Sensorless rotor position estimation of PMSM by full-order and sliding mode EMF observers with speed estimate

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Abstract: The paper discusses the problem of rotor position estimation for the nonsalient PMSM and presents a Full-Order (FO) observer and a Sliding Mode (SM) observer; both are constructed using a speed estimate. The observers are developed using the PMSM model in the stationary reference frame and use the motor voltages, currents and speed as inputs. The PMSM is treated as a time-varying plant. Convergence is analysed assuming that the speed signal is inaccurate - this corresponds to the situation when a speed estimate is used to obtain a sensorless observer. Under improper speed, the FO observer gives errors; however, the SM observer can be forced to converge by increasing the gains. The SM structure proposed allows design of a variety of sensorless observers.

Keywords: PMSM; permanent magnet synchronous motor; rotor position estimation; full-order observer; sliding mode observer; sensorless control; electric vehicles; hybrid electric vehicles.

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Biographical note: Mihai Comanescu received a BS from Bucharest Polytechnic Institute in 1992 and his MS and PhD in Electrical Engineering from The Ohio State University, in 2001 and 2005 respectively. Currently, he is an Assistant Professor of Electrical Engineering at Penn State, Altoona. His research interests are in the area of AC motor drives, control theory, power electronics and renewable energy applications.

1 Introduction

The Permanent Magnet Synchronous Motor (PMSM) is an excellent candidate for high performance power conversion applications because of its high efficiency, high power density and low noise. The PMSM is widely used in electric, hybrid electric and traction applications (Bazzi and Krein, 2010; Finken et al., 2008; Afjei et al., 2006; Chan et al., 1996). The PMSM is sometimes preferred over the induction motor because the absence of the rotor winding reduces heat dissipation and allows for an enclosed design (Nasiri, 2007). Estimation and control methods for the PMSM have been studied extensively (Acarnley and Watson, 2006; Bose, 2002; Jahns, 1994; Harnefors et al., 2003; Monajemy
and Krishnan, 2001; Morimoto et al., 2002; Perera et al., 2003; Seok et al., 2006; Shyu et al., 2002; Vas, 1998). Generally, the PMSM can be controlled using the traditional V/Hz method – this works well at steady-state; however, the transients are not very well controlled. In high-performance applications or when rapid and precise acceleration is needed (e.g., electric vehicles, high-speed drives), the PMSM is controlled using the field-oriented method (vector control). In this case, the angle of rotor position must be known. Rotor position can be measured with a shaft–mounted encoder; however, this increases the cost, reduces the drive’s ruggedness, is sensitive to vibrations and does not work well at high speeds. To eliminate the encoder, the rotor position angle should be estimated based on the system measurements.

In sensorless PMSM methods, the rotor position is estimated using measured electrical quantities. It is standard to measure the motor voltages and currents and to estimate (or compute directly) all the subsequent quantities that are needed in the control algorithm (usually fluxes, speed and rotor position).

Several rotor position estimation methods are available. Generally, they belong to one of the following two classes: model-based methods or magnetic saliency methods.

Magnetic saliency methods are based on the variation of the inductance between the d-q axes and are constructed using signal injection (Corley and Lorenz, 1996; Schroedl, 1996; Lim et al., 1994). They are relatively complicated for real-time implementation, but they work well at low speed. Using high-frequency injection, the rotor position can also be estimated at standstill (Hu et al., 2006). As a result, since the orientation of the N and S poles is known, there is no need to align the rotor and the motor can be started in the desired direction, without oscillations, right from the beginning (Ostlund and Brokemper, 1996).

Model-based methods use one of the PMSM models (in either the stationary or the rotating reference frame) to construct observers for state estimation. For a given plant model, several methods can be used to design observers, depending on the type of plant (linear or nonlinear, time-varying or time-invariant) and the type of observer (full-order, reduced-order, sliding-mode, adaptive, etc). For the PMSM, many state observers have been published – some authors treat the PMSM plant as time-varying, while others consider that the speed varies slowly and design observers using time-invariant methods (eigenvalue placement).

Depending on the approach used, several PMSM observers are available: full-order observers (Batzel and Lee, 2005; Yamamoto et al., 2004) use continuous feedback and the entire set of plant equations to estimate the state vector. Reduced-order observers (Comanescu and Batzel, 2009; Kim and Sul, 1995; Krichen et al., 2007; Tatematsu et al., 1998) are easier to implement, since only a subset of the plant equations are involved. The states of the PMSM model can also be estimated using the Extended Kalman Filter (EKF) method (Bolognani et al., 1999; Huang et al., 2006). In the EKF, the speed is appended as an additional state in the state-space model of the motor and is estimated. However, the method is relatively difficult to implement and accuracy is problematic, because the EKF involves the entire set of equations and motor parameters.

Depending on the PMSM model used, an observer may estimate either the fluxes or the EMFs of the motor.

Finally, several sliding mode (SM) observers were presented by Utkin et al. (1999), Yan and Utkin (2002), Li and Elbuluk (2002, 2003), Chi et al. (2007), Chi and Sun (2008), Paponpen and Konghirun (2006), Liu et al. (2009), Foo and Rahman (2010). Other recent SM methods have been published by Xie (2006), Lascu and Andreescu
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In particular, the observer by Li and Elbuluk (2003) and its versions published by Chi et al. (2007), Chi and Sun (2008), Paponpen and Kongsirun (2006) is very popular and simple to implement – it uses only the current equations of the PMSM and allows direct estimation of the EMFs. However, the EMFs are obtained by filtering the switching terms of the observer and, if a wide speed range is intended, filtering leads to error in the rotor position estimate. The error may be reduced by using a linear correction term (Li and Elbuluk, 2003); however, accurate correction based on real-time frequency estimation (Comanescu and Xu, 2005) requires significant effort and increases the complexity.

This paper presents two methods to estimate the rotor position of the PMSM: the first is a Full-Order EMF observer; the second is a SM EMF observer. They are both developed based on the motor’s model in the stationary reference frame. The input quantities of the observers are the motor voltages and currents (which are measured) and a speed estimate (which is used instead of the real speed). The speed of the PMSM can be routinely estimated with various methods based on electrical measurements (Bose, 2002; Vas, 1998). In development, the PMSM is treated as a time-varying plant and convergence is investigated using Lyapunov’s stability theory.

The paper first develops the observers under the assumption that the speed signal is available and shows how to design the feedback gains. Second, these structures are analysed assuming an incorrect speed signal. This corresponds to the situation where an inaccurate speed estimate is used instead of the real speed in an attempt to obtain a sensorless observer. The paper investigates convergence under improper speed and the nature of the errors obtained. For the FO observer, it is shown that estimation errors are not influenced by the observer’s design gains – the errors cannot be reduced or eliminated using these gains. Using the FO structure, the paper develops an observer that uses a speed estimate which is obtained by differentiating the estimated (output) rotor position. Its performance is demonstrated experimentally.

Second, the paper discusses a SM EMF observer. The gains of this observer are designed and convergence is analysed based on Lyapunov’s theory. It is shown that under improper speed, the SM observer has an important property: its gains can be used to reduce the estimation errors that appear because of the speed mismatch.

Another important advantage is that the SM structure allows ideal integration of the observer’s equations (due to the discontinuous feedback). Therefore, there is no need to substitute integration with low pass filtering when the equations are implemented; as a result, in the low speed range, the method outperforms the linear or nonlinear observers that use continuous feedback. Also, the SM-based estimation method does not involve low pass filtering, and does not require rotor position correction.

2 Modelling of nonsalient PMSM

The equations of the PMSM in the stationary reference frame with the state vector \( \begin{bmatrix} \lambda_a & \lambda_b & \lambda_{\alpha \mu} & \lambda_{\beta \mu} \end{bmatrix} \) are:
The output equations are:

\[
\begin{align*}
    i_a &= \frac{1}{L} \dot{\lambda}_a - \frac{1}{L} \dot{\lambda}_{PMa} \\
    i_b &= \frac{1}{L} \dot{\lambda}_b - \frac{1}{L} \dot{\lambda}_{PMb}
\end{align*}
\]  

(2)

The permanent magnet fluxes \( \lambda_{PMa}, \lambda_{PMb} \) are obtained by projecting the vector of the permanent magnet flux on the stationary reference frame:

\[
\begin{align*}
    \lambda_{PMa} &= \lambda_{PM} \cos \theta \\
    \lambda_{PMb} &= \lambda_{PM} \sin \theta
\end{align*}
\]

(3)

where \( \theta = \omega t \) is the rotor position angle and \( \lambda_{PM} = K_E \). The motor EMFs \( e_a, e_b \) are defined as the derivatives of the permanent magnet fluxes. System (1) is manipulated to include the currents in the state vector and the equations with respect with the new state vector \([\lambda_a, \lambda_b, i_a, i_b]^T\) are:

\[
\begin{align*}
    p\dot{\lambda}_a &= -Ri_a + \frac{1}{L} V_a \\
    p\dot{\lambda}_b &= -Ri_b + \frac{1}{L} V_b \\
    p\dot{i}_a &= \frac{1}{L} \omega_b \lambda_b - \frac{R}{L} i_a - \omega_b i_b + \frac{1}{L} V_a \\
    p\dot{i}_b &= -\frac{1}{L} \omega_a \lambda_a + \omega_a i_a - \frac{R}{L} i_b + \frac{1}{L} V_b
\end{align*}
\]

(4)

In permanent magnet motors with rotor-mounted magnets, the stator winding sees a large effective air gap (the magnet permeability is close to that of air); therefore, armature reaction is small and these motors have low inductance. As a result, there is generally a wide separation between the electrical and mechanical time constants (Batzel and Lee, 2005) – the electrical signals vary much faster than the speed and it can be assumed that \( \dot{\omega}_e = 0 \). If we differentiate the fluxes (3) with this assumption, the result is:

\[
\begin{align*}
    e_a &= p\dot{\lambda}_{PMa} = -K_E \omega_e \sin \theta \\
    e_b &= p\dot{\lambda}_{PMb} = K_E \omega_e \cos \theta
\end{align*}
\]

(5)

The equations with respect to the currents and EMFs are:
Sensorless rotor position estimation of PMSM by full-order

\[
\begin{align*}
\dot{e}_a &= -\omega e_\beta \\
\dot{e}_\beta &= \omega e_a \\
\dot{i}_a &= -\frac{R}{L}i_a - \frac{1}{L}e_a + \frac{1}{L}V_a \\
\dot{i}_\beta &= -\frac{R}{L}i_\beta - \frac{1}{L}e_\beta + \frac{1}{L}V_\beta 
\end{align*}
\] (6)

In matrix form, the model of the PMSM can be written as:

\[
\begin{pmatrix}
e_a \\
e_\beta \\
i_a \\
i_\beta
\end{pmatrix} =
\begin{bmatrix}
0 & -\omega & 0 & 0 \\
\omega & 0 & 0 & 0 \\
-\frac{1}{L} & 0 & -\frac{R}{L} & 0 \\
0 & -\frac{1}{L} & 0 & -\frac{R}{L}
\end{bmatrix}
\begin{pmatrix}
e_a \\
e_\beta \\
i_a \\
i_\beta
\end{pmatrix} + \begin{pmatrix}
0 & 0 \\
0 & 0 \\
\frac{1}{L} & 0 \\
0 & \frac{1}{L}
\end{pmatrix}
\begin{pmatrix}
V_a \\
V_\beta
\end{pmatrix}
\] (7)

\[
\begin{pmatrix}
i_a \\
i_\beta
\end{pmatrix}
= \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix} \begin{pmatrix}
e_a \\
e_\beta \\
i_a \\
i_\beta
\end{pmatrix}^T .
\] (8)

3 Full-order observer for EMF estimation

The full-order observer proposed is developed based on the model above. Equations (7) and (8) are already in the standard form:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\] (9)

and the matrices \( A, B, C \) can be directly identified.

Since the speed \( \omega \) appears twice in matrix \( A \), the PMSM is treated as a time-varying system (\( A \) is time-varying). The observer is designed using continuous feedback of the current mismatches. The equations are:

\[
\begin{pmatrix}
\dot{e}_a \\
\dot{e}_\beta \\
\dot{i}_a \\
\dot{i}_\beta
\end{pmatrix} =
\begin{bmatrix}
0 & -\omega & 0 & 0 \\
\omega & 0 & 0 & 0 \\
-\frac{1}{L} & 0 & -\frac{R}{L} & 0 \\
0 & -\frac{1}{L} & 0 & -\frac{R}{L}
\end{bmatrix}
\begin{pmatrix}
\dot{e}_a \\
\dot{e}_\beta \\
\dot{i}_a \\
\dot{i}_\beta
\end{pmatrix} + \begin{pmatrix}
0 & 0 \\
0 & 0 \\
\frac{1}{L} & 0 \\
0 & \frac{1}{L}
\end{pmatrix}
\begin{pmatrix}
V_a \\
V_\beta
\end{pmatrix}
\]

\[
\begin{pmatrix}
i_{11} \\
i_{21} \\
i_{31} \\
i_{41}
i_{12} \\
i_{22} \\
i_{32} \\
i_{42}
i_{13} \\
i_{23} \\
i_{33} \\
i_{43}
i_{14} \\
i_{24} \\
i_{34} \\
i_{44}
i_{15} \\
i_{25} \\
i_{35} \\
i_{45}
i_{16} \\
i_{26} \\
i_{36} \\
i_{46}
i_{17} \\
i_{27} \\
i_{37} \\
i_{47}
i_{18} \\
i_{28} \\
i_{38} \\
i_{48}
i_{19} \\
i_{29} \\
i_{39} \\
i_{49}
i
\end{pmatrix} + \begin{pmatrix}
\tau_a \\
\tau_\beta
\end{pmatrix}
\] (10)
where \( l_{11} \) to \( l_{42} \) are the eight design gains and

\[
\begin{bmatrix}
\tau_w \\
\tau_p
\end{bmatrix} = \begin{bmatrix}
l_{11}
\\
l_{12}
\\
l_{21}
\\
l_{22}
\\
l_{31}
\\
l_{32}
\\
l_{41}
\\
l_{42}
\end{bmatrix} - \begin{bmatrix}
iu
\\
i\rho
\end{bmatrix}.
\]

(11)

In the development, \( \nu, \nu_p \) and \( iu \) and \( i\rho \) are considered known (measured quantities); the speed \( \omega \) will initially be considered known.

After subtracting equation (7) from equation (10), the mismatch equations are:

\[
\begin{bmatrix}
\varepsilon_a \\
\varepsilon_p
\end{bmatrix} = \begin{bmatrix}
0 & -\omega \\
\omega & 0
\end{bmatrix} \begin{bmatrix}
\varepsilon_a \\
\varepsilon_p
\end{bmatrix} = \begin{bmatrix}
l_{11} & l_{12} \\
l_{21} & l_{22}
\end{bmatrix} \begin{bmatrix}
\tau_w \\
\tau_p
\end{bmatrix}.
\]

(12)

To study the convergence of this observer, the method based on the analysis of eigenvalues cannot be used (the system is not time invariant). Instead, select the Lyapunov function:

\[
V = \frac{1}{2} (\varepsilon_a^2 + \varepsilon_p^2 + \tau_w^2 + \tau_p^2).
\]

(13)

After differentiation and after replacing the derivatives from equation (12), the expression of \( \dot{V} \) is:

\[
\dot{V} = \varepsilon_a ( -\omega \varepsilon_p + l_{11} \tau_w + l_{12} \tau_p ) + \varepsilon_p ( \omega \varepsilon_a + l_{21} \tau_a + l_{22} \tau_p ) + \tau_a \left( -\frac{1}{L} \varepsilon_a - \frac{R}{L} \varepsilon_a + l_{31} \tau_a + l_{32} \tau_p \right) + \tau_p \left( -\frac{1}{L} \varepsilon_p + l_{41} \tau_a + l_{42} \tau_p \right).
\]

(14)

This becomes:

\[
\dot{V} = -\omega \varepsilon_a \varepsilon_p + l_{11} \varepsilon_a \tau_w + l_{12} \varepsilon_a \tau_p + l_{31} \varepsilon_a \tau_a + l_{32} \varepsilon_a \tau_p + l_{41} \varepsilon_p \tau_a + l_{42} \varepsilon_p \tau_p - \frac{R}{L} \varepsilon_a^2 - \frac{1}{L} \varepsilon_a \tau_w - \frac{R}{L} \varepsilon_p^2 - \frac{1}{L} \varepsilon_p \tau_p + l_{41} \varepsilon_a \tau_p + l_{42} \tau_p^2.
\]

(15)

Based on equation (15), the design gains must be selected to make \( \dot{V} \) negative definite. First, to cancel the terms that contain \( \varepsilon_a \tau_a \) and \( \varepsilon_p \tau_p \), the gains \( l_{11} \) and \( l_{22} \) are chosen as:

\[
l_{11} = l_{22} = \frac{1}{L}.
\]

(16)

Then, the terms that contain \( \varepsilon_a \tau_a \) and \( \varepsilon_p \tau_p \) are eliminated with:

\[
l_{12} = l_{21} = 0.
\]

(17)

The terms with \( \tau_a \tau_p \) are not useful towards making \( \dot{V} \) negative and they are also eliminated:
Sensorless rotor position estimation of PMSM by full-order

\( l_{\alpha} = l_{\beta} = 0. \) (18)

The terms in \( T_\alpha^2 \) and \( T_\beta^2 \) are retained, and the gains \( l_{\alpha} \) and \( l_{\beta} \) are chosen as:

\( l_{\alpha} = l_{\beta} = -k, \quad k > 0. \) (19)

With the above gains, the expression of \( \dot{V} \) becomes:

\[
\dot{V} = -\left( \frac{R}{L} + k \right) \left( T_\alpha^2 + T_\beta^2 \right).
\] (20)

Therefore, as long as the current mismatches \( \tau_\alpha \) and \( \tau_\beta \) are different from zero, \( \dot{V} \) is negative and \( V \) decays. When \( \tau_\alpha = 0 \) and \( \tau_\beta = 0 \), the Lyapunov function stops decaying and settles into an equilibrium point. If we replace \( \tau_\alpha = 0, \tau_\beta = 0 \) in equation (12), it follows that, at this equilibrium point, \( \tau_\alpha = 0 \) and \( \tau_\beta = 0 \).

In conclusion, all mismatches are zero and the observer is asymptotically stable. With the gains (equations (16)–(19)), the equations of the observer are:

\[
\begin{align*}
\dot{p}_\alpha &= -\omega_e \dot{e}_\beta + \frac{1}{L} \tau_\alpha, \\
\dot{p}_\beta &= \omega_e \dot{e}_\alpha + \frac{1}{L} \tau_\beta, \\
p_\alpha &= -\frac{R}{L} \dot{p}_\alpha - \frac{1}{L} e_\alpha + \frac{1}{L} V_\alpha - k \tau_\alpha, \\
p_\beta &= -\frac{R}{L} \dot{p}_\beta - \frac{1}{L} e_\beta + \frac{1}{L} V_\beta - k \tau_\beta
\end{align*}
\] (21)

where \( k \) is to be selected, \( k > 0. \)

Some observations about the properties of this observer: first, note that the derivative of the Lyapunov function in equation (20) is negative even if \( k = 0. \) Second, since the squares of \( e_\alpha \) and \( e_\beta \) do not appear on the right side of equation (20), it is not possible to write a relationship of the type \( \dot{V} = -\frac{R}{L} + k \dot{V} \).

This would lead to a closed form solution

\[
V(t) = V(0) e^{-\left( \frac{R}{L} + k \right) t}
\]

in which case the decay of \( V \) could be accelerated by increasing \( k. \) Finally, note that implementation of the observer equation (21) requires accurate knowledge of the PMSM speed \( \omega_e. \)

Next, the observer’s behaviour is studied under the assumption that the speed signal fed in the observer is different from the real speed of the motor. The intention is to use a speed estimate instead of the real speed in order to obtain a sensorless observer (however, a speed estimate is likely to be inaccurate).

This section investigates the observer’s convergence under this condition, derives the analytical expressions of the estimation errors and explores whether the errors are influenced by the design gains.

Consider that the speed signal fed in the observer is:

\[
\dot{\omega}_e = \omega_e + \Delta \omega_e
\] (22)
where $\Delta \omega_e$ is the difference between the speed estimate and the real speed and is unknown. Then, the observer equations will correspond to equations (10) or (21), except that they use equation (22). After subtracting the original equations, the mismatches are:

$$
\begin{align*}
\rho \bar{e}_\alpha &= -\omega_{\bar{e}_\beta} + \frac{1}{L} \bar{e}_\alpha - \Delta \omega \dot{\bar{e}}_\beta \\
\rho \bar{e}_\beta &= \omega_{\bar{e}_\alpha} + \frac{1}{L} \bar{e}_\beta + \Delta \omega \dot{\bar{e}}_\alpha \\
p \bar{e}_\alpha &= -\frac{R}{L} \bar{e}_\alpha - \frac{1}{L} \bar{e}_\alpha + \frac{1}{L} V_a - k \bar{e}_\alpha \\
p \bar{e}_\beta &= -\frac{R}{L} \bar{e}_\beta - \frac{1}{L} \bar{e}_\beta + \frac{1}{L} V_\beta - k \bar{e}_\beta
\end{align*}
$$

(23)

Using the Lyapunov function (13) and the gains in equations (16)–(19), the derivative of $V$ is:

$$
\dot{V} = -\left( \frac{R}{L} + k \right) (\bar{e}_\alpha^2 + \bar{e}_\beta^2) + \Delta \omega \left( \dot{\bar{e}}_\alpha \bar{e}_\beta - \dot{\bar{e}}_\beta \bar{e}_\alpha \right).
$$

(24)

In equation (24), the first term is always negative, the second term has an unknown sign. As long as the current mismatches are large, function $V$ decays; however, $V$ stops decaying when:

$$
\left( \frac{R}{L} + k \right) (\bar{e}_\alpha^2 + \bar{e}_\beta^2) = \Delta \omega \left( \dot{\bar{e}}_\alpha \bar{e}_\beta - \dot{\bar{e}}_\beta \bar{e}_\alpha \right).
$$

(25)

Using the notation $\bar{e}_\beta = m \bar{e}_\alpha$, ($m$ is unknown but close to 1 since $\bar{e}_\alpha$ and $\bar{e}_\beta$ should be of the same order of magnitude), equation (25) can be rewritten to give the values of the mismatches $\bar{e}_\alpha$, $\bar{e}_\beta$ at which the Lyapunov function stop decaying:

$$
\begin{align*}
\bar{e}_\alpha &= \frac{\left( \frac{R}{L} + k \right) (\bar{e}_\alpha^2 + \bar{e}_\beta^2)}{\Delta \omega (m \bar{e}_\alpha - \dot{\bar{e}}_\beta)}; \\
\bar{e}_\beta &= m \frac{\left( \frac{R}{L} + k \right) (\bar{e}_\alpha^2 + \bar{e}_\beta^2)}{\Delta \omega (m \bar{e}_\alpha - \dot{\bar{e}}_\beta)}.
\end{align*}
$$

(26)

The expressions in equation (26) are not a good result; this is because the design parameter $k$ appears in the numerator. Had $k$ been in the denominator, it would have been possible to increase this gain to reduce the estimation errors.

In conclusion, for the FO observer presented, estimation errors due to the speed difference $\Delta \omega_e$ cannot be reduced using $k$. The magnitude of the mismatches $\bar{e}_\alpha$, $\bar{e}_\beta$ and the size of the vicinity where the Lyapunov function settles will be investigated through simulations.

4 Simulations of the full-order observer

The FO observer is simulated using Matlab/Simulink. The parameters of the PMSM used are given in Table 1.
In the simulation, the PMSM drive is operated in speed control mode. The motor is started towards a speed of 1000 rpm with a load torque of 0.1 Nm. The simulation model uses PI controllers to regulate the $d, q$ axis currents. A PI controller is also used for speed regulation (this produces the reference current $\dot{c}_f$). On the $d$ axis, $\dot{c}_d = 0$. The simulation model neglects the PWM switching of the inverter; also, operation in the flux weakening region is not investigated. The simulation is done in discrete time with a sampling time of 50 µs.

In the initial investigation, the measured speed of the PMSM is fed in the observer. The motor is oriented with the correct rotor position (at start-up, $\theta_0 = 0$). The observer runs in parallel with the PMSM model and its outputs are captured.

Figures 1 and 2 show the real and estimated $\alpha, \beta$ currents and EMFs when the correct speed is fed in the observer; $k = 1000$ and $R/L = 13,888$ in equation (20). Note that the estimates converge, the observer is stable and this result confirms the theoretical analysis and the proposed gain design.

**Figure 1** Real and estimated currents of the FO observer with correct speed, $k = 1000$

<table>
<thead>
<tr>
<th>Table 1</th>
<th>PMSM parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance ($R$)</td>
<td>2.5 Ω</td>
</tr>
<tr>
<td>Synchronous inductance ($L$)</td>
<td>1.8 mH</td>
</tr>
<tr>
<td>Rated voltage ($V_n$)</td>
<td>18.2 V</td>
</tr>
<tr>
<td>Rated continuous torque ($T_c$)</td>
<td>50 oz-in</td>
</tr>
<tr>
<td>Rated speed ($n$)</td>
<td>6000 rpm</td>
</tr>
<tr>
<td>Number of poles ($P$)</td>
<td>4</td>
</tr>
</tbody>
</table>

In the simulation, the PMSM drive is operated in speed control mode. The motor is started towards a speed of 1000 rpm with a load torque of 0.1 Nm. The simulation model uses PI controllers to regulate the $d, q$ axis currents. A PI controller is also used for speed regulation (this produces the reference current $\dot{c}_f$). On the $d$ axis, $\dot{c}_d = 0$. The simulation model neglects the PWM switching of the inverter; also, operation in the flux weakening region is not investigated. The simulation is done in discrete time with a sampling time of 50 µs.

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Figures 1 and 2 show the real and estimated $\alpha, \beta$ currents and EMFs when the correct speed is fed in the observer; $k = 1000$ and $R/L = 13,888$ in equation (20). Note that the estimates converge, the observer is stable and this result confirms the theoretical analysis and the proposed gain design.

**Figure 1** Real and estimated currents of the FO observer with correct speed, $k = 1000$
Figures 2–6 were obtained by feeding an improper speed in the observer. The speed used is ±30% different from the real speed, \( k = 1000 \). Note that the estimates do not converge anymore. Interestingly, the current errors are significant, while the EMF errors are small.

The estimated rotor position is shown vs. the real rotor position – the error is approximately 15–20°. Several other values of \( k \) were tried – it was found that they do not reduce the errors and they do not improve the quality of the estimates. When \( k \) is increased, the accuracy of the EMF estimates is maintained, while the errors in the current estimates increase.

Figure 2  Real and estimated EMFs of the FO observer with correct speed, \( k = 1000 \)

Figure 3  Real and estimated currents of the FO observer with –30% improper speed, \( k = 1000 \)
Figure 4  Real and estimated EMFs of the FO observer with −30% improper speed, $k = 1000$

![Figure 4](image1.png)

Figure 5  Real and estimated currents of the FO observer with improper speed (+30%), $k = 1000$

![Figure 5](image2.png)

Figure 6  Real and estimated EMFs of the FO observer with improper speed (+30%), $k = 1000$

![Figure 6](image3.png)
5 Experimental results of the full-order observer

The PMSM used in the experiment is a MCG motor (parameters correspond to Table 1). The motor is powered by a Spectrum Digital MOSFET inverter. The DSP used for controller implementation is TMS320F2812 (32-bit fixed-point). The control algorithm is executed in one 50 µs interrupt (the inverter switches at 20 kHz). The currents are measured using sensing resistors placed in the lower legs of the inverter bridge. The stator voltages are computed using the duty cycles of the Space Vector PWM algorithm and the measured dc bus voltage. The motor used has an optical encoder with 512 pulses/revolution, and this is used to sense the rotor position angle.

Experimental waveforms are obtained using the second PWM unit of the F2812 DSP chip. The variables of interest are used as PWM duty cycles and the resulting output waveforms are filtered with hardware RC filters and displayed on a scope (three signals can be displayed at a time).

The motor is initially started and operated in open loop mode (V/Hz method). The observer studied runs in parallel with the control algorithm – the estimated currents, EMFs and rotor position are captured. A diagram of the experimental algorithm is shown in Figure 7.

Figure 7  Block diagram of the algorithm used in the experimental testing of the FO observer
In the experiment, the speed of the PMSM is computed by differentiating the measured rotor position followed by low pass filtering.

\[ \omega_t = \frac{1}{T_s} (\theta_{t+1} - \theta_t). \]  

(27)

The speed obtained with equation (27) is the measured speed.

For the experimental setup, the base quantities (peak values) for voltage, current and speed are: \( V_b = 18.6 \, V \), \( I_s = 2.86 \, A \), \( \omega_b = 377 \, \text{rad/s} \).

In the real-time implementation, the observer equations are discretised using Euler’s method. The ideal integrators are replaced with low pass filters (the \( 1/s \) transfer function is replaced with \( 1/ \omega_c s + 1 \)). The equations are:

\[
\begin{align*}
\hat{e}_{\alpha, k+1} &= \mu \hat{e}_{\alpha, k} - T_S \omega_b \hat{\theta}_t \hat{e}_{\beta, k} + \frac{T_e}{L} \frac{i_{\alpha}}{V_b} \\
\hat{e}_{\beta, k+1} &= \mu \hat{e}_{\beta, k} + T_S \omega_b \hat{\theta}_t \hat{e}_{\alpha, k} + \frac{T_e}{L} \frac{i_{\beta}}{V_b} \\
i_{\alpha, k+1} &= \left( \mu - \frac{R}{L} T_S \right) i_{\alpha, k} - \frac{V_b}{L} T_e \frac{i_{\alpha, k}}{I_s} (\hat{e}_{\alpha, k} - V_a + k^* \tau) \\
i_{\beta, k+1} &= \left( \mu - \frac{R}{L} T_S \right) i_{\beta, k} - \frac{V_b}{L} T_e \frac{i_{\beta, k}}{I_s} (\hat{e}_{\beta, k} - V_\beta + k^* \tau) \\
\mu &= 1 - \omega_b T_s.
\end{align*}
\]  

(28)

(29)

The gain \( k' \) is expressed in per unit, its relationship to the gain \( k \) in equation (21) is \( k' = \frac{L_c}{k} \). In the experiment, \( \mu = 0.999 \).

The waveforms in Figures 8–10 are looking to validate the observer implementation; they were obtained by feeding the measured speed in the observer. The motor runs at 900 rpm, \( k' = 0.5 \) and the load torque is 0.1 Nm. Figure 8 shows the real and estimated \( \alpha \) axis currents – note that they approximately correspond (the same for the \( \beta \) currents). Figure 9 shows the estimated EMFs and the estimated rotor position. In Figure 10, the estimate is shown vs. the measured rotor position. The estimates are of good quality and the estimated rotor position is very close to the measured one.

Next, the observer is studied under detuned speed signal. The measured speed is detuned by \( \pm 30\% \) (similar to the simulations) and the resulting signal is fed in the observer.

Figures 11 and 12 were obtained with a speed detuning of \( -30\% \); \( k' = 0.5 \) and 0.1 Nm of load torque. Figure 11 shows that the \( \alpha \) current \( i_{\alpha} \) differs from \( i_{\alpha} \) in both magnitude and phase. This experimental result corresponds to the simulation in Figure 3. Figure 12 shows the real vs. estimated rotor position – despite the inaccurate speed used, the error is relatively small.

The experiment is repeated and the speed is detuned by \( \pm 30\% \) \( (k' = 0.5) \). Figure 13 shows the estimated current and the real current on the \( \alpha \) axis. The difference in magnitude/phase shown corresponds to the simulation result in Figure 7. The estimated rotor position is shown vs. the measured rotor position in Figure 14 – the error is less than 20°.
Figure 8  Current $i_a$ and estimated current $\hat{i}_a$ of the FO observer, 20 ms/div, 0.86 A/div

Figure 9  Estimated EMFs and rotor position of the FO observer, 20 ms/div, 5.6 V/div

Figure 10  Real vs. estimated rotor position of the FO observer, 20 ms/div, 300 deg./div, no speed detuning
During the experimentation with the speed detuning, some other conditions were tried. First, the load torque was varied – it was found that this does not affect the behaviour of the observer and the accuracy of $\hat{\theta}$. Second, the gain $k'$ was varied: this also does not influence accuracy. Therefore, the theoretical analysis and the simulations are validated.

**Figure 11** Current $i_\alpha$ and estimated current $\hat{i}_\alpha$ of the FO observer, 20 ms/div, 0.86 A/div –30% speed detuning

**Figure 12** Real vs. estimated rotor position of the FO observer, 20 ms/div, 300 deg./div, –30% speed detuning
6 Full-order observer with speed estimate

From the analysis in Section 5, it follows that the observer presented produces an accurate rotor position estimate when the correct speed signal is used. It is also important to note that, if the speed signal is off by a small amount, this leads to only a small error in the estimated rotor position.
For the PMSM, since the rotor electrical speed is equal to the frequency of the electrical signals applied, a couple of excellent speed estimation methods (accurate and reliable) are available – these could be used in conjunction with the observer presented.

The first method is based on a Phase Locked Loop that is synchronised with the voltage vector applied on the motor (Kaura and Blasko, 1997). The idea is to construct a rotating reference frame that is aligned with the vector of the three-phase voltages applied by the inverter; this is done by forcing the projection $V_q$ to zero using a PI controller. The frequency of this rotating vector is obtained as a by-product of the process that generates the reference frame. Since the motor model and the parameters are not involved, the method offers excellent accuracy both at steady-state and in transient.

A second estimation method that is accurate obtains the speed of the PMSM by differentiating the observer’s own estimated rotor position. With this approach, the speed estimate is slightly discontinuous – this is because the derivative of $\theta$ is taken only for $\theta \in [0.1 \, 0.9] \, \text{pu}$ (to avoid the jump at the $0 - 2\pi$ transition). Note that if the speed signal fed in the observer is different from the real speed, the estimated EMFs and rotor position still have the correct frequency. The estimate $\hat{\theta}$ may lead or lag the real $\theta$; however, its frequency is always correct (this corresponds to the frequency of the stator voltages). Therefore, this signal can be processed and a good speed estimate is obtained; note that the model of the motor is not involved. As this speed estimate is fed in the observer, an accurate rotor position is expected.

A system diagram of the proposed observer is shown in Figure 15. The performance of this observer is shown next. Figure 16 shows the real and estimated rotor position of the observer for a 0.05 to 1 pu step in speed command. The experiment is done with a load of 0.1 Nm. Figure 17 shows the measured and the estimated speed under the same conditions. The estimated speed is used in the observer and the resulting rotor position is quite accurate. Overall, the observer performs well both at steady-state and in transient. Figure 18 shows a picture of the experimental setup used.

**Figure 15** Block diagram of the FO observer with speed estimate proposed
Figure 16  Real and estimated rotor position using the FO observer with speed estimate, 0.05-1 pu speed step command, 200 ms/div, $k' = 0.5$

Figure 17  Real and estimated speed of the FO observer with speed estimate, 0.05-1 pu step command, 200 ms/div, 1800 rpm./div, $k' = 0.5$

Figure 18 Experimental setup (see online version for colours)
7 Sliding mode observer for EMF estimation

Considering the model of the PMSM given by equation (6), the EMFs can also be estimated using a SM observer. A simple method that can be used in a sensorless drive is to construct a SM observer using only the current equations in equation (6). This observer is of the form:

\[
\begin{align*}
pl_\alpha &= -\frac{R}{L} i_a + \frac{1}{L} V_a - \frac{1}{L} M \cdot \text{sign}(s_\alpha) \\
npl_\beta &= -\frac{R}{L} i_\beta + \frac{1}{L} V_\beta - \frac{1}{L} M \cdot \text{sign}(s_\beta) \\
\end{align*}
\]

(30)

After subtracting the original current equations from equation (30), the dynamics of the manifolds, \( s_\alpha, s_\beta \) are:

\[
\begin{align*}
\dot{s}_\alpha &= -\frac{R}{L} s_\alpha - \frac{1}{L} e_\alpha - \frac{1}{L} M \cdot \text{sign}(s_\alpha) \\
\dot{s}_\beta &= -\frac{R}{L} s_\beta - \frac{1}{L} e_\beta - \frac{1}{L} M \cdot \text{sign}(s_\beta) \\
\end{align*}
\]

(31)

In system equation (31), if the gain \( M \) is selected high enough, the manifolds and their derivatives have opposite signs; the manifolds tend to zero and SM occurs. Once SM starts, \( s_\alpha, s_\beta \) and their derivatives are identically equal to zero. It follows that the equivalent controls (which represent the average values of the discontinuous terms \( M \cdot \text{sign}(s_\alpha) \) and \( M \cdot \text{sign}(s_\beta) \)) are equal to the EMFs \( e_\alpha, e_\beta \). In practice, equations (30) are implemented and the EMFs are obtained by low pass filtering the switching terms. This is the main weakness of the method: if the drive operates in a wide speed range, the estimates of \( e_\alpha, e_\beta \) will differ in phase (and magnitude) from the real EMFs due to the LPF operation. The errors are frequency dependent and become quite significant as the frequency of the EMFs approaches the filter’s cutoff. The phase errors in the EMFs propagate in the rotor position estimate. It is possible to correct the phase of the EMFs prior to calculating \( \theta \). The correction requires to estimate the applied frequency; however, the method is quite complex.

The SM observer proposed in this paper is developed based on system equation (6), and is a full-order, sensorless observer. The EMFs are estimated directly and low pass filtering is not needed – thus, it works in wide speed range.

The voltages \( V_\alpha, V_\beta \) and currents \( i_\alpha, i_\beta \) are measured. Since the first two equations of (6) involve the motor speed, the observer uses a speed signal that is obtained from a general purpose speed estimator.

The analysis considers that the speed estimate corresponds to equation (22) – the paper will show that the estimated EMFs tend to the real EMFs with the proposed gain design.

The method presented allows development of a family of PMSM observers using the proposed SM structure. The main advantage is that the designer does not need to worry about the accuracy of the speed estimate used in the observer. It will be shown that the approach is robust to errors in the speed signal used.

The proposed SM observer is designed based on the original plant equation (6) and uses the speed estimate (equation (22)).
The equations are:

\[
\begin{align*}
\dot{\varphi}_a &= -(\alpha + \Delta \alpha)\hat{\varphi}_a + l_1i_a \\
\dot{\varphi}_\beta &= (\alpha + \Delta \alpha)\hat{\varphi}_\beta + l_2i_\beta \\
\dot{i}_a &= \frac{R}{L}\hat{i}_a - \frac{1}{L}\hat{\varphi}_a + \frac{1}{L}V_a - \frac{1}{L}u_a, \\
\dot{i}_\beta &= \frac{R}{L}\hat{i}_\beta - \frac{1}{L}\hat{\varphi}_\beta + \frac{1}{L}V_\beta - \frac{1}{L}u_\beta.
\end{align*}
\]  
\tag{32}

The switching (SM) controls \( u_a, u_\beta \) are:

\[
\begin{align*}
u_a &= M \cdot \text{sign}(s_a) \\
u_\beta &= M \cdot \text{sign}(s_\beta).
\end{align*}
\]  
\tag{33}

Note that \( M \) is a design gain, \( M > 0 \); the feedback gains \( l_1 \) and \( l_2 \) are also design parameters. After the original equation (6) are subtracted from equation (32), the result is:

\[
\begin{align*}
\dot{\varphi}_a &= -\alpha \varphi_a - \Delta \alpha \hat{\varphi}_a + l_1i_a \\
\dot{\varphi}_\beta &= \alpha \varphi_\beta + \Delta \alpha \hat{\varphi}_\beta + l_2i_\beta \\
\dot{s}_a &= -\frac{R}{L}s_a - \frac{1}{L}\varphi_a - \frac{1}{L}M \cdot \text{sign}(s_a) \\
\dot{s}_\beta &= -\frac{R}{L}s_\beta - \frac{1}{L}\varphi_\beta - \frac{1}{L}M \cdot \text{sign}(s_\beta).
\end{align*}
\]  
\tag{34}

where \( \varphi_a = \hat{i}_a - e_a, \varphi_\beta = \hat{i}_\beta - e_\beta \). In the last two equations of (34), if \( M \) is chosen high enough, the manifolds \( s_a, s_\beta \) and their derivatives have opposite signs. Therefore, the manifolds tend to zero and SM occurs; i.e., \( s_a \to 0 \) and \( s_\beta \to 0 \). This means that the estimates of the currents converge to the real currents, i.e., \( i_a \to i_a \) and \( i_\beta \to i_\beta \). Once SM starts, \( s_a, s_\beta \) and their derivatives are equal to zero. According to the equivalent control method of Utkin et al. (1999), the equivalent controls are:

\[
\begin{align*}u_{a,eq} &= \varphi_a \\
\dot{u}_{\beta,eq} &= \varphi_\beta.
\end{align*}
\]  
\tag{35}

The behaviour of the mismatches \( \varphi_a, \varphi_\beta \) is studied next; for this, the switching terms \( u_a, u_\beta \) are replaced with the equivalent controls in the upper equations of (34). The resulting dynamics of the EMF mismatches is:

\[
\begin{align*}
\dot{\varphi}_a &= -\alpha \varphi_a - \Delta \alpha \hat{\varphi}_a + l_1\varphi_a \\
\dot{\varphi}_\beta &= \alpha \varphi_\beta + \Delta \alpha \hat{\varphi}_\beta + l_2\varphi_\beta.
\end{align*}
\]  
\tag{36}

Note that system (36) is time-varying and is also affected by disturbance. Stability analysis cannot be done based on eigenvalues, instead, time-varying methods are used.

To study the convergence of the EMF estimates, select the positive definite candidate Lyapunov function:

\[
V = \frac{1}{2}(\varphi_a^2 + \varphi_\beta^2).
\]  
\tag{37}
Sensorless rotor position estimation of PMSM by full-order

After differentiation and after replacing the derivatives from equation (36), this becomes:

\[
\dot{V} = l_1 \ddot{\tau}_u^2 + l_2 \ddot{\tau}_p^2 - \Delta \omega_e \dot{\tau}_\rho \dot{\tau}_u + \Delta \omega_e \dot{\tau}_u \dot{\tau}_p.
\]  

(38)

If \( l_1 = l_2 = -k \) with \( k > 0 \), the derivative of \( V \) is:

\[
\dot{V} = -k(\ddot{\tau}_u^2 + \ddot{\tau}_p^2) + \Delta \omega_e (\dot{\tau}_\rho - \dot{\tau}_u). \quad (39)
\]

Equation (39) is an important result and is used to study the convergence of the observer. Note that \( \dot{V} < 0 \) if \( \Delta \omega_e = 0 \) and the observer is asymptotically stable. However, the paper will show what can be done when \( \Delta \omega_e \neq 0 \). Regarding the two terms in equation (39): the first one is always negative (and can be increased using the design parameter \( k \)) while the second one has an unknown sign. As long as the mismatches \( \tau_u \) and \( \tau_p \) are significant, the first term overcomes the second and \( \dot{V} \) is negative (as a result, \( V \) decays).

The Lyapunov function stops decaying when \( \dot{V} = 0 \). and this is equivalent to:

\[
k(\ddot{\tau}_u^2 + \ddot{\tau}_p^2) = \Delta \omega_e (\dot{\tau}_\rho - \dot{\tau}_u). \quad (40)
\]

Since the mismatches \( \tau_u \), \( \tau_p \) should be of the same order of magnitude, using the notation \( m\tau_u = \tau_u \) (\( m \) is unknown), equation (40) is used to obtain the values of \( \tau_u \), \( \tau_p \) at which \( \dot{V} \) stops decaying:

\[
\tau_u = \Delta \omega_e \frac{m\dot{\tau}_\rho - \dot{\tau}_u}{k(1+m^2)} \quad \tau_p = \Delta \omega_e \frac{m(\dot{\tau}_\rho - \dot{\tau}_u)}{k(1+m^2)} \quad (41)
\]

The Lyapunov function settles to the vicinity given by the mismatches (equation (41)). More importantly, note that the size of this vicinity and the mismatches can be reduced by increasing \( k \).

In conclusion, the estimated EMF be made to approximately tend to the real EMFs and the estimation error can be reduced using a design gain. The result shows that the influence of the speed mismatch \( \Delta \omega_e \) can be made irrelevant since, theoretically, the estimation errors in equation (41) can be made as small as desired.

Once the EMFs are found, the rotor position is computed directly with:

\[
\dot{\theta} = \tan^{-1}\left(-\frac{\dot{\tau}_u}{\dot{\tau}_p}\right). \quad (42)
\]

The diagram of the proposed SM observer is shown in Figure 19.

Figure 19  Block diagram of the sliding mode observer based on speed estimate
8 Simulations of the sliding mode observer

The SM observer is simulated using Simulink. The parameters of the PMSM correspond to Table 1. In the simulation, the PMSM drive is operated in speed control mode. The motor is started towards a speed of 1000 rpm with a load of 0.2 Nm. The estimated currents, EMFs and rotor position are recorded. The motor is oriented with the correct rotor position and the SM observer runs in parallel with the PMSM model. The same scheme is used as in Section V regarding PI controllers. The simulation is run with a sampling time of 50 µs and does not use a PWM state machine.

The model of the proposed SM observer uses the speed estimate (equation (22)) – this is generated by detuning the real (measured) speed in the ±25% range.

Figure 20 shows the real and estimated \( \alpha, \beta \) currents of the SM observer with \( M = 15 \); note that they correspond. Figure 21 shows the real and estimated EMFs when the speed fed in the observer is 25% smaller than the real one; \( k = 50 \) (note the estimation error). Figure 22 shows the same EMFs for \( k = 100 \); the error has been reduced. Finally, Figure 23 shows the EMFs for \( k = 300 \); with this gain, the estimates approximately tend to the real EMFs. If \( k \) is increased even more, the error can be further reduced.

Figures 24 and 25 show the real and estimated EMFs when the speed fed in the observer is 25% bigger than real. In Figure 24, \( k = 50 \) (note the error) and this increases to \( k = 300 \) in Figure 25 – there, the EMF estimates tend to the real EMFs.

The results show an interesting relationship between the real and estimated EMFs: when the speed fed in the SM observer is smaller than real (Figures 21–23), the estimates lag the real EMFs. Otherwise, the estimates lead the real EMFs as in Figures 24 and 25). The lead/lag relationship is also true for the rotor position estimates (Figures 21 and 24).

Simulations confirm that the EMF estimates tend to the real EMFs if the gains \( l_1, l_2 \) are properly designed; \((l_1 = l_2 = -k)\), \(k \) should be high. Thus, the theoretical analysis of the observer is validated. It was also verified through simulation that if the uncertainty range of the speed estimate is extended to ±50%, the estimation errors are still very small and can be controlled with the design gains.

Figure 20  Real and estimated currents of the SM observer (see online version for colours)
Sensorless rotor position estimation of PMSM by full-order

Figure 21 Real vs. estimated EMFs of the SM observer with a –25% speed mismatch, $k = 50$
(see online version for colours)

Figure 22 Real vs. estimated EMFs of the SM observer with a –25% speed mismatch, $k = 100$
(see online version for colours)

Figure 23 Real vs. estimated EMFs of the SM observer with a –25% speed mismatch, $k = 300$
(see online version for colours)
9 Experimental results of the sliding mode observer

The same experimental setup as in Section 6 is used. The control algorithm is executed with a sampling time of 50 µs. The motor voltages and currents are obtained with the same approach as before. The output waveforms are obtained using the second set of PWM channels of the DSP chip.

In the experiment, the motor is first operated in open loop (V/Hz) mode. The observer runs in parallel with the control algorithm and the estimates are captured.

The speed of the PMSM is computed by differentiating the measured rotor position with equation (27), followed by low pass filtering. The measured speed is detuned in a −25% to +25% range (similar to the simulations) and the resulting signal is fed in the observer.
Sensorless rotor position estimation of PMSM by full-order

The equations of the SM observer (equation (32)) are discretised using Euler’s method; the discrete-time equations are:

\[
\begin{align*}
\hat{e}_{\alpha,k+1} &= \hat{e}_{\alpha,k} + T_s \omega_s (\hat{\omega}_\beta \hat{e}_{\beta,k} - k'u_\alpha) \\
\hat{e}_{\beta,k+1} &= \hat{e}_{\beta,k} + T_s \omega_s (\hat{\omega}_\alpha \hat{e}_{\alpha,k} - k'u_\beta) \\
i_{\alpha,k+1} &= \left(1 - \frac{R}{L} T_s\right)i_{\alpha,k} + \frac{V_k}{I_\alpha} T_s \frac{T_s}{I_\alpha} (V_\alpha - \hat{e}_{\alpha,k} - u_\alpha) \\
i_{\beta,k+1} &= \left(1 - \frac{R}{L} T_s\right)i_{\beta,k} + \frac{V_k}{I_\beta} T_s \frac{T_s}{I_\beta} (V_\beta - \hat{e}_{\beta,k} - u_\beta)
\end{align*}
\] (43)

The SM gain $M$ is expressed in per unit; the gain $k'$ in equation (43) corresponds to $k' = k'\omega_s$; $(k')$ is also in p.u.). During the experiment, the load torque is 0.15 Nm. The SM terms in equation (8) are implemented using the linearisation of the sign function (Utkin et al., 1999):

\[
M \cdot \text{sign}(s) = \begin{cases} 
+M & \text{if } s > E_0 \\
M \frac{s}{E_0} & \text{if } |s| \leq E_0 \\
-M & \text{if } s < -E_0
\end{cases}
\] (44)

where $E_0$ is a design parameter ($E_0 = 0.05\text{ pu}$).

The discrete-time implementation equations in equation (43) show a significant advantage of the SM approach: the equations of the observer equations are integrated using ideal integration. This is possible because the discontinuous feedback terms $M \cdot \text{sign}(s_\alpha), M \cdot \text{sign}(s_\beta)$ prevent the outputs from diverging in the presence of dc offsets or imperfections. As a result, low pass filtering is not needed and the estimation errors due to LPFs that occur at low speed (Finch and Giaouris, 2008) are avoided. In conclusion, the proposed SM observer outperforms traditional observers at low speed – traditional observers with continuous feedback cannot avoid low pass filtering.

The experimental waveforms in Figures 26–28 are looking to validate the observer implementation. The motor runs at 900 rpm; the gains are $M = 0.8, k' = 0.9$. There is no speed detuning, Figure 26 shows the real and estimated $\alpha, \beta$ currents – note that they correspond.

Figure 27 shows the estimated EMFs and the estimated rotor position which is obtained with equation (42). The EMFs are smooth and the rotor position is of good quality. The accuracy of the observer is shown in Figure 28 – note that the estimated rotor position matches closely the measured rotor position.

Next, the observer is studied under detuned speed signal and for various values of $k'$. Figure 29 was obtained at a speed detuning of $+25\%$; the waveforms are obtained with three different values of $k'$ ($k' = 0.01; k' = 0.1; k' = 4$). Note that the rotor position error is eliminated by increasing $k'$.

The experiment is repeated in Figure 30 with a speed detuning of $-25\%$. The same values are used for $k'$. The results confirm the theoretical analysis and validate the simulations.
The proposed observer was also studied in the lower speed range. In Figures 31 and 32, the applied frequency is approximately 6 Hz (180 rpm). Figure 31 shows the real and estimated rotor position for $k' = 0.01$ under $-25\%$, $0\%$ and $+25\%$ speed detuning. Note the estimation error and the slight asymmetry between the errors at $-25\%$ and $+25\%$. Also, there is a small error even when the speed fed in the observer is correct (this is due to the low gain used). In Figure 32, under the same conditions but with a higher value of $k'$ ($k' = 4$), the estimation improves and the rotor position is less influenced by the speed estimate fed in the observer.

**Figure 26** Real and estimated currents of the SM observer, 20 ms/div, 0.86 A/div

**Figure 27** Estimated EMFs and rotor position of the SM observer, 20 ms/div, 5.6 V/div

**Figure 28** Real vs. estimated rotor position of the SM observer, 20 ms/div, 300 deg./div, no speed detuning
Sensorless rotor position estimation of PMSM by full-order

**Figure 29** Real vs. estimated rotor position, 20 ms/div, 300 deg./div, +25% speed detuning

**Figure 30** Real vs. estimated rotor position of the SM observer, 20 ms/div, 300 deg./div, –25% speed detuning

**Figure 31** Real and estimated rotor position of the SM observer at low speed, 50 ms/div, 300 deg./div, –25%, 0%, +25% speed detuning, \( k' = 0.01 \)
Figure 32 Real and estimated rotor position at low speed, 50 ms/div, 300 deg./div, –25%, 0%, +25% speed detuning, $k' = 4$

It should be noted that the estimated rotor position leads the real one in Figure 29 (where the speed is detuned by –25%) – this confirms the simulation result of Figure 21. Similarly, in Figure 30, the rotor position estimate lags the real rotor position and this result corresponds to the simulation in Figure 24.

Finally, the observer was tested with a speed detuning of ±50% and similar results were obtained. Therefore, the method is not restricted to the ±25% speed range initially considered, although the accuracy of most speed estimators should fall in this category.

A somewhat similar observer for the PMSM that uses adaptation was presented by Chen et al. (2000a, 2000b, 2001). The method is more complicated than the one shown here: it is a full-order SM observer which uses adaptation to estimate the speed; the speed estimate is fed in the observer and is also used for feedback. A robustness analysis for operation with improper parameters and speed is done based on the sensitivity function and eigenvalue placement.

10 Comparison of the two observer methods

While both methods obtain the rotor position of the PMSM drive, there are a few subtle differences between them.

First, the SM observer exhibits a strong robustness property, since estimation is highly insensitive to the errors in the speed estimate used. On the other hand, the Full Order observer requires an accurate speed signal for correct estimation.

In terms of ease of tuning, the FO observer only involves one feedback gain and is clearly easier to tune. With the SM method, the SM gain and the feedback gains need to be found. Tuning of the SM observer should be done by first selecting $M$ and then examining the current estimates (if the ripple is too high, $M$ should be reduced).

Regarding low-speed operation, both observers have performed well down to a frequency of 2–3 Hz. The range can be extended even more if the voltage and current acquisition hardware has very low offset, or if a more accurate floating point processor is used.

If operation in very wide range is intended, given a certain (fixed) tuning, the FO observer outperforms the SM observer. This is because, with the SM design, the gain $M$ needs to be high at high speed according to equation (31). Then, with $M$ fixed and high,
if low speed operation is intended, the estimates have high ripple. To solve this problem, a speed-dependent (and time-varying) gain $M$ should be adopted.

The two observer methods are compared in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>FO observer</th>
<th>SM observer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robustness to speed errors</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Low speed limit</td>
<td>2–3 Hz</td>
<td>2–3 Hz</td>
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<tr>
<td>Ease of tuning</td>
<td>Easy</td>
<td>More difficult</td>
</tr>
<tr>
<td>Wide speed range operation</td>
<td>Easy</td>
<td>More difficult</td>
</tr>
</tbody>
</table>

11 Conclusions

The paper discusses the problem of estimating the rotor position of the PMSM drive and presents two sensorless observer methods that use a speed estimate. The paper studies their behaviour and the estimation accuracy when the speed signal fed in the observer is improper. The observers are developed based on the PMSM model in the stationary reference frame and use the motor voltages, currents and speed as inputs. The PMSM is treated as a time-varying plant and the design is done using Lyapunov’s stability theory. First, a Full-Order observer structure is presented: it is shown that this yields inaccurate estimates under improper speed and the estimation accuracy is not influenced by the design gains. Based on this structure, an observer with a speed estimate obtained from differentiating the output rotor position is developed. Second, a SM observer structure is proposed. For this, it is shown that the estimation errors under improper speed can be reduced by increasing the observer’s gains. This is a significant advantage of the sliding-mode structure; as a result, a variety of sensorless observers can be constructed with less than accurate speed estimates. In addition, the SM observer performs well at low speed because it avoids the errors due to low pass filtering. In the paper, the FO and SM observers are first analysed assuming that the speed is available and the observer’s gains are designed. Then, the speed is replaced with a speed estimate (assumed to be inaccurate) to obtain sensorless observers. The paper shows that both methods can be used to provide field orientation in a sensorless PMSM drive and work well in a wide speed range.

References


Sensorless rotor position estimation of PMSM by full-order


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Derivative operator</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Rotor electrical speed</td>
</tr>
<tr>
<td>$i_\alpha,i_\beta$</td>
<td>Currents in stationary reference frame</td>
</tr>
<tr>
<td>$V_\alpha,V_\beta$</td>
<td>Voltages in stationary reference frame</td>
</tr>
<tr>
<td>$\lambda_\alpha,\lambda_\beta$</td>
<td>Fluxes in stationary reference frame</td>
</tr>
<tr>
<td>$\lambda_{pm\alpha},\lambda_{pm\beta}$</td>
<td>Magnet fluxes in stationary reference frame</td>
</tr>
<tr>
<td>$e_\alpha,e_\beta$</td>
<td>EMFs in stationary reference frame</td>
</tr>
<tr>
<td>$R$</td>
<td>Stator resistance</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>$L$</td>
<td>Synchronous inductance</td>
</tr>
<tr>
<td>$K_e$</td>
<td>EMF constant</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Rotor position angle</td>
</tr>
<tr>
<td>$\hat{\omega}_e$</td>
<td>Estimated electrical speed</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Sampling time</td>
</tr>
</tbody>
</table>