Design for Efficiency Optimization and Voltage Controllability of Series–Series Compensated Inductive Power Transfer Systems

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Abstract—Inductive power transfer (IPT) is an emerging technology that may create new possibilities for wireless power charging and transfer applications. However, the rather complex control method and low efficiency remain the key obstructing factors for general deployment. In a regularly compensated IPT circuit, high efficiency and controllability of the voltage transfer function are always conflicting requirements under varying load conditions. In this paper, the relationships among compensation parameters, circuit efficiency, voltage transfer function, and conduction angle of the input current relative to the input voltage are studied. A design and optimization method is proposed to achieve a better overall efficiency as well as good output voltage controllability. An IPT system design procedure is illustrated with design curves to achieve a desirable voltage transfer ratio, optimizing between efficiency enhancement and current rating of the switches. The analysis is supported with experimental results.

Index Terms—Inductive power transfer (IPT), loosely coupled transformer, resonance power converter, series–series compensation, voltage transfer function.

I. INTRODUCTION

In the application of magnetic induction to transfer electric power wirelessly, good coupling is the fundamental factor for effective operation. Development in modern power electronics, however, enables many new loosely coupled energy transmission applications. An inductive power transfer (IPT) system is designed to deliver power to a load over relatively large air gap via magnetic coupling. The fundamental principles of such systems are identical to the widely used electromechanical devices with good coupling such as transformers and induction motors, where the leakage inductance is much lower than the mutual inductance. Because of electrical isolation and the absence of mechanical contact between the power supply side and the load, the IPT system has advantages of low maintenance cost, high reliability, and the ability to operate in ultraclean or ultradirty environment. Common applications of this technology include wireless power supply to home appliances such as electric toothbrush, wireless charging of mobile phones using a charging platform [1]–[3], and in medical use such as wireless power supply to implantable devices [4], [5]. Medium- to high-power applications of this technology include continuous power transfer to people movers [6], and contactless battery charging for moving actuator [7], or electric vehicles [8]–[10].

It is common that IPT involves a large separation between the primary and secondary winding [11], [12]. Therefore, the electric characteristics of the transformers used are very different from those of conventional transformers which have good coupling between the windings. Due to large winding separation, IPT has a relatively large leakage inductance, as well as increased proximity-effect and winding resistances. Furthermore, for an IPT system where the primary and secondary winding are separated by an air gap, the magnetizing flux is significantly reduced. Thus, a higher magnetizing current and circuit compensation for the larger leakage inductances are needed.

Compensation is often required for both primary and secondary of the loosely coupled transformer to enhance the power transfer capability, to minimize the voltampere (VA) rating of the power supply, and to regulate the value of current in the supply loop and the voltage of the receiving loop with a higher efficiency [2], [4], [13]–[15]. Several studies have been carried out in order to select the most appropriate compensation topology for the IPT systems, depending on the application and taking into account the stability of the configuration [15], [17], [18]. However, these studies focused on either system efficiency or system controllability but rarely considered both. Usually, the compensation technique is selected to pursue maximum power transfer capability, whereas the voltage controllability is mostly neglected, leading to significant fluctuation in the output voltage under load change. An additional secondary power regulator is therefore needed for applications requiring a stable output voltage.

In this paper, a compensation technique considering both good output voltage controllability and high system efficiency is proposed. A systematic design procedure for the circuit and loosely coupled transformer parameters are elaborated.
Fig. 1. S–S compensation topology.

Fig. 2. Equivalent circuit model of Fig. 1.

II. CIRCUIT CHARACTERISTICS

A. Circuit Model and Efficiency

A commonly-used loosely coupled transformer model, as shown in Fig. 1, with series–series (S-S) external capacitors compensation is analyzed in this paper. Transformer inductances \( L_P \) in the primary, \( L_S \) in the secondary, and mutual inductance \( M \) are components of the transformer model shown in Fig. 1. \( R_P \) and \( R_S \) are the winding resistances (which are varying with frequency) of the transformers primary and secondary, respectively. \( C_P \) and \( C_S \) are the primary and secondary external compensation capacitors, for enhancing energy transfer from an ac source \( v_{in} \) to an output loading resistance \( R_L \). The ac source is generally an equivalent voltage generated from a half- or full-bridge switching circuit operating at an angular frequency \( \omega \).

In the subsequent analysis, a frequency-domain equivalent circuit is adopted and only the fundamental component is considered here for simplicity [9], [14], [15]. The fundamental component approximation is sufficiently accurate for a high-quality factor resonant circuit that works near resonance. Fig. 2 gives an equivalent circuit of Fig. 1 for the analysis of steady-state solutions. The current-dependent source \( \omega M i_S \) in Fig. 2 can be replaced by an equivalent impedance \( Z_r \) which is calculated by dividing \( \omega M i_S \) with \( i_P \). In this way, the primary loop is decoupled from the secondary loop. We have

\[
Z_r = \frac{\omega^2 M^2}{Z_s + R_L}
\]

where \( Z_s \) is the impedance of the secondary network given by

\[
Z_s = j\omega L_S + \frac{1}{j\omega C_S} + R_S.
\]

An expression for power transfer efficiency can be obtained by solely considering active power and thus the efficiency \( \eta_P \) in the primary loop and the efficiency \( \eta_S \) in the secondary loop are calculated separately as

\[
\eta_P = \frac{\mathbb{R}(Z_r)}{R_P + \mathbb{R}(Z_r)}, \quad \text{and} \quad \eta_S = \frac{R_L}{R_L + R_S}
\]

where the operator “\( \mathbb{R} \)” represents the real component of the corresponding variable, and

\[
\mathbb{R}(Z_r) = \frac{\omega^2 M^2 R_O}{R_O^2 + L_S^2} + \frac{\omega^2 k^2 L_P L_S R_O}{R_O^2 + X_S^2}
\]

where \( R_O = R_S + R_L, X_S = \omega L_S - \frac{1}{\omega C_S} \), and the coupling coefficient is given by

\[
k = \frac{M}{\sqrt{L_P L_S}}.
\]

The transfer efficiency \( \eta_T \) is therefore given by

\[
\eta_T = \eta_P \eta_S.
\]

Using normalized quantities, we have

\[
\eta_T(\omega) = \frac{1 - \frac{Q_O}{Q_S}}{1 + \frac{1 + Q_P^2}{k^2 \frac{Q_O}{Q_S} - \frac{Q_P}{Q_S}}}
\]

where

\[
Q_S = \sqrt{\frac{L_S}{C_S R_S}},
\]

\[
Q_O = \sqrt{\frac{L_S}{C_S R_O}},
\]

\[
Q_P = \sqrt{\frac{L_P}{C_P R_P}},
\]

\[
\omega_P = \frac{1}{\sqrt{L_P C_P}},
\]

\[
\omega_S = \frac{1}{\sqrt{L_S C_S}}.
\]

In practice, the quality factors in (9)–(11) are varying with frequency. The approximate Dowell’s equation [16] which is valid for frequencies much lower than the self-resonant frequency of the inductor is used in this paper. The quality factors can be written as follows:

\[
Q_S(\omega) = \frac{2Q_{S,\max}}{\frac{Q_S}{\omega} + \frac{Q_S}{\omega S}}
\]

\[
Q_P(\omega) = \frac{2Q_{P,\max}}{\frac{Q_P}{\omega} + \frac{Q_P}{\omega P}}
\]

\[
Q_O(\omega) = \frac{Q_S(\omega)Q_L}{Q_S(\omega) + Q_L}
\]

\[
Q_L = \sqrt{\frac{L_S}{C_S R_L}}.
\]
The quality factor $Q_S$ maximizes at $Q_{S,\text{max}}$ when $\omega = \omega_{Q_S}$, likewise for $Q_P$. Substituting (14) to (16) into (8), we have

$$\eta_f(\omega) = \frac{1 - a(\omega)}{1 + b(\omega)}$$

where

$$a(\omega) = \frac{QL}{QL + Q_S(\omega)}$$

and

$$b(\omega) = \frac{1 + Q_D^2(\omega) \left(\frac{\omega}{\omega_{Q}} - \frac{\omega}{\omega_{Q}}\right)^2}{k^2 \frac{\omega}{\omega_{Q}} Q_D(\omega) \frac{\omega}{\omega_{P}} Q_P(\omega)}.$$  

(19)

(20)

**B. Maximum Efficiency and Operating Frequency**

The power loss caused by winding resistances of the loosely coupled transformer has been considered by (18). The power loss imposed on the switching components should be considered in order to achieve an accurate calculation for the overall efficiency. Switching loss is composed of conduction loss and transient loss. We will show in Section IV-A that conduction and switching losses of the switching components can be modeled as equivalent resistances of $R_{DS(on)}$ and $R_{f,\text{equiv}}$, respectively, of which $R_{DS(on)}$ does not depend on frequency and $R_{f,\text{equiv}}$ increases linearly with the operating frequency. In practice, it can be designed such that switching devices satisfy $R_{P} \gg R_{f,\text{equiv}}$. Therefore, $R_{DS(on)} + R_{f,\text{equiv}}$ can be readily absorbed into $R_{P}$ without significantly affecting the frequency property of the $Q_P(\omega)$. From (18), the power transfer efficiency $\eta_T$ can be maximized at a particular operating frequency $\omega = \omega_M$ given a load quality factor $Q_L$. Qualitatively, $\eta_T$ is maximized for a minimum $a$ and a minimum $b$ as $a$ and $b$ are both nonnegative. It can be readily observed that $a$ is minimized when $\omega = \omega_S$. The numerator of $b$ is minimized as 1 when $\omega = \omega_S$ and denominator of $b$ is maximized when $\omega$ is a bit higher than $\omega_Q$ or $\omega_{Q_P}$. So, it is common to design $\omega_{Q_S}$ close to $\omega_{Q_P}$ and compensate the circuit by choosing $\omega_M = \omega_S \approx \omega_{Q_S} \approx \omega_{Q_P}$.

When the fixed operating frequency $\omega_S$ is used, the corresponding efficiency can be obtained by substituting $\omega = \omega_S$ into (18), i.e.,

$$\eta_T(Q_L)\bigg|_{Q_S > Q_O > Q_{0,\text{min}}} = \frac{1 - \frac{Q_{S,\text{max}}}{\omega_S}}{1 + \frac{1 + Q_{S,\text{max}}}{\omega_S Q_{P,\text{max}}}}$$

(21)

where $\eta_T$ has a maximum value when $Q_L = Q_{L_P}$, i.e.,

$$\eta_{T,\text{max}} = \frac{c}{(1 + \sqrt{1 + c})^2}$$

(22)

$$Q_{L_P} = \frac{Q_{S,\text{max}}}{\sqrt{1 + c}}$$

(23)

where $c = k^2 \frac{\omega}{\omega_{P}} Q_{S,\text{max}} Q_{P,\text{max}}$. It is readily observed that $\eta_{T,\text{max}}$ increases with increasing $c$.

Essentially, a power converter should be designed for a suitable range of loading for best efficiency. It can be achieved by introducing a design parameter $0 < \gamma < 1$, such that the converter efficiency $\eta_T$ is always larger than $\gamma \eta_{T,\text{max}}$ for the desired range of loading within $[Q_{L,\text{min}}, Q_{L,\text{max}}]$, where $Q_{L,\text{min}} < Q_{L_P} < Q_{L,\text{max}}$. The boundaries of $Q_{L,\text{min}}$ and $Q_{L,\text{max}}$ are found by equating the RHS of (21) to $\gamma \eta_{T,\text{max}}$ and solving for the roots, which are given as follows:

$$Q_{L,\text{min}} = \frac{Q_{S,\text{max}}}{A + B},$$

(24)

$$Q_{L,\text{max}} = \frac{Q_{S,\text{max}}}{A - B}$$

(25)

where

$$A = 2 + 2c_1 (1 - \gamma) + c_1 c_2 (1 - \gamma)$$

(26)

$$B = \frac{\sqrt{(1 - \gamma)(1 + c) c_2}}{2 c_1}$$

(27)

$$c_1 = \sqrt{1 + c}$$

(28)

$$c_2 = 8 (1 + c_1) + 4 c (2 + c_1) + c^2 (1 - \gamma).$$

(29)

For illustration and verification, SPICE simulations using the transformer model in Fig. 1 are conducted. In the simulation, we use $Q_P = Q_S = 100$ and $k = 0.2$. The operating frequency is $\frac{\omega}{2\pi} = 150$ kHz. For this given set of parameters, $\eta_{T,\text{max}}$ is obtained as 0.90 at $Q_L = 4.99$ using (23). Using $\gamma = 0.9$, $Q_{L,\text{min}}$ and $Q_{L,\text{max}}$ are calculated as 1.17 and 17.39, respectively, using (23)–(25). The simulation results shown in Fig. 3 confirm the theoretical findings in this section.

**C. Output to Input Voltage Transfer Ratio**

For converters with conventional transformers, the ratio of the output voltage compared with the input voltage is determined by the turns ratio of the transformer. For IPT applications using loosely coupled transformer which has compensation in both primary and secondary sides, the compensation capacitors are involved in the voltage transfer function. Therefore, the voltage transfer function will be studied subsequently for optimal operation and circuit design.

From Fig. 2, the output voltage across $R_L$ can be calculated using KVL as

$$v_{out} = j \omega M \frac{R_L}{Z_s + R_L} i_P$$

(30)
where \( i_p \) can be calculated from the decoupled primary loop of Fig. 2 using (1) as
\[
i_p = \frac{v_{in}}{Z_p + \frac{\omega^2 M^2}{Z_p + R_p}}
\]  
where
\[
Z_p = j\omega L_p + \frac{1}{j\omega C_P} + R_p.
\]  

The output-to-input voltage transfer function \( G_v \) is determined by substituting (31) into (30) as
\[
G_v = \frac{v_{out}}{v_{in}} = \frac{j\omega M R_L}{Z_p(Z_s + R_L) + \omega^2 M^2}.
\]  

From Section II-B, the converter can operate at \( \omega = \omega_S \). It will be interesting to know the voltage transfer function operating at \( \omega_S \) when the load changes. We have plotted \( G_v \) using (33) for the set of parameters used in Section II-B and compared with a simplified \( G_v \) with zero \( R_P \) and \( R_S \), and found that there is no significant difference. Therefore, \( R_P \) and \( R_S \) can be assumed zero for simplicity for the subsequence analysis of \( G_v \) in this section.

The curves shown in Fig. 4 are approximated values of \( |G_v| \) with zero \( R_P \) and \( R_S \) at various loads. Usually \( \omega_M \) is assigned as \( \omega_M \) [14], [15] by selecting a suitable \( C_P \), such that \( Z_p \) is zero according to (32). Therefore, for maximum efficiency and operating at \( \omega = \omega_S \),
\[
|G_v(\omega_S)| = \frac{R_L}{\omega_S M}.
\]  

The \( |G_v| \) at \( \omega_S \) is directly proportional to \( R_L \), making it difficult to have good voltage regulation without an additional power converter. In Fig. 4, however, there are two frequency points where the voltage transfer ratio is constant when load changes. To calculate the two frequency points, further manipulation of (33) [19] is conducted to give
\[
G_v = \frac{1}{\frac{Z_p}{j\omega M} + \frac{1}{j\omega M C_P C_P R_L}}
\]  

where
\[
\delta = \frac{\omega^4}{\omega^2 \omega_S^2} (k^2 - 1) + \omega^2 \left( \frac{1}{\omega_P^2} + \frac{1}{\omega_S^2} \right) - 1. \tag{36}
\]

It can be readily seen from (36) that if \( \delta = 0 \), then \( G_v \) is independent of \( R_L \). Solving for roots \( \omega_L \) and \( \omega_H \) of (36), the normalized frequencies at which \( G_v \) is independent of \( R_L \) and can be obtained as
\[
\omega_L = \frac{\omega_L}{\omega_S} = \sqrt{\frac{\omega^2_n + 1 - \lambda_5}{2(1 - k^2)}} \tag{37}
\]
\[
\omega_H = \frac{\omega_H}{\omega_S} = \sqrt{\frac{\omega^2_n + 1 + \lambda_5}{2(1 - k^2)}}. \tag{38}
\]

where \( \lambda_5 = \sqrt{(\omega^2_n - 1) + 4k^2 \omega^2_n} \) and \( \omega_n = \frac{\omega}{\omega_S} \). Operating at frequency \( \omega_H \) or \( \omega_L \) (shown in Fig. 4), the converter has
\[
|G_v(\omega_L)| = k \sqrt{\frac{L_S}{L_P}} \frac{\omega^2_n + 1 - \lambda_5}{(1 - 2k^2) \omega^2_n - 1 + \lambda_5} \tag{39}
\]
\[
|G_v(\omega_H)| = k \sqrt{\frac{L_S}{L_P}} \frac{\omega^2_n + 1 + \lambda_5}{1 - 2k^2 \omega^2_n + 1 + \lambda_5}. \tag{40}
\]

The value of \( |G_v| \) at \( \omega_L \) or \( \omega_H \) has no relationship with the loading resistance. Therefore, if the converter is adjusted to operate at frequency \( \omega_H \) or \( \omega_L \), the voltage controllability is significantly improved when load changes.

D. Input Phase Angle

The input phase angle between the input voltage and current \( (\theta_{in}) \) when working at \( \omega_H \) and \( \omega_L \) is studied to guarantee an inductive input impedance. For H-bridge converters, which is adopted in our experiment, an inductive input impedance is necessary to realize soft switching for better efficiency.

The inductive or capacitive characteristic of \( Z_{in} \) is decided by the sign of its imaginary part, which is given by
\[
\Im(Z_{in}) = Z_p \left( \omega_n - \frac{k^2 \omega^2}{\omega^2 + \omega_S^2} \omega_n \right) \tag{41}
\]
where \( \omega_n = \frac{\omega}{\omega_S} \) and \( \omega_n = \frac{\omega}{\omega_S} \). We take the derivative of \( \Im(Z_{in}) \) to obtain
\[
\frac{d\Im(Z_{in})}{dQ_O} = -\frac{2k^2 \omega^2 - \omega_S^2 \omega_n}{(\omega_S^2 + \frac{1}{Q_O^2})^2 Q_O^2} \tag{42}
\]
which is always smaller (greater) than zero for all \( \omega > \omega_S \) (\( \omega < \omega_S \)). Therefore, the theoretical minimum (maximum) of \( \Im(Z_{in}) \) is at \( Q_O \to \infty \). Substituting either (37) or (38) into (41), we obtain \( \lim_{Q_O \to \infty} \Im(Z_{in}) = 0 \). Hence, \( Z_{in} \) is always inductive (capacitive) when it operates at \( \omega_H \) (\( \omega_L \)). As a result, operating at \( \omega_H \) can provide soft switching when \( Z_{in} \) is driven by a half-bridge or full-bridge switching circuit.

The value of \( \theta_{in} \) is calculated as
\[
\theta_{in} = \frac{180}{\pi} \tan^{-1} \frac{\Im(Z_{in})}{\Re(Z_{in})} \tag{43}
\]
where
\[ \Re(Z_{in}) = \frac{\omega_n^2(1-k^2) + \omega_n^2(k^2-2) + \omega_n^2}{k^2\omega_n^2} + 1 - \frac{(\omega_n^2 - 1)^2 + 2}{k^2\omega_n^2}. \] (44)

Fig. 5 shows the simulation results which confirm the input phase angle analysis. \( \omega_H \) is suitable for H-bridge converters for its inductive input phase angle. In the next section, we will study the tuning of \( C_S \) and \( C_P \) for voltage controllability and best efficiency.

III. PARAMETER OPTIMIZATION AND CIRCUIT DESIGN

A. Compensation Parameter Optimization

In Section II, we have studied three operational angular frequencies \( \omega_S, \omega_L, \) and \( \omega_H \) for the S–S tuned loosely coupled transformer circuit. It has been proven in Section II-B that the converter can give optimal efficiency by operating at the fixed frequency \( \omega_S \) for a load range of \([Q_0, min, Q_0, max]\). On the contrary, operating at \( \omega_S \) as described in Section II-C, the converter gives the worst load voltage regulation. Meanwhile, operating at \( \omega_H \), the converter achieves the best output voltage controllability with soft switching. It will be desirable to have both good efficiency and good load voltage regulation.

Essentially, operation frequency cannot deviate from \( \omega_H \) for good load voltage regulation. It will be desirable to design \( \omega_H \) as close to \( \omega_S \) as possible. From (38), we have
\[ \frac{\partial \omega_h}{\partial \omega_n} = \frac{\omega_n}{(1-k^2)\omega_h} \left( \frac{\lambda_6}{\lambda_5} \right). \] (45)
where
\[ \lambda_6 = \lambda_5 + \omega_n^2 + 2k^2 - 1. \] (46)

We are now going to show that \( \frac{\partial \omega_h}{\partial \omega_n} > 0 \) for all \( \omega_n \geq 0 \). With \( 1 \geq k \geq 0, \omega_h \) will be decreasing and has a minimum of \( \frac{1}{\sqrt{1-k^2}} \) at \( \omega_h = 0 \).

The sign of (45) is determined by \( \lambda_6 \) whose slope is given as
\[ \frac{\partial \lambda_6}{\partial \omega_n} = \omega_n \frac{\lambda_7}{\lambda_5}. \] (47)
where \( \lambda_7 = 2\lambda_5 + 4k^2 + 2(\omega_n^2 - 1) \). The sign of (47) is determined by \( \lambda_7 \) whose slope is given as
\[ \frac{\partial \lambda_7}{\partial \omega_n} = \frac{16k^2(1-k^2)\omega_n}{\lambda_5^3}. \] (48)

It can be readily seen from (48) that \( \frac{\partial \lambda_7}{\partial \omega_n} > 0 \). Therefore, \( \lambda_7 \) has a minimum of \( 4k^2 \) at \( \omega_n = 0 \). Therefore, \( \frac{\partial \lambda_6}{\partial \omega_n} > 4k^2 > 0 \) and \( \lambda_6 \) has a minimum of \( 2k^2 \) at \( \omega_n = 0 \). Hence, \( \frac{\partial \omega_h}{\partial \omega_n} > 0 \) and
\[ \omega_{h,min} = \frac{1}{\sqrt{1-k^2}} \text{ at } \omega_h = 0. \] (49)

From (49), \( \omega_h \) cannot be designed for maximum efficiency at \( \omega_{h,min} \) as \( \omega_n \) cannot be zero. As a compromise, \( \omega_h \) can be chosen as \( \omega_{h,\gamma_1} = \gamma_1 \omega_{h,min} \) where \( \gamma_1 \geq 1 \). The corresponding \( \omega_{n,\gamma_1} \) can be solved using (38) as
\[ \omega_{n,\gamma_1} = \frac{\gamma_1 (\gamma_1 - 1)}{\gamma_1 + k^2 - 1}. \] (50)

A plot of \( \omega_h \) versus \( \omega_n \) using (38) at various coupling coefficients is shown in Fig. 6 which illustrates that \( \omega_h \) decreases toward \( \frac{1}{\sqrt{1-k^2}} \) as \( \omega_h \) decreases. The data points in Fig. 6 indicate the position of \( \omega_h \) when \( \omega_h = \omega_{n,\gamma_1} \). We will therefore expect that the converter operating at \( \omega_H \) for better voltage controllability can have a better efficiency by choosing the compensation \( \omega_n = \omega_{n,\gamma_1} \) rather than the conventional compensation of \( \omega_n = 1 \). The efficiency equation (8) can be rewritten as
\[ \eta_r = \frac{\frac{1}{1 + Q_0 \left( \omega_h - \frac{\omega_n}{\omega_h} \right)^2}}{1 + \frac{1 + Q_0^2 \left( \omega_h - \frac{\omega_n}{\omega_h} \right)^2}{k^2 + \frac{Q_0}{Q_P}}}. \] (51)

Therefore, a transfer efficiency enhancement \( \eta_{r_en} \) can be calculated using the difference of efficiency designed with \( \omega_n = \omega_{n,\gamma_1} \) and \( \omega_n = 1 \), where \( 0 < \omega_{n,\gamma_1} \leq 1 \). The actual enhancement operating at \( \omega_h \) is given as
\[ \eta_{r_en} = \eta_r (\omega_n = \omega_{n,\gamma_1}) - \eta_r (\omega_n = 1). \] (52)
Fig. 7 shows the efficiency enhancement versus \( k \) and \( Q_O \) with various values of \( \omega_{n, \gamma_1} \). Fig. 7 gives us the information that:

1) the efficiency enhancement is increasingly obvious with larger \( Q_O \) and smaller \( \omega_{n, \gamma_1} \);

2) the enhancement peaks at smaller \( k \) and vanishes when \( k = 0 \) or 1.

### B. Circuit Design

From the analysis in Section III-A, the design with \( \omega_{n, \gamma_1} \) can improve transfer efficiency and operating at \( \omega_H \) can keep the voltage transfer ratio constant. In this section, we focus on choosing the value of \( \omega_{n, \gamma_1} \) and give design procedures for the system considering higher efficiency and voltage controllability.

A design example is illustrated in this section. The specifications are given as follows. The converter has a rated power \( P_{out} \) of 18 W, an output voltage of 12 V, and an input voltage of 48 V.

The size of the transformer is application specific. In this design example, the air gap between the two coils \( g \) is 35 mm. It has outer diameter \( d_1 = 90 \) mm and inner diameter \( d_2 = 9 \) mm. More detailed dimensions are shown in Fig. 8.

The coupling coefficient can be calculated according to the dimensions shown in Fig. 8 by using Neumann’s formula [20],

\[
\begin{align*}
L & = \frac{\mu_0}{4\pi} N^2 \oint \oint dl \cdot dl' \\
M & = \frac{\mu_0}{4\pi} N_P N_S \oint \oint dl \cdot dl' \\
\end{align*}
\]

which are applied to the sample transformer in Fig. 8, giving [22]

\[
L = \frac{\mu_0}{8\pi} N^2 d \cdot \Psi \\
M = \frac{\mu_0}{4\pi} N_P N_S d \cdot \Phi
\]

where

\[
\begin{align*}
d & = \frac{d_1 + d_2}{2} \\
\Psi & = \frac{(1 + \rho)^3}{\rho^2} \left( 1.7424 + 3.29\kappa^3 \ln \kappa - 2.27\kappa^3 + 0.3702\kappa^5 + 0.0826\kappa^7 + 0.0312\kappa^9 \right) \\
\rho & = \frac{r}{d} \\
\kappa & = \frac{d_2}{d_1}
\end{align*}
\]

and the value of \( \Phi \) can be determined by the table look-up scheme in [22] to be 1.4 for the example transformer. Substituting (54) into (6), we obtain

\[
k = \frac{2\Psi}{\Phi} = 0.18.
\]

Quality factors \( Q_P \) and \( Q_S \) of the transformer windings can be estimated according to the wire material and manufacturer data sheets. For accurate efficiency estimation, quality factors can be measured after the completion of the design of the transformer physical dimensions. In our design, \( Q_{P, max} \) and \( Q_{S, max} \) are both estimated as 100 when working at 200 kHz for the Litz wires used.

Using the estimated quality factors, the converter transfer efficiency and the input phase angle at various \( Q_O \) and \( \omega_{n, \gamma_1} \) are plotted using (51) and (43). The sample design curves are as shown in Fig. 9.

The selection of \( \omega_{n, \gamma_1} \) should take into account the efficiency enhancement and the amount of inactive power which affects
The current rating of switches. We select $\omega_{n,1}$ as 0.7. When $\omega_{n,1}$ is chosen, optimal $Q_O$ is selected according to Fig. 9(a).

The inductance of the secondary coil can be calculated using (60). In the example design, $Q_O$ is approximately 5, and $L_S = 33.51 \, \mu H$. The number of turns $N_S$ is calculated to be around 29 turns based on (55). Using the current in the secondary loop, which can be calculated by $i_s = \frac{v_{out}}{R_L}$ to choose the secondary total wire sectional area

$$L_S = \frac{R_L}{\omega_S \left( \frac{1}{Q_O} - \frac{1}{Q_S} \right)}.$$  

The compensation capacitor in the secondary is calculated using (13) as $C_S = 18.9 \, nF$.

The value of $L_P$ is determined by substituting the selected $\omega_{n,1}$ and $L_S$ into (40), according to the desired voltage transfer ratio. In our example, $|G_v| = 0.25$ and $L_P = 38.21 \, \mu H$. As (40) is calculated without considering $R_P$ and $R_S$, which may result in a lower voltage transfer ratio. Therefore, in transformer fabrication, $L_P$ should be designed slightly smaller to satisfy the requirement of $G_v$. Based on (55), the number of turns of $L_P$ is calculated as 30. Using the output power and the approximate efficiency, the input active power can be calculated. The input phase angle in Fig. 9 indicates the proportion of input active power to inactive power. Therefore, the apparent power as well as input current can be estimated for the determination of sectional area of the Litz wire used in the primary. In our design, we use ready-made Litz wire in our laboratory that can surely satisfy the current requirement.

Finally, the compensation capacitor in the primary can be calculated according to (12), (13) at the selected $\omega_{n,1}$ as $C_P = 33.15 \, nF$.

### IV. Evaluation

#### A. Overall Efficiency Calculation

A full-bridge circuit shown in Fig. 10 with four MOSFETs is used for driving the converter circuit. Besides the power loss caused by winding resistances of the loosely coupled transformer, the power loss imposed on the switching components cannot be neglected in order to have an accurate overall efficiency calculation. Switching loss is composed of conduction loss and transient loss, which are calculated separately in the following.

Switching transient loss calculation is conducted first. Due to the impedance experienced by the full-bridge circuit is inductive, zero-voltage-on of the four MOSFET switches can be realized. The turn-on loss is assumed zero. When turned OFF, however, the voltage and current stresses on the four MOSFETs overlap, as shown in Fig. 11. The figure assumes that the current through MOSFET reaches zero before its voltage rises up to $v_{IN}$ when it turns OFF. It is easy to realize by increasing the capacitance between drain and source by external capacitor.

The switching loss of each MOSFET in unit time can be expressed as

$$P_{LS} = f \frac{I_{in}^2}{16C_{DS}}t_f^2.$$  

where $C_{DS}$ is the drain-to-source capacitance and $t_f$ is the fall time upon switching of the MOSFETs selected. $I_{in}$ is the MOSFET drain current just before the switching transient, which is given as

$$I_{in} = \sqrt{2}v_{in} \sin \theta_{in}.$$
Hence,

\[ P_{Ls} = i_{in}^2 f \left( \frac{t_f \sin \theta_{in}}{8C_{DS}} \right)^2 = i_{in}^2 R_{f,eqv} \]  

(63)

where \( R_{f,eqv} \) is the equivalent switching loss resistance of the MOSFET.

For conduction loss of the MOSFETs, it can be calculated as \( i_{in}^2 R_{DS(on)} \), where \( R_{DS(on)} \) is the conduction resistance of the MOSFET selected. The power loss of the rectifier bridge diode in the secondary loop is calculated by multiplying its voltage drop and an output current.

According to the design parameters in Section III-B, the transformer is built and shown in Fig. 12. In addition, the quality factors of the windings are measured under 1-A current, which is shown in Fig. 13.

Table I gives the measured parameters of the prototype and the components used in experiment.

### B. Experimental Evaluation

A prototype is constructed using the components in Table I to verify the analytical results. The measured \( \omega_H \) and \( G_v \) for \( \omega_n, \gamma = 0.7 \) is depicted in Fig. 15(a), showing the good voltage regulation at \( \omega_H \) with respect to load change. This agrees well with the calculation result in Fig. 14(a). Efficiency versus \( Q_O \) has been measured and compared with calculation results for the two \( \omega_n, \gamma \) values of 0.7 and 1, as Fig. 15(b) shows. By decreasing the \( \omega_n, \gamma \) value, there is an obvious improvement in efficiency especially at heavy load.

In addition, more experiments are conducted by decreasing the air gap. Fig. 16 shows the measured efficiency results of \( g = 27 \) mm \( (k = 0.264) \) and \( g = 20 \) mm \( (k = 0.355) \) with \( \omega_n, \gamma = 1 \) and \( \omega_n, \gamma = 0.7 \), respectively.

**TABLE I**

<table>
<thead>
<tr>
<th>Circuit components</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power MOSFET ( Q_1-Q_4 )</td>
<td>IRF640N</td>
</tr>
<tr>
<td>Capacitance ( C_f )</td>
<td>33.12 nF/630V</td>
</tr>
<tr>
<td>Capacitance ( C_S )</td>
<td>18.9 nF/630V</td>
</tr>
<tr>
<td>Schottky diode ( D_1-D_4 )</td>
<td>STPS20HI00CG</td>
</tr>
<tr>
<td>Capacitance ( C_f )</td>
<td>220 ( \mu )F</td>
</tr>
<tr>
<td>Loosely-coupled transformer</td>
<td>( N_p=30 ) turns of Litz wire</td>
</tr>
<tr>
<td></td>
<td>( L_P=32.78 \mu H ) Q(_P)=94.9@200 kHz</td>
</tr>
<tr>
<td></td>
<td>( N_S=29 ) turns of Litz wire</td>
</tr>
<tr>
<td></td>
<td>( L_P=31.46 \mu H ) Q(_P)=89.0@200 kHz</td>
</tr>
<tr>
<td></td>
<td>air gap = 35 mm</td>
</tr>
<tr>
<td></td>
<td>( k=0.182 )</td>
</tr>
</tbody>
</table>

The calculated overall efficiency using MOSFET IRF640N with \( R_{DS(on)} = 0.18 \Omega \), \( C_{DS} = 1.96 \) nF, \( t_f = 35 \) ns, as well as the voltage transfer ratio at \( \omega_n, \gamma = 0.7 \) are shown in Fig. 14. The efficiency at \( \omega_n, \gamma = 1 \) is also shown in Fig. 14(b) for comparison. With \( R_P + 2R_{DS(on)} = 0.794 \) \( \Omega \) and \( 2R_{f,eqv} = 0.032 \) \( \Omega \), conduction loss is dominating when operating at around 200 kHz. The switching loss may be higher if higher working frequency is selected.
V. Conclusion

The operating frequency of an inductive power transfer system that achieves an optimal efficiency, a load independent voltage transfer ratio and an inductive input phase angle is studied in this paper. A compensation technique to optimize the system efficiency and output voltage controllability of an inductive power transfer system is proposed. The value of $\omega_P$ compared to $\omega_S$ is selected as a design parameter for the efficiency enhancement. A circuit design procedure is explicated with an example experimental prototype. Experimental measurements confirm the analytical results and the design procedures in achieving both efficiency optimization and voltage controllability.

REFERENCES


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