REAL vs. FINANCIAL INVESTMENT
Can Tobin Taxes Eliminate the Irreversibility Distortion?

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In the recent past several developing countries have failed to achieve significant real capital investment despite episodes of large capital inflows. Although there are real projects with seemingly high returns, investors prefer to wait for the correct time to invest. In this paper we address this issue by considering a two-sector economy where investment in real capital is irreversible and debt-financed. Furthermore, the interest rate, which is determined in the financial sector, is random as a result of volatile expectations. In this economy the expected return on real capital is above the expected interest rate. This is because the option to wait for lower interest rates has a positive value. In the presence of rumors, taxes on international financial transactions (Tobin taxes) reduce the variance of the domestic interest rate, while leaving its mean unchanged. As a result, they induce more investment in irreversible real capital. The model borrows from the irreversible investment literature. A difference with other models is that the source of noise is a mean-reverting process not just a geometric Brownian motion. We solve for the optimal decision rule using the Girsanov theorem.

Of the maxims of orthodox finance none, surely, is more anti-social than the fetish of liquidity ... It forgets that there is no such thing as liquidity of investment for the society as a whole.

The introduction of a substantial government transfer tax on all transactions might prove the most serviceable reform available, with a view to mitigating the predominance of speculation over enterprise ...

Keynes, The General Theory.

1. Introduction

In the recent past several developing countries have failed to achieve significant real capital investment despite episodes of large capital inflows. During the episodes when capital flight has been reversed (for example, in the aftermath of a stabilization) the capital that returned has taken a very liquid form. This is paradoxical, for real projects with seemingly high marginal returns are not being undertaken. Although fundamentals are correct, investors prefer to 'wait for the correct time to invest' in these highly profitable real projects. Since everyone is waiting with their capital in liquid...
form, the economy is kept in a state of 'corto-plazismo', with capital engaging in frequent round-tripping. Since 'in the aggregate there is no such thing as liquidity of investment for the society as a whole', this round-tripping generates great volatility in the prices of domestic assets (e.g., interest and exchange rates), which in turn justifies the investors' cautiousness. Given the current debate on the liberalization of international financial markets, it is important to address this issue in order to understand better the pros and cons of liberalization.

A common argument made by businessmen in Latin America is that, given the uncertain environment, real investment does not occur because real capital is irreversible, while financial capital is reversible, in the sense of being more certainly realizable at short notice. Consequently, when expectations about the home country become more optimistic, there will indeed be a capital inflow. However, since there exists uncertainty, investors will be cautious and 'wait' before committing to a real investment. This is because: (a) in case a pessimistic rumor were to arrive, those who invested in irreversible real capital would regret it, since the holders of financial assets would be in a better position to exchange them for foreign assets; (b) in case expectations were to remain optimistic in the future, those who did not invest in real capital would not regret it. They will still be able to invest in real capital. This asymmetry, which confers an option value to financial capital, leads to underinvestment in irreversible real capital.

In this paper, we present a model which rationalizes this behavior, and we address the issue of whether it is possible and desirable to intervene in the financial market in order to induce more real investment. We show that even when the fundamentals are correct (there is no macroeconomic mismanagement), and it is expected that rumors regarding the profitability of domestic investment will vanish (there is no pessimistic long-run view of the economy), there still exists underinvestment in real capital. We also show that underinvestment can be reduced with the introduction of a sequence of taxes on international financial transactions, which vary inversely with the state of expectations. These taxes are generally known as Tobin taxes.

The model we present differs from the capital flight literature. In this literature just one type of capital is considered, and the focus is on the issue of why very little capital is held domestically (i.e., why capital flight occurs), not on the issue of why this capital is not invested in real projects, instead being kept in the form of financial capital. The basic idea is that investors

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¹"Corto-plazismo" is the curse of Latin America. Literally "short-termism", the expression captures that pervasive mix of chronic anxiety and skepticism that leads to an inability to plan beyond next week. The result is the region's scarce investment, casino-like financial markets and capital flight. All this in turn spawns economic stagnation and national frustration". From the Wall Street Journal (September 30, 1988, p. 27).

²Those who own machines will find it more difficult to sell them, at short notice, than owners of domestic bonds.
shift their resources abroad to 'save-havens' to avoid higher taxes. A move by one investor increases the tax obligations of the remaining investors, and capital flight then spreads like an epidemic. In these models, under-investment is caused by a low (steady state) expected return on domestically held capital. This is not the case in our model, since in the steady state rumors vanish.

In section 2 we present casual evidence from the recent Mexican experience. In section 3 we present an overview of the model. In section 4 we present the model. It is a continuous-time, stochastic two-sector model, and it borrows from the irreversible investment literature. A difference with other models is that the source of noise is a mean-reverting process not just a geometric Brownian motion. We solve for the optimal decision rule using the Girsanov theorem. In section 5 we show how Tobin taxes bring the level of real capital closer to its first-best level. Finally, in section 6 we present the conclusions.

2. The recent Mexican experience

In December 1987 a stabilization program was implemented. Although the operational budget and the current account were in surplus (1.4% and 2.9% of GDP), there were rumors that the program was going to fail, and investment in real capital did not take place. According to the quarterly survey of firms' expectations made by the National Institute of Statistics, during the first quarter of 1988 expected inflation was 150%. As rumors of failure diminished, it fell to 60% in the last quarter of 1988. During this period, fundamentals such as the price of oil or the prime interest rate did not move significantly.

During 1988, 46% of respondents were unwilling to invest. Among the reasons that limited investment were: exchange rate uncertainty (35% of respondents) and insufficient future demand (45%). In 1989, the proportion of those unwilling to invest fell to 32%, this reduction is explained basically by the first reason (it fell to 14%). The second reason did not vary significantly (it just fell from 45% to 40%). Since Mexico is a small open economy, interest rates are mainly determined by expected changes in the exchange rate. Thus, this casual evidence gives some support to the assumption made below that interest rate volatility generated by rumors is the cause of underinvestment.

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3 See Eaton (1987), Giovannini (1987), and Khan and Ul Haque (1985). Obstfeld (1988) develops a similar argument in a monetary economy. He emphasizes the connection between capital flight, currency depreciation, and public finances. He shows that there may be equilibria driven by inflationary expectations, in which capital flight causes expectations to be validated by government policies.

4 Inflation was actually reduced from an annualized rate of more than 400% in December of 1987 to 28% in December of 1988, and 13% in August 1989.
Next, we analyze how nominal interest rates responded to rumors. This is shown in fig. 1. When the stabilization program was implemented, interest rates were around 160%. Only two months later they started to fall, as rumors of failure vanished. However, this trend was reversed in July 1988 after the presidential elections because the PRI got only 51% of the votes (lower than expected), and also because it was rumored that at the end of the presidential period there had to be a maxidevaluation (as it happened with the last two presidents). Thus, between July and December, interest rates rose from 40% to 52% with an annualized inflation of just 19%. More striking is the fall in reserves, from $14 billion in July to $7.0 billion in December. It turned out that those expectations were just rumors and no devaluation took place. President Salinas took office in December 1988, and he soon adopted measures which cleared doubts about his leadership, like the imprisonment of the head of the powerful oil workers’ union, ‘La Quina’, which, with the announcement of the Brady Plan in March contributed to the decline of the
pessimistic rumors and of interest rates as well. However, since a debt agreement was not reached when expected, and as the renewal date of the 'Pacto' (price control agreement) was getting close (July) rumors grew that authorities had lost bargaining power vis-à-vis the banks because authorities had to have an agreement before that date. Thus, interest rates increased 10% between March and June (with an annual inflation of less than 20%). However, at the end of June, the 'Pacto' was unexpectedly renewed and interest rates fell by almost 10 points. In July an agreement was reached with the banks and interest rates fell by another 15%. During this period fundamentals remained basically unchanged, interest rates were mainly driven by rumors of whether debt reduction will be 10% higher or 10% lower.

3. Overview of the model

To formalize the argument in the introduction, we need two ingredients. First, there must be a distinction between financial and real capital. However, in the aggregate they must be linked. Second, operating profits in the real sector should be affected by events in the financial sector. We introduce these elements by assuming that real investment is debt-financed. In particular, we assume that investors in the financial sector can invest their funds in a foreign asset or they can deposit them in a domestic financial intermediary. This intermediary, in turn, makes loans which are used to install irreversible real capital.

To focus on the essentials we assume that the only source of uncertainty is rumors regarding the return on domestically held assets. In the short run, these rumors behave randomly, independently of fundamentals. However, they vanish in the long run. This reflects the fact that even when the correct economic measures have been taken (e.g., the public deficit has been eliminated), there is not much room for maneuvering. Therefore, there is fertile ground for 'a growth of semispeculative psychology among capitalists, who have become aware of risks which they never before considered, and thus have become prepared to export capital at the slightest provocation, e.g., of a political sort', as described by Ohlin (1937, p. 148).

The following explains the link between events in the financial market and profits in the real sector. Since each investor can freely shift between foreign

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5There are other causes for underinvestment and other sources of uncertainty than the ones considered here. However, in order to present a simple model we just focus on interest rate volatility.

6To facilitate calculations we assume that rumors are driven by a diffusion process with two components: a Brownian motion component and a mean-reverting drift. The first component is a source of volatility, the second ensures that rumors will vanish in the long run. Brownian motion is a random walk in continuous time (i.e., rumors might become more or less optimistic; however, the expected value of the change is zero).
and domestic financial assets and there is no aggregate liquidity, rumors about taxes on deposits held at home are reflected in the domestic interest rate. Given that real capital is debt-financed, rumors affect operating profits in the real sector, through interest payments on the outstanding debt.

In this model, more volatile rumors translate into a higher variance of the domestic interest rate. Since real capital is irreversible and debt-financed, this leads to lower real investment (at each level of the domestic interest rate). As the variance increases, there is a greater probability of facing either extremely low or extremely high interest payments in the future. If investment takes place, extremely high interest rate realizations will hurt because real capital is irreversible. In contrast, if investment does not take place, extremely low realizations will not hurt because the firm is not precluded from investing in real capital in the future. In short, the higher the interest rate’s variance, the higher the value of the option to wait, and less likely that it will be exercised.⁷

Now we turn to the policy issues. In the models considered in the capital flight literature there are multiple equilibria. Thus, capital flight is stemmed (and the capital stock is increased) by altering the steady-state return to capital. This is done by coordinating investors through the implementation of obstacles to capital outflows. In the model we present, real capital can be increased by just reducing the variability of the domestic interest rate, without changing its long-run expected level. This can be done with a sequence of taxes on all international financial transactions (Tobin taxes) that vary inversely with the state of expectations. When expectations turn more pessimistic, these taxes act as capital outflow levies. When expectations turn more optimistic, they act as taxes on capital inflows. Thus, this sequence of taxes produces a mean-preserving reduction in the variance of the real

⁷To illustrate this idea consider a two-period world, and suppose that the second period’s interest rate (i₁) can take one of two values: i < i. The firm must decide whether or not and when to install a unit of irreversible capital. Once installation has occurred, the firm gets during each operating profits (n), which depend only on the interest rate. For the sake of argument we assume that n(i₀) > 0, n(i) > 0 and n(i) < 0. The firm has two strategies:

(a) Make the installation at i = 0. The expected payoff is: \( \Pi₀ = \pi(i₀) + \pi(Eₙ) \)
(b) Wait and make the installation decision at i = 1. The expected payoff is: \( \Pi₁ = 0 + E \max [0, \pi(i₁)] \)

By waiting, the firm loses the first period’s profits. However, it acquires the right (an ‘option’) to decide at i = 1 whether or not to have the capital installed. This option has a positive value since at i = 1 if i₁ were equal to i, the firm would be able to choose not to install capital, and thus save \( \pi(i) \), which is negative.

The value of this option increases with a mean-preserving spread of i₁. To see this, let the possible realizations of i₁ be i < i and i > i, with (i, i) such that the expected value of i₁ remains unchanged. The value of the option increases because \( \pi_n - \pi_s \) increases:

(1) since \( Eₙ₁ \) remains unchanged, \( \Pi₁ \) remains the same.
(2) However, since i < i and i > i, \( \Pi₁ \) increases. This is because: first in case i₁ were equal to i, \( \Pi₁ \) would increase because \( \pi(i) > \pi(i) \). Second, in case i₁ were equal to i, \( \Pi₁ \) would remain unchanged because no installment would take place. That is, the option to wait is an insurance against high realizations of i.
sector's interest payments and of its operating profits. As a result, higher investment in real capital is induced (at each level of the interest rate).\textsuperscript{8} In order to implement these taxes, there is no need for authorities to learn every period the state of expectations. They can be implemented through a dual exchange rate system, as explained in section 5.

The question as to whether it is optimal to intervene in order to reduce the profitability of the round-tripping of financial capital is part of a broader debate concerning intervention in the international financial markets. On the one side, based on the view that factors other than fundamentals are the driving force in financial markets, economists such as Dornbusch (1986), Liviatan (1980) and Tobin (1978) have proposed to put some 'sand into the wheels', by taxing international financial transactions. On the other side, Adams, Greenwood, Stockman and Van Wijnbergen (among others) see these interventions as welfare-reducing distortions in an otherwise perfectly competitive economy (since they interfere with the agents' intertemporal decisions).\textsuperscript{9} Other critics [e.g., Cuddington (1986)] argue that limiting financial mobility: (a) keeps capital out of the country for fear of not being able to leave in the future, and (b) gives freedom to governments to implement bad policies, i.e., policies that would otherwise induce capital flight.

If there is a case for obstructing the free mobility of financial capital, it must rest on the existence of a distortion. In the economy we consider, the distortion is the 'option to wait' (which is lost when a real investment is undertaken). This option gives its owner the right to wait until the realization of uncertainty and decide whether to invest in real capital or in foreign assets. Even in a risk-neutral world, this option has a positive value. This option value forces the marginal return on real capital to be above the domestic interest rate (resulting in underinvestment). Worth noting is that the distortion is not merely the irreversibility of real capital nor the existence of rumors, but the combination of the two.\textsuperscript{10}

There would be no reason for rumors regarding the return on domestically held assets if the country had solved all of its problems (so that it had high

\textsuperscript{8}The experience of countries with controls such as Italy, suggest that they are indeed effective at reducing the volatility of domestic interest rates. However, they are not effective at keeping their mean for long periods below the international interest rates. For example, Giavazzi and Pagano (1985) document that between November 1980 and August 1984 these controls have reduced the variability of domestic rates to about one third of the variability of the corresponding Eurorates.


\textsuperscript{10}In the capital flight literature, the distortion stems from a lack of coordination. From a social perspective, the decentralized outcome is inefficient since capital is less productive abroad than domestically. In this situation, the introduction of obstacles to international financial mobility might improve the allocation.
reserves, low debt, a budget surplus and low taxes). In this situation there would be plenty of room to absorb shocks. Therefore, the value of the option to wait would be nil, and no underinvestment would exist. But of course that is not the case with most LDCs. Even good policies can at best stabilize these economies, leaving little room for maneuvering.

The preceding discussion suggests that, when factors other than fundamentals are the driving force in financial markets, and insurance schemes that would de facto eliminate the irreversibility of real capital are absent, the introduction of Tobin taxes is a second-best policy in a risk-neutral world. On the one hand, the value of the option to wait (the distortion) is reduced. On the other hand, the speculators' long-run expected returns in the financial market remain unchanged, since the mean of the interest rate is unaffected.

It is important to note that Tobin taxes are not generalized capital controls. They are imposed solely on international financial transactions, not on purchases of real capital nor on the repatriation of its dividends. Therefore, the criticism that Tobin taxes will keep capital out of the country (for fear of not being able to leave in the future) is unfounded. Also, note that these taxes do not give authorities the freedom to pursue systematically bad policies, i.e. they do not allow authorities to systematically alter the domestic interest rate.

The conclusions reached in this paper result merely from the theory of the second best. The argument made here is similar to those made in other branches of economics, such as banking and trade. In the banking literature, an example is the paper of Diamond and Dybvig (1983) in which the role of banks is to offer demand deposits, and finance investment projects which are illiquid in the short run. This transformation from illiquid assets into liquid liabilities makes banks susceptible to runs caused by nothing else but rumors. A run forces banks to sell their illiquid assets at a loss, thus reducing the productive capacity of the economy. In this context, a suspension of convertibility allows a more efficient transformation of liquidity.

In the trade literature, examples of welfare-improving deviations from free trade are: the Pareto-inferior free-trade argument of Newbury and Stiglitz (1984), and the paper of Eaton and Grossman (1985), according to which the imposition of tariffs is a substitute for insurance markets when these are incomplete and when agents must specialize in their use of a factor of production; and the argument in Helpman and Razin (1978), and Edwards and Van Wijnbergen (1986) showing that liberalizing the capital account might be welfare-reducing when trade distortions are present.\textsuperscript{11}

\textsuperscript{11}Helpman and Razin (1978) offer two other arguments for intervention in financial capital markets. In the first the country possesses monopoly power in security trade. In the second, foreign investment faces confiscation risk. Therefore, a subsidy should be given to exported securities.
4. The model

In order to formalize the above argument, we need two ingredients: a two-sector economy where investment decisions are more reversible in one sector than in the other, and where expectations in the reversible sector affect the operating profits of the irreversible sector. To this end we consider a small open economy formed by a financial sector and a real sector. This economy is populated by three types of agents: workers, financial investors and real investors. All these agents consume an imported good, which is the numeraire.

Investors in the financial sector can invest their resources in a foreign asset with a rate of return $i^*$ (in terms of the consumption good), or they can invest domestically in a financial intermediary and get a rate of return $i$ (in terms of the consumption good). The foreign financial asset is used as the means of exchange. The intermediary uses the resources it receives to make loans to investors in the real sector. These investors spend the loans just received in hiring labor services from the workers. Each unit of these labor services produces instantaneously one unit of productive capital, which does not depreciate and is irreversible, i.e. it cannot be put into alternative uses. Workers, in turn, spend all their income in the imported consumption good.

To close the model we assume that the stock of productive capital is used to produce a good which is exported. The revenues obtained are used by investors in the real sector to pay the interest on their debts to the financial intermediary. The rest is spent in the imported consumption good. The intermediary, in turn, transfers these interest payments to the investors in the financial sector. These investors also spend this income and their income from abroad in the imported consumption good. For consistency we note that in this economy every capital account imbalance is exactly matched by a current account imbalance (i.e., the balance of payments is always zero). First, since during each period the workers' consumption is identical to the loans received by investors in the real sector, there is a trade deficit which is equal to the capital account surplus. Second, since investors' consumption equals exports plus interest from abroad, the effect of these flows on the current account is zero. In Appendix C we present with more detail the production and consumption flows of this economy.

4.1. The financial sector

Investment in the foreign financial asset is perceived by investors as

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12 The dichotomy between financial and real investors is made for convenience. It will allow us to formalize Tobin taxes in a simple way, as explained in footnote 15.

13 We assume away intertemporal consumption-smoothing considerations in order to focus on the investment decisions. This assumption might be justified on the grounds that empirical estimates of the intertemporal elasticity of substitution in consumption are very low.
riskless, while investment in the domestic financial intermediary is perceived by investors as risky. As explained above, this just reflects the existence of extrinsic uncertainty, i.e. rumors not related to fundamentals. We will represent these rumors with a random process $Z_t$, and the expected return on domestic financial investment as $i_t - Z_t$. The process $Z_t$ increases when rumors become more pessimistic, and decreases in the opposite case. Since there are no transactions costs for buying or selling financial assets, arbitrage implies that

$$i_t = i^* + Z_t, \quad i^* > 0, \quad Z_0 = 1. \tag{1}$$

The process $Z_t$ is composed of a deterministic and a stochastic component. It is assumed to take the form

$$dZ_t = a[i^* - i_t]i_t dt + \sigma i_t dW_t, \quad 0 < a < 1. \tag{2}$$

The first component in (2) is the drift, a mean-reverting process. This captures the fact that since fundamentals are correct, the long-run expected value of $i$ is equal to $i^*$. It also ensures that the process $Z$ does not blow up: when $Z_t$ is high, the drift component $a[i^* - i_t]$ becomes negative, thus driving $Z$ towards zero and $i$ towards $i^*$.

The second component of (2) is stochastic. $dW_t$ is a Wiener process (standard Brownian motion) and $\sigma$ is the variance of its increments. A Wiener process is a stochastic process that has continuous sample paths with increments that are independent and normally distributed [i.e. $dW$ is $N(0, 1)$]. That is, at each point in time, the process is as likely to go up as to go down and the expected value of the increment is zero. Furthermore, the variance of the process $W$ increases with the length of the time interval. The use of Brownian motion is very convenient since it will allow us, in the next section, to replace the optimal value function of investors in the real sector by a differential equation, and this will facilitate the solution.

By substituting (2) into (1), we obtain that the domestic interest rate follows the process

$$di_t/i_t = a[i^* - i_t] dt + \sigma dW_t. \tag{3}$$

As can be seen, under perfect capital mobility, the variance of $Z$ is translated one for one into the domestic interest rate. It will be shown in the next section that an increase in this variance has contractionary effects on real investment, because it increases the value of the option to wait. This option is precisely the distortion that Tobin taxes are supposed to eliminate. In the remainder of this section we model these taxes.

Tobin's proposal is:
'an internationally uniform tax on all spot conversions of one currency into another, proportional to the size of the transaction. The tax would particularly deter short-term financial round trip excursions into another currency... Moreover, it is desirable to obstruct as little as possible international movements of capital responsive to long-run portfolio preferences and profit opportunities.' [Tobin (1978).]

In order to introduce these taxes into the model, suppose that there exists a constitutional rule according to which a non-negative tax $T_i$ is imposed on all swaps between the domestic and the foreign financial assets. It will turn out that in order for this tax to reduce the variance of the domestic interest rate it must be continuously adjusted. Thus, suppose that the rule specifies that $T$ must be adjusted each period by a fixed proportion $\mu$ of the change in the investors’ expectations. To illustrate how this sequence of taxes is adjusted, suppose that at time $t=0$, $Z_o=0$ and $i_o=i^*$, so that $T_0=0$. If a pessimistic rumor were to arrive (i.e., $dZ>0$), then the tax would increase by an amount $\mu dZ$. Although the tax applies to both, outflows and inflows of financial capital, in this case it would just be effective for capital outflows, since no capital inflows would occur. If instead, an optimistic rumor were to arrive (i.e., $dZ<0$), the tax would increase by $-\mu dZ$. However, since no capital outflow would occur, it would just apply to capital inflows. It follows from (1) and (2) that the domestic interest rate would follow the process

$$\frac{di}{i} = dZ - dT,$$

$$= dZ - \mu dZ,$$

$$= \sigma [1-\mu] dt + \sigma [1-\mu] dW_t. \quad (3')$$

As can be seen from (3'), Tobin taxes reduce the variance of the domestic interest rate, while keeping its expected long-run value unchanged (i.e., equal to $i^*$), as shown in fig. 2. When expectations about the future worsen, the

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14 In order to obtain a simple expression for the process governing the domestic interest rate [as in (3')] we will assume that investors in the financial sector are short-lived (i.e. live for only one period). At birth, the representative investor receives as a bequest an amount $K$ of the domestic asset, and $K^*$ of the foreign asset. He must choose whether to keep this portfolio or to reshuffle it. At the end of the period, he consumes the proceeds $i, K + i^* K^*$. He leaves $K$ and $K^*$ as a bequest for the next generation.

15 If agents in the financial market were long lived then $di$ would not be given by a simple expression such as (3'). The imposition of the Tobin tax would generate a region (around $Z=0$) where $i$ would be insensitive to changes in $Z$. This is because a long-lived investor knows that in the near future he will want to buy back the asset he is selling today. Since he will need to pay the Tobin tax then, it is optimal to be cautious and wait for a sufficiently high level of $Z$ before selling the domestic asset. In contrast, if the investor is short-lived, he will never buy back the asset he sells. Therefore instantaneous returns must be equalized period by period and the process for $di$ is given by (3').
Tobin tax increases. This dampens the desired net capital outflow and thus reduces the increase in the domestic interest rate. Conversely, when expectations improve, the Tobin tax is reduced. Two points are important to note. First, the expected value of $T$ at time $t=0$ is zero. This is because since at time $t=0 W_0-Z_0=0$, and $W$ follows a diffusion process (random walk in continuous time), the expected value of $Z$ is zero. Second, this sequence of Tobin taxes does not introduce a systematic wedge between the domestic and foreign interest rates. This is because the process governing $Z$ has a mean-reverting drift. Therefore, in the long run $Z$ and $T$ converge to zero.

Since the sequence of Tobin taxes does not allow the domestic interest rate to deviate systematically from its long-run level, it does not give freedom to the authorities to pursue systematically 'wrong policies'. Note also that these taxes are not generalized capital controls. They are imposed only on swaps between domestic and foreign financial assets, not on the repatriation of dividends from real capital. Their purpose is to reduce the excess mobility of financial capital or, equivalently, to reduce the variability of the domestic interest rate, not to lock-in domestically held capital.

It seems a difficult task for the authorities to calculate each period what the state of expectations is, in order to adjust the Tobin tax. Note, however, that this is not necessary: the sequence of continually adjusting Tobin taxes can be implemented through a dual exchange rate system, as discussed in section 3.

4.2. The real sector

In contrast to the financial sector, this sector is populated by infinitely
lived investors. During each period the representative investor must choose whether to wait or to borrow in the financial market in order to hire labor services and install the marginal unit of productive capital. If he decides to wait, he gets no return, but he keeps the option to invest this unit in the future. If he decides to borrow and invest, then he will need to pay during each period, ad-infinitum (because productive capital is irreversible), the annuity value of the amount borrowed.\textsuperscript{16}

Since no variable factors enter in the production of the exportable good, the production decision is trivial: produce the maximum amount possible.\textsuperscript{17} We assume that labor is elastically supplied, that the unit price of labor services and of the exportable good (in terms of the consumption good) are one, and that the production function is given by \( f(K) = K^\alpha \) \((0 < \alpha < 1)\). It follows that the operating profits are

\[
\pi(i, K, t) = K^\alpha - i, K, t, \tag{4}
\]

We further assume that the long-lived investors are risk neutral with a discount rate \( r \). It follows that the problem solved by the representative investor in the real sector is

\[
\begin{align*}
\max_{i, t} & \int_{t+1}^{\infty} e^{-r(t-s)} \pi(i, K, t) ds, \\
\text{s.t.} & \quad K_t = I_t, \\
& \quad I_t \geq 0 \quad \text{(irreversibility)},
\end{align*}
\]

where

\( K \) is the stock of productive capital,

\( I_t \) is the amount of new capital installed during period \( t \).

Since variations in the domestic interest rate just reflect rumors and are not related to fundamentals, the first-best allocation is the one that sets \( i_* = i \) in (4) for the infinite future. This allocation cannot be achieved in a decentralized system due to the existence of irreversibility and rumors, which confer value to the 'option to wait'. This option is the market distortion that keeps real capital below its first-best level. In the remainder of this section we

\textsuperscript{16}The fact that real capital is irreversible just means that it cannot be put into alternative uses, it does not mean that its owner cannot sell it. However, the irreversibility will be reflected in a lower sale price.

\textsuperscript{17}Since during each period, the investor consumes the difference between his revenue and the interest payments, it is possible that consumption will be negative during periods of very high interest rates. If negative consumption were not allowed, then the investor would go bankrupt in this situation.
solve (5) and in the next section we show how Tobin taxes can bring the capital stock nearer to its first-best level.

The irreversible investment problem is very similar to the option valuation problem. For the case in which the position of the demand curve follows a geometric Brownian motion, it has been solved by Bertola (1987), McDonald and Siegel (1986), and Pindyck (1986). In the case we are considering, the interest rate does not follow a geometric process, but a mean-reverting process. We will solve this problem by transforming it into a geometric Brownian process using the Girsanov theorem.

We will solve (5) using an optimal stopping argument as in Van Moerebeke (1976). Consider the decision to install the marginal unit of capital. The investor's problem is to determine the interest rate at which it is optimal to stop waiting and invest. At each instant, the investor has two choices: to invest the marginal unit capital or to wait. The payoff of investing is the present value of future profits:

\[ J(i,t,K) = E_i \int_t^\infty e^{-i(t-s)} \pi_k(i,s) ds, \]  

where

\[ \pi_k(i,s) = \alpha K^{s-1} - i. \]  

If instead the investor decides to wait for an interval of length \( \tau \), he does not receive any profits. Thus, the payoff associated with waiting for a period of length \( \tau \), and then investing, is just the expected increase in the value of the project:

\[ E_i[J(i_{\tau+}, t+\tau; K) \mid i = i_\tau] - J(i,t,K). \]  

The problem is to determine the waiting time interval that maximizes (8), i.e. to find the optimal stopping time over all \( \tau \in [t, \infty) \):

\[ V(i_t,t; K) = \max_{\tau} E_i[J(i_{\tau+}, t+\tau; K) \mid i = i_\tau]. \]  

We now turn to find the optimal value of waiting \( V \), and the stopping time \( \tau \). Note that since \( J \) is continuous, we can confine our attention to the local solution. Thus the optimal strategy is to wait as long as the expected value of waiting is higher than the value of investing the marginal unit of capital, and to invest as soon as this ordering is reversed. This strategy partitions the space \((i,t)\) in two regions: a continuation region (that is open) in which it is
optimal to continue waiting, and a stopping region in which it is optimal to invest. That is,

wait as long as: \( E_i V(i_{t+dt}, t+dt; K) > J(i_t, t; K) \) (continuation region), (10a)

invest as soon as: \( E_i V(i_{t+dt}, t+dt; K) = J(i_t, t; K) \) (stopping region). (10b)

Since the horizon is infinite, time only enters the value function through the discount factor. Therefore, the optimal boundary which separates the continuation and stopping regions is time-independent. Let us denote the optimal boundary by \( i(K) \). Then, the continuation region consists of all interest rates higher than \( i(K) \), and the stopping region consists of all interest rates lower or equal to \( i(K) \). That is, the optimal boundary is just an interest rate at or below which it is optimal to invest the marginal unit of capital. Given this boundary, the optimal strategy (10) implicitly defines the optimal value function \( V \):

\[
V(i_t; K) = \begin{cases} 
  e^{-\alpha dt} E_i V(i_{t+dt}; K) & \text{in the continuation region,} \\
  J(i_t; K) & \text{in the stopping region.} 
\end{cases}
\] (11)

Up to now, the argument is circular. To obtain the value function \( V \) we need to know the optimal boundary \( i \). However, \( i \) is itself determined by the equality of \( J \) and the yet unknown function \( V \). In order to find \( i \) and \( V \) in closed form we will assume that \( V \) is sufficiently differentiable so that we can take a Taylor expansion of (11) around \( i_t \). Using Itô's lemma it follows that we can approximate the first equation in (11) by

\[
E_i V(i_{t+dt}; K) = E_i \{ V(i_t; K) - r V(i_t; K) + V'(i_t; K) di_i + \frac{1}{2} V''(i_t; K)[di_i]^2 \}.
\]

From (3') it follows that

\[
E(di_i) = a[1-\mu](i^*/i - 1) dt \quad \text{and} \quad E(di_i)^2 = (\sigma[1-\mu]i)^2 dt.
\]

Thus, (11) can be rewritten as

\[
0 = H \equiv -i^* V + a[1-\mu](i^*/i - 1) \cdot V' + \frac{1}{2}(\sigma[1-\mu]i)^2 \cdot V''
\]

for \( i > i(K) \), (12a)
\[ V(i; K) = \bar{J}(i; K); K) \quad \text{for } i \leq \bar{i}(K). \quad (12b) \]

Eq. (12a) is the differential equation that \( V \) must satisfy as long as investment has not taken place. It represents the expected gain from waiting. Eq. (12b) states that once investment takes place \( V \) is just the value of the marginal unit invested. In order to find \( V \) and the free boundary \( i \) we need to add two more boundary or initial conditions. First, since the free boundary \( \bar{i}(K) \) is optimally chosen, it must be impossible to gain by waiting a bit longer for a reduction in \( i \). Therefore, \( \bar{i}(K) \) must be such that there is a 'smooth fit' between \( V \) and \( \bar{J}(i; K) \).

\[ \frac{\partial V(i; K)}{\partial i} = \frac{\partial \bar{J}(i; K)}{\partial i}. \quad (12c) \]

Second, we impose a boundedness condition on \( V \):

\[ \lim_{i \to \pm \infty} V(i) = 0. \quad (12d) \]

The solution to (12) will give us the value function of waiting \( V(i; K) \) and the interest rate below which it is optimal to invest [i.e., \( i(K) \)]. To ensure that this is indeed the solution we need to verify that it pays to wait as long as \( i > \bar{i}(K) \) [i.e., condition (10a) holds], and that the payoff from waiting when \( i < \bar{i}(K) \) is nonpositive. That is,

\[ V > J \quad \text{if} \quad i > \bar{i}(K), \quad (13a) \]

\[ H \leq 0 \quad \text{if} \quad i < \bar{i}(K). \quad (13b) \]

Let us now solve (12). Note that the term containing \( V' \) makes it impossible to find a simple closed form solution for \( V \). If it were eliminated, then (12) would become a Cauchy–Euler equation and a simple solution could be found. Surprisingly, this term can be removed by means of the Girsanov Theorem. The trick consists in considering instead of \( \bar{W} \) another Brownian motion \( \hat{W} \) under a different probability measure which effectively equates \( i \) to \( i^* \). The statement of this theorem and its application to the problem we are concerned with is in Appendix A. Here we just show the mechanics. Note that our objective is to eliminate the drift term \( a[1 - \mu][i^* - i_t] \, dt \) appearing in (12). In order to do this consider the process

\[ \hat{W}_t \equiv W_t + \int_0^t \frac{a[i^* - i_s]}{\sigma} \, ds. \]

This is the 'high-impact' condition of Merton (1973) and the 'smooth-pasting' condition of Krylov (1980) and Dixit (1988).
The Girsanov theorem establishes that this process is Brownian motion under a new probability measure $\tilde{P}$ defined in Appendix A. We will write eq. (3') in terms of this new process. First, note that

$$d\tilde{W}_t = dW_t + \left[ a[i^* - i_t]/\sigma \right] dt.$$

Second, substitute for $dW_t$ in (3') and note that the two terms in $a[i^* - i_t]$ cancel out. Thus we get

$$di_t/i_t = \sigma[1 - \mu]d\tilde{W}_t.$$

This is precisely what we wanted! Now, we can write eq. (12a), in terms of the new process $\tilde{W}_t$ as

$$0 = -rV + \frac{1}{2}(\sigma[1 - \mu]i_t)^2 \cdot V'' \quad \text{in the continuation region.}$$

This differential equation has a very simple general solution:\textsuperscript{19}

$$V(i) = c_1 i^{\lambda_1} + c_2 i^{\lambda_2}$$

with

$$\lambda_1 = \frac{1}{2} - \frac{(2i^*/\sigma^2 - 1)}{2} < 0, \quad \lambda_2 = \frac{1}{2} + \frac{(2i^*/\sigma^2 - 1)}{2} > 1.$$

In order to determine the two constants $c_1$ and $c_2$, and the free boundary $i$ we use conditions (12b)-(12d) and the closed form for $J$ obtained in Appendix B:

$$J(i_t) = [\alpha K^{i_t} - i_t]/i^*.$$  \hspace{1cm} (15)

First, since $\lambda_2 > 0$, (12d) implies that $c_2 = 0$. Second, by substituting (12c), (14) and (15) in (12b) we get

$$i(K) = [\lambda/(\lambda - 1)] \cdot \alpha K^{i_t}$$

(henceforth we will denote $\lambda_1$ by $\lambda$). \hspace{1cm} (16)

and

$$V(i_t; K) = \begin{cases} (-1/\lambda i^*) \cdot i(K)^{1 - \lambda} \cdot i_t^\lambda & \text{for } i_t > i(K), \\ (\alpha K^{i_t} - i_t)/i^* & \text{for } i_t \leq i(K). \end{cases}$$ \hspace{1cm} (17)

The first expression in (17) is the value of the option to wait and the second

\textsuperscript{19}The solution is obtained using the same procedure as the one used in Appendix B to obtain $J$. 
is just the expected present value of profits derived from the marginal unit invested.

Let us now derive the optimal investment rule. Recall that we casted the problem in terms of the marginal unit of irreversible capital. This marginal unit should be invested if the net present value of marginal profits exceeds the expected value of waiting, i.e. if $EJ(i_t, t; K) > EV(i_{t+dt}, t + dt; K))$. We then showed that this is the case whenever the interest rate is lower than the critical level $i(K)$. To go from here to the optimal investment rule, just note that since $i$ is monotonically decreasing in the amount of fixed capital, as shown in fig. 3, we can define a desired capital level $K$ as a function of the current interest rate:

$$K(i_t) = \left[\frac{(\lambda - 1)}{\sigma} \cdot (i_t/\sigma)\right]^{1/(\alpha - 1)}$$

(18)

And the investment rule becomes:

- invest as long as $K_t < K(i_t, \sigma [1 - \mu])$,
- wait if $K_t \geq K(i_t, \sigma [1 - \mu])$.

(19)

Let us now compare the stock of capital that would result in this economy with the first-best allocation. Suppose that initially $K_0 = 0$ and $i_0 = i^*$ and recall that the first-best level of $K$ is the one chosen by a central planner who disregards rumors and faces a constant interest rate equal to $i^*$. The first-best allocation is the one that equates to zero the expected net present value of marginal profits:

$$\pi_K(i^*, K^{fb})/i^* = 0.$$ 

(20)
As can be seen in (10) or (17), if the value of the option to wait ‘V’ were zero, the representative investor would invest according to (20). Thus we might consider the option to wait as the distortion. Alternatively, we can invert (20) and get the first-best capital stock:

$$K^{fb} = \left[ \frac{i^*}{\sigma^2} \right]^{1/(\alpha - 1)}.$$  (21)

Comparing (21) with (18) it follows that there exists underinvestment, i.e. $K(i^*) < K^{fb}$ because $\lambda < 0$. The important point to notice is that when capital is irreversible, even infinitely lived investors that are risk neutral, and face an expected rate of interest $i^*$ will invest less than $K^{fb}$. The reason is that it pays to wait for lower interest rates.

The intuition for this is the following: when investment is irreversible, the installation of capital involves giving up the option of waiting for new information about $i$, and using this information to re-evaluate the convenience and timing of installation. Thus, the value of the investment must equal the purchasing cost plus the value of the option to wait. A mean-preserving increase in the variance of the interest rate reduces the desired capital stock (at each level of $i$) because it increases the value of the option to wait (i.e. the distortion), while leaving unchanged the value of installed capital:

$$\frac{\partial V}{\partial \sigma^2} = \frac{\partial V}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma^2} = -V \cdot \log \left( \frac{i}{i_h} \right) \cdot \frac{i^*}{\sigma^2 [1 - \mu]^2} \left[ \frac{1}{4} + \frac{2i^*}{\sigma^2 [1 - \mu]^2} \right]^{-1/2} \geq 0.$$

This expression is non-negative since the capital stock is continuously adjusted so that $i(K)$ is never higher than $i_h$. Thus $\log (i/i_h) \leq 0$. The intuition for this result is clarified by relating the investment decision to option pricing. The decision to install the marginal unit of capital is equivalent to exercising a call option on a stock that pays dividends. The price of the stock is the value of an installed unit of capital ($J$), the dividends are the operating profits, the call’s exercise price is the purchasing cost of capital.

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20This point was made by Bernanke (1983) and further developed by Bertola (1987), McDonald and Siegel (1986) and Pindyck (1986).

21A call option is a right to buy an asset at a specified price (called exercise price) during the period of the contract. There are two types of calls: European and American. A European call can only be exercised at maturity, whereas an American can be exercised before. The price of the call when exercised is $C = \max [0, S - X]$, where $S$ is the stock price and $X$ is the exercise price. Heuristically, in a risk-neutral world, the price of a European call $t$ periods before maturity is $C_{T-t} = e^{-rT} \max [0, S - X]$.
(which in this case is zero since capital is debt financed) and the value of the call is \( V \). The decision to exercise the option is the same as the optimal strategy (10): exercise it if \( V \leq J \). Now, for illustrative purposes consider a European call that matures at time \( T \). Its value at \( T-t \) is \( e^{-r(T-t)} \max\{0, J\} \). This value increases with the variance of \( i \). With a higher variance, the likelihood of a low \( i \) (and thus a high \( J \) and high profits derived from selling the option) at the time of maturity (\( T \)) is greater. Of course, the likelihood of a high \( i_T \) (low \( J \)) is also greater. But since the call price at maturity cannot be lower than zero, the first effect dominates and the value of the option at time \( T-t \) increases.\(^{22}\)

5. Tobin taxes as a second-best policy

Consider an economy in the constitution-design stage, i.e., before the realization of uncertainty and when no capital has been installed yet, and ask the question: which rule would bring the amount of irreversible real capital as near as possible to its first-best level, subject to the restriction of not reducing expected welfare in the financial sector? The first-best policy is the creation of an insurance scheme that would allow real capital to be reversible de facto. In the absence of this scheme, a second-best policy is a mean-preserving reduction in the variance of operating profits in the real sector. This is precisely what Tobin taxes do. They reduce the variance of the domestic interest rate, while keeping its mean constant. Since real investment is irreversible, this variance reduction induces more investment at each level of the interest rate. Thus the Tobin tax will improve the allocation of resources. Moreover, if investors in the financial sector are risk neutral these taxes will not reduce their welfare since the expected value of domestic interest rates remains unchanged.

We now turn to formalize this idea. Consider an economy at time \( t=0 \), before the realization of uncertainty, with \( i_0 = i^* \) and \( K_0 = 0 \). Suppose that a central planner has to choose the value of \( \mu \), i.e., the proportion by which the Tobin tax will respond to changes in the expectations of financial investors. The objective function of the central planner is\(^{23}\)

\[
\min_{\mu} \mathbb{E}[K(i_0; \sigma(1-\mu)) - K^{fb}]^2, \tag{22}
\]

\(^{22}\) Another way to put it is that the option to wait is killed only when the interest rate is low enough (i.e., \( i < \bar{i}(K) \)). With a higher variance, the likelihood of very low interest rates is greater. This increases the value of the option to wait for a lower \( i \). Of course, the likelihood of extremely high interest rates is also greater. But these realizations have no effect on the value of the option. This is because once \( i \) is higher than \( \bar{i}(K) \), no investment takes place regardless of the level of \( i \).

\(^{23}\) We assume that the central planner can precommit not to alter \( \mu \) in the future.
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\[ \text{s.t. } E_{i,0}[i_t - i^*] = 0 \text{ for all } t, \]
\[ \mu \in [0, 1]. \]

The first restriction implies that the sequence of Tobin taxes will not induce systematic deviations of the domestic interest rate from the international interest rate. Since investors in the financial market are risk neutral, this implies that their welfare is not affected.

Given the results of the preceding sections, the solution to (22) is straightforward: set \( \mu = 1 \). The argument is as follows:

(i) Since initially \( K_0 = 0 \), during the first instant \( t \), \( K \) will jump to \( K(i, \sigma[1 - \mu]) \).

(ii) Initially \( i_0 = i^* \), thus by setting \( \mu = 1 \), we ensure that \( i_t = i^* \) for all \( t \), and that the variance of operating profits in the real sector becomes zero \( (\sigma[1 - \mu]) = 0 \).

(iii) From (i) and (ii) it follows that \( K \) will be equal to \( K(i^*, 0) \), which is in turn equal to the first best level of real capital. That is,

\[ \lim_{\mu \to 1} K(i^*) = \left\{ \lim_{\lambda \to \infty} [1 - 1/\lambda] \cdot i^*/2 \right\}^{1/(2 - 1)} = K^{fb}. \]

Note that the reduction in the variance of the domestic interest rate reduces the value of the option to wait. We did not include the value of the option to wait in the central planner's objective function because, from a social perspective, variations in the domestic interest rate do not represent changes in the opportunities of the real sector. They just represent rumors.

Although it is desirable to introduce a sequence of Tobin taxes that would insulate the domestic interest rate from rumors, it must be recognized that there are important difficulties with its implementation. These difficulties have to do with enforcement and with information about the state of expectations. Regarding the first, it is clear that the high tax rates that might be sometimes needed are not cheating-proof. The higher the tax rate, the higher the incentives to avoid paying it. Consequently, it will in general be optimal not to insulate completely the domestic interest rate from rumors (to set \( \mu \) at a level lower than one). Quoting Tobin: 'Doubtless there would be difficulties of administration and enforcement. Doubtless there would be ingenious patterns of evasion. But since these will not be costless either, the main purpose of the plan will not be lost'.

Regarding the second difficulty, it is quite idealistic to expect that the authorities have perfect knowledge about the evolution of rumors, so as to be able to adjust the tax rate every period. Note, however, that this task can be done by the market, through a dual exchange rate system. Under this
system, commercial and financial transactions take place at different exchange rates. If the financial exchange rate is freely floating, then, when expectations turn pessimistic and investors try to buy foreign assets, the financial exchange rate depreciates, reducing the profitability of investing financial capital abroad. Conversely, when expectations turn optimistic, the financial exchange rate appreciates, reducing the extent of capital inflows [see Tornell (1988)]. Therefore, a dual exchange rate system acts as a sequence of continuously-adjusting Tobin taxes.

6. Conclusions

In the last years, countries like Mexico and Argentina have not been able to induce investment in real capital. Capital inflows have indeed occurred, but have taken the form of financial investment, and have been followed by massive capital outflows. In many cases this round-tripping was just responding to rumors. The question arises as to whether free mobility of financial capital is optimal.

We analyzed this issue by considering a two-sector economy where investment in real capital is irreversible and debt-financed. Furthermore, the interest rate, which is determined in the financial sector, is random as a result of volatile expectations. In this economy the expected return on real capital is above the expected interest rate. This is because the option to wait for lower interest rates has a positive value. This option is the distortion.

In the presence of rumors, taxes on international financial transactions (Tobin taxes) reduce the variance of the domestic interest rate, while leaving its mean unchanged. As a result, they induce more investment in irreversible real capital. In other words, they reduce the distortion. In a risk-neutral world this is welfare-improving since the allocation of investment is improved and expected returns in the financial sector remain unchanged.

Appendix A: The Girsanov theorem

In this section we follow Karatzas and Shreve (1987) in constructing an auxiliary probability measure \( \hat{P} \) under which the process \( \hat{W} \) of the text is Brownian motion. Set

\[
X_t = \exp \left\{ - \int_0^t L(s) dW(s) - \frac{1}{2} \int_0^t L(s)^2 ds \right\}, \quad \mathcal{F}_t, \quad 0 \leq t \leq \infty,
\]

where

\[
L(t) \equiv a \left[ i^* - i_t \right]/\sigma.
\]
Note that $X$ is a martingale, thus we can define the following auxiliary probability measure on $\mathbb{F}_T^W$:

$$\tilde{P}(A) \equiv \mathbb{E}(X_t 1_A); \quad A \in \mathbb{F}_T^W, \quad 0 \leq T \leq \infty.$$ 

The auxiliary probability measure $\tilde{P}$ is related to $P$ by $d\tilde{P} = X_t dP$, where $X$ is known as the Radon–Nikodym derivative.

If $X(L)$ is a martingale and $L(t)$ is progressively measurable and satisfies

$$\mathbb{E} \left[ \int_0^T (L_s)^2 ds < \infty \right] = 1, \quad 0 \leq T \leq \infty,$$

Then a corollary to the Girsanov theorem stated in Karatzas and Shreve (1987, p. 192) establishes that the process

$$\tilde{W}(t) \equiv W(t) + \int_0^t L(s) ds, \quad 0 \leq t \leq \infty,$$

is Brownian motion on $(\Omega, \mathbb{F}_t^W, \tilde{P})$, whenever the process $W(t)$ is Brownian motion on $(\Omega, \mathbb{F}, P)$ with $P[W_0 = 0] = 1$.

Appendix B: Closed form for the value function $J$

We can rewrite (6) as

$$J(i_t; K) = \mathbb{E}_i \left( \int_t^{t+dt} e^{-i[s-t]} \pi_k(i_s; K) ds + \int_t^{t+dt} e^{-i[s-t]} \pi_k(i_s; K) ds \right).$$

(6')

$$J(i_t; K)/dt = \mathbb{E}_i \left[ \left( \int_t^{t+dt} e^{-i[s-t]} \pi_k(i_s; K) ds + E_{it+dt} e^{-i*dt} J(i_{t+dt}; K) \right)/dt \right].$$

Next, note that

$$\lim_{dt \to 0} \mathbb{E}_i \left[ \left( \int_t^{t+dt} e^{-i[s-t]} \pi_k(i_s; K) ds \right)/dt \right] = \pi_k(i_t; K).$$

Taking a Taylor expansion of $J(i_{t+dt}; K)$ around $i_t$ as we did with $V$, and taking the limit as $dt$ tends to zero we get

$$-i^*J + a(i^*/i_t - 1) \cdot J' + \frac{1}{2} (\sigma[1-\mu]i_t)^2 \cdot J'' = \alpha K^{x-1} - i_t,$$

which in terms of the new process $\tilde{W}$ can be written as

$$-i^*J + \frac{1}{2} (\sigma[1-\mu]i_t)^2 \cdot J'' = \alpha K^{x-1} - i_t.$$
This is a second-order linear ordinary differential equation with variable coefficients of the Cauchy–Euler type. Its general solution is the sum of a complementary and a particular solution. To obtain the complementary solution we will transform the equation into one with constant coefficients using the transformation \( i = e^h \). It follows that \( dJ/dh = V'i \), and \( d^2J/dh^2 = J'i + J''i^2 \). Substituting back we get

\[
-i^*J + \frac{1}{2}(\sigma[1-\mu])^2 \cdot (d^2J/dh^2 - dJ/dh) = 0.
\]

The characteristic equation is: \( \lambda^2 - \lambda - 2i^*/\sigma^2[1-\mu]^2 = 0 \), and the eigenvalues are:

\[
\lambda_1 = \frac{1}{2} - \sqrt{1/4 + 2i^*/\sigma^2[1-\mu]^2}, \quad \lambda_2 = \frac{1}{2} + \sqrt{1/4 + 2i^*/\sigma^2[1-\mu]^2}.
\]

Note that \( \lambda_1 < 0 \) and \( \lambda_1 + \lambda_2 = 1 \). Thus \( \lambda_2 > 1 \).

The complementary solution is then \( J^c(h) = c_1 e^{\lambda_1 h} + c_2 e^{\lambda_2 h} \) or, making the substitution \( i = \log h \): \( J^c(i) = c_1 i_{\lambda_1} + c_2 i_{\lambda_2} \).

The particular solution is given by

\[
J^p(i) = \int\sum_{i=0}^{\infty} e^{-(s-i)\mu} [\alpha K^{s-1} - i_i] \, ds = [\alpha K^{s-1} - i_i]/i^*.
\]

Therefore, the general solution is

\[
J(i) = c_1 i_{\lambda_1} + c_2 i_{\lambda_2} + [\alpha K^{s-1} - i_i]/i^*.
\]

Finally, we impose the following boundary conditions: that \( J(i) \) is bounded when \( i \) goes to zero and that \( J(i)/i \) is bounded when \( i \) goes to infinity. Since \( \lambda_1 < 0 \), the first condition implies that \( c_1 = 0 \), and since \( \lambda_2 > 1 \), the second condition implies that \( c_2 = 0 \). Hence,

\[
J(i) = (\alpha K^{s-1} - i_i)/i^*.
\]

**Appendix C: National income and balance of payments accounts**

The objective of this appendix is to check the consistency of the model. In this economy two goods are produced: productive capital \( K \) and an exportable good \( X \). Productive capital is produced using only labor services as an input, and the exportable good is produced using only productive capital as an input. The consumption good is imported, and we consider it as the numeraire. Since the unit price of labor and the exportable good are one,
and productive capital does not depreciate, the gross (and net) domestic product is given by

\[ Y = \dot{K} + X \quad \text{where} \quad \dot{K} \] is new productive capital. \hfill (C.1)

Total consumption \((C)\) is the sum of the financial and real investors' consumption \((C_f\) and \(C_r\)) plus the workers' consumption \((C_w)\).

\[ C = C_f + C_r + C_w. \quad \hfill (C.2) \]

Since we assumed that productive capital is debt-financed and that during each period all proceeds are consumed, it follows that

\[ C_r = X - Ki, \quad \hfill (C.3) \]
\[ C_f = Ki + K^*i^*, \quad \hfill (C.4) \]
\[ C_w = \dot{K}, \quad \hfill (C.5) \]

Since the only import is the consumption good, the trade balance is given by

\[ T = X - C = -K^*i^* - \dot{K}. \quad \hfill (C.6) \]

Note that in this economy, the basic identity between the trade balance and output net of absorption is satisfied \((Y - A \equiv T)\), where absorption \((A)\) is the sum of consumption \((C)\) and investment \((K)\).

Next we will show that in this economy every capital account imbalance has a corresponding current account imbalance, i.e., the balance of payments is always zero. When financial capital flows into the country, it goes, through the financial intermediary, to the real sector in the form of a loan. This loan is in turn used to hire labor services and install new productive capital. Therefore, the capital account is given by

\[ KA = \dot{K}. \quad \hfill (C.7) \]

The current account is

\[ CA = T + K^*i^*. \quad \hfill (C.8) \]

It follows from \((C.2)-(C.6)\) that the current accounts can be expressed as

\[ CA = \{Y - [(Y - Ki) + (Ki + K^*i^*) + \dot{K}]\} + K^*i^* = -\dot{K}. \]

Therefore, the capital and the current accounts add up to zero.
References

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