white color indicating the highest intensity. These images are presented to illustrate that by visual inspection it is impossible to detect the existence and infer the locations of the targets with lower SNR.

Figure 2 shows the number of estimated targets based on a single run. Here the number of estimated targets is calculated by dividing the total number of particles by the number of particles representing one target. This is justified by the fact that the particles are of equal weight after the resampling step. The figure shows that targets with lower SNR, i.e., target 2 and target 3, take more time to get detected by the algorithm.

Figure 3 shows the target position samples at times 19 s, 39 s and 75 s. The cross indicates current true target position. The initial samples are distributed uniformly within cells that have intensity levels above a predefined threshold value. Eventually, several time steps after targets have appeared, particles begin to cluster around the target.

5. CONCLUSIONS

In this paper, we presented a sequential Monte Carlo probability hypothesis density filter for recursive Bayesian track-before-detect. The algorithm can detect and track multiple weak targets in noise. It does not have any restriction on the maximum possible number of targets and offers flexibility in the target and noise models. Through simulation of a 20×20 grid of data, it was shown that multiple targets with SNR as low as 6 dB can be detected and tracked.

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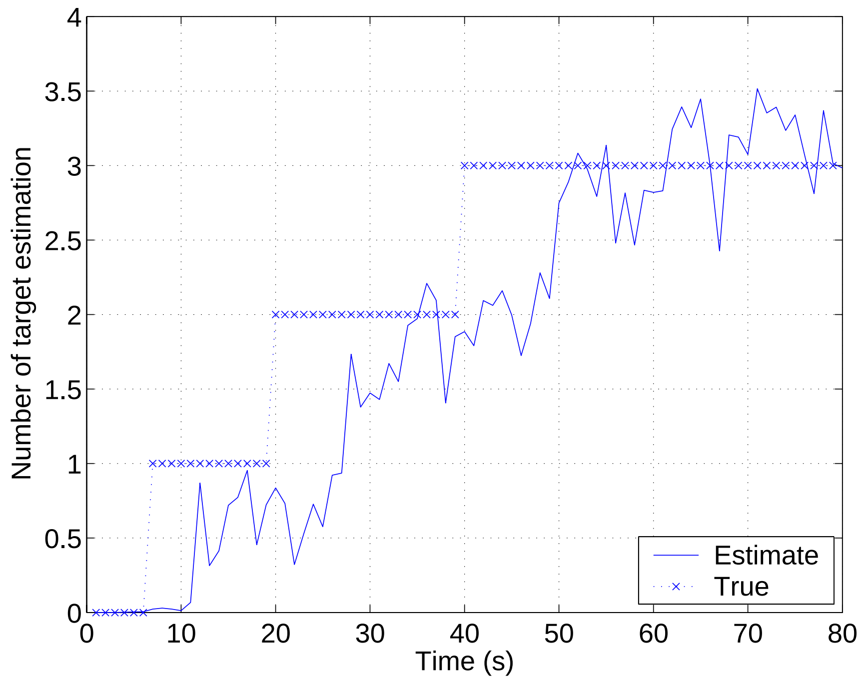

Figure 2. Number of estimated targets (single run)

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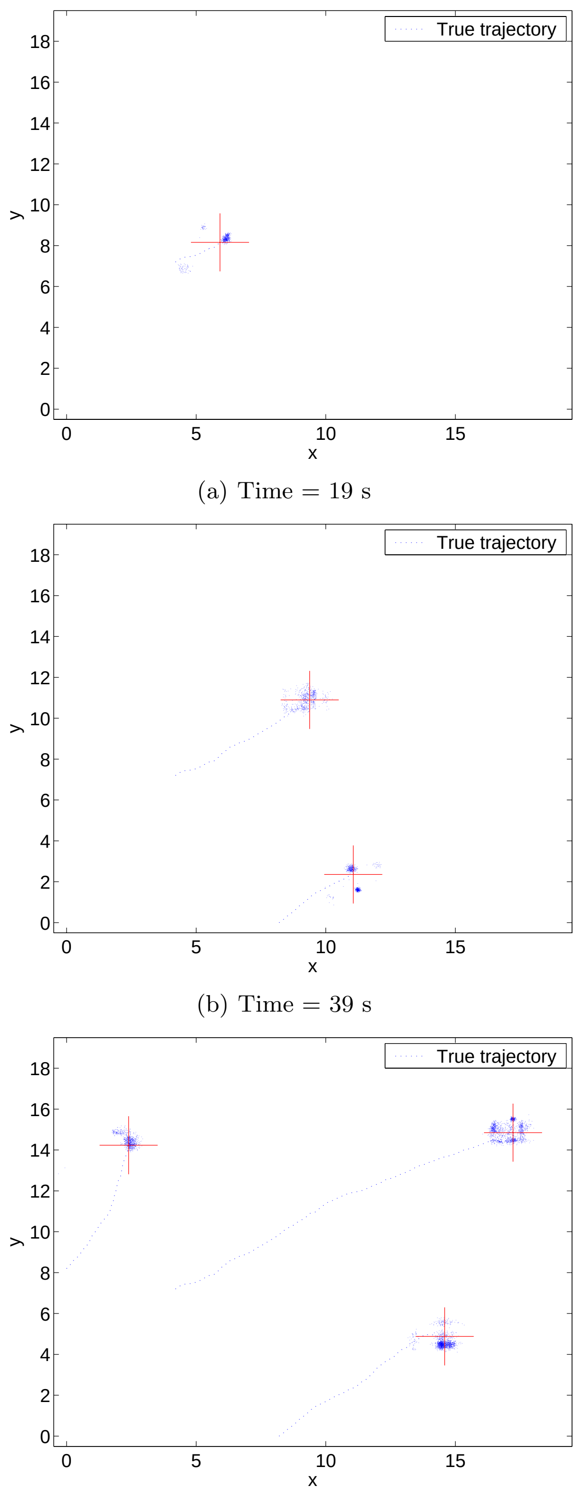

(a) Time = 19 s

(b) Time = 39 s

(c) Time = 75 s

Figure 3. Target position particles