Joint Transceiver Beamforming Design for End-to-End Optimization in Full-Duplex MIMO Relay System With Self-Interference

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***Abstract*— In this letter, a novel transceiver beamforming design is proposed for improving the end-to-end performance of a full-duplex amplify-and-forward relay system. Different from previous schemes based on nulling out residual self-interference (RSI), our proposed schemes can capture a balance between useful signal improvement and RSI suppression for further achievable rate improvement. To solve the nonconvex optimization problem, an iterative algorithm with proven convergence is developed. Specifically, at each iteration, an optimal solution derived by reformulating the optimization problem as a fractional program problem and an efficient sub-optimal closed-form solution are presented, respectively. The simulation results show that the proposed design outperforms the traditional schemes in all signal-to-noise ratio regime.**

***Index Terms*— Full-duplex, residual self-interference, achievable rate, fractional programming, closed-form solution.**

I. Introduction

FULL-DUPLEX (FD) communication system which supporting the concurrent transmission and reception over the same frequency can double the spectral efficiency compared with the half-duplex communication system. However, the implementation of FD communication encounters a serious challenge since it suffers from the loopback self-interference (SI) in practice.[1] SI mitigation methods, which make FD communication a very promising technique for the 5G wireless networks [1], have been intensively investigated in [2], [3].

In recent researches, instead of purely SI suppression, the e2e performance has received significant interest as an important metric of FD AF relay systems [4]–[6]. In [4], a joint interference-nulling algorithm is derived to maximize the e2e achievable rate. A low complexity joint precoding/decoding design for e2e SNR maximization based on nulling out RSI is proposed in [5]. Moreover, closed-form solutions for the scheme's outage probability as well as high SNR simple expressions are provided. In [6], a joint beamforming design is presented to maximize the e2e spectrum efficiency for a two-way transmission with the FD source, relay and destination.

It is worth noting that the traditional schemes tackle the problem of maximizing e2e performance in a limited scope: restricting the system to nulling out SI. This restriction results in sub-optimal solutions because the proposed beamformers subjected to nulling out SI might also accidentally suppress the useful signal as well. This phenomenon is analyzed in [7]–[10]. A new closed-form expression for the outage probability is obtained in [7] based on taking the effect of RSI into account. The scheme in [8] examined the effect of RSI in cooperative CP-SC spectrum sharing with FD AF relay and obtained the exact and asymptotic outage probability for relay selection policies in frequency selective fading channels. In [9], the instantaneous throughput is maximized by optimizing beamformers at the decode-and-forward (DF) FD relay. The results in [9] reveal that the zero-forcing (ZF)-based schemes have preferable performance only in high SNR regime. In [10], improper Gaussian signaling (IGS) is firstly employed in an attempt to alleviate the non-negligible RSI adverse effect in DF FD relaying. The results show that IGS yields a promising e2e performance over conventional proper Gaussian signaling.

Inspired by the above observations, in this letter we develop a general optimal beamforming design for FD AF relay, which jointly considers the effects of SI suppression and useful signal improvement. Our main contributions are twofold. 1) A general transceiver beamforming scheme is proposed to maximize the e2e achievable rate by capturing the balance between SI suppression and useful signal improvement. 2) The challenging problem is solved by an iterative algorithm with proven convergence. At each iteration, an optimal solution derived by transforming the nonconvex subproblem into a fractional program form and an efficient suboptimal closed-form solution with low complexity are presented. At last, simulation results confirm that the proposed design is more general and efficient than the ZF-based methods.

Notation: The absolution value of a scalar x is denoted by $|x|$. The Euclidean norm of a column vector $\mathbf{x}$ is denoted by $\|\mathbf{x}\|$. The conjugate transpose and trace of a matrix $\mathbf{X}$ are denoted by $\mathbf{X}^H$ and $\mathrm{Tr}(\mathbf{X})$, respectively. The expectation operator is denoted by $\varepsilon(\cdot)$. An identity matrix is denoted by $\mathbf{I}$. The set of N-by-Mcomplex matrices is denoted by $\mathbb{C}^{N\times M}$. The complex Gaussian distribution with mean m and variance υ is denoted by $\mathcal{CN}(m,\upsilon)$.

II. System Model and Problem Statement

The block diagram of the FD AF relay system is illustrated in Fig. 1. The FD relay node (R) is equipped with N receive antennas and M transmit antennas. Both S and D are equipped with a single antenna. The channels S-R, R-D and R-R are denoted by $\mathbf{h}_{\mathrm{SR}} \in \mathbb{C}^{N\times 1}$, $\mathbf{h}_{\mathrm{RD}} \in \mathbb{C}^{M\times 1}$ and $\mathbf{H}_{\mathrm{RR}}$, respectively. The RSI channel $\mathbf{H}_{\mathrm{RR}}$ can be modeled as $\sqrt{\rho}\mathbf{H}_{\mathrm{loop}}$ where a scalar $\rho \in [0,1]$ is used to parameterize the effect of the imperfect SI cancellation[2] and $\mathbf{H}_{\mathrm{loop}} \in \mathbb{C}^{N\times M}$ denotes a

Manuscript received May 23, 2016; revised July 1, 2016; accepted July 10, 2016. Date of publication July 18, 2016; date of current version September 8, 2016. This paper was supported by the National Natural Science Foundation of China (No. 61271232). The associate editor coordinating the review of this letter and approving it for publication was K. Tourki.

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Digital Object Identifier 10.1109/LCOMM.2016.2592502

[1]The SI can not be simply cancelled out by subtracting the known transmitted signal form the received signal because of he limited ADC dynamic-range, nonlinearities associated with amplifiers, oscillator phase-noise, etc.[3].

[2]To reduce the effects of SI on system performance, we assume that an imperfect SI cancellation scheme has been applied at R which can effectively remove the dominant line-of-sight component in SI.

non-selective independent Rayleigh block fading channel [9], [11]. Each element of $\mathbf{h}_{\text{SR}}$, $\mathbf{h}_{\text{RD}}$ and $\mathbf{H}_{\text{loop}}$ can be modeled as $\mathcal{CN}(0, \sigma_X^2)$, where $X \in \{\text{SR}, \text{RD}, \text{loop}\}$, respectively. The additive noises $\mathbf{n}_{\text{R}} \in \mathbb{C}^{N\times 1}$ and $n_{\text{D}} \in \mathbb{C}$ at the receiving side are modeled as $\mathcal{CN}(0, \sigma_Y^2)$,where $Y \in \{\text{R}, \text{D}\}$, respectively. We assume that the perfect channel state information (CSI) is known at the relay.[3] The direct channel from S to D is neglected because of sufficiently large path loss between them. At the time i instance, the signal after the normalized receive beamforming vector $\mathbf{d}^H \in \mathbb{C}^{1\times N}$ at the relay is denoted by $r_{\text{in}}[i]$. The signal before the normalized transmit beamforming vector $\mathbf{p} \in \mathbb{C}^{M\times 1}$ at the relay is denoted by $r_{\text{out}}[i]$. They can be expressed as: $r_{\text{in}}[i] = \mathbf{d}^H(\mathbf{h}_{\text{SR}}x_{\text{s}}[i] + \mathbf{H}_{\text{RR}}\mathbf{p}r_{\text{out}}[i] + \mathbf{n}_{\text{R}}[i])$ and $r_{\text{out}}[i] = \beta r_{\text{in}}[i-\tau]$, where $x_{\text{s}}[i]$ denotes the signal from S with the power $P_{\text{S}} = \varepsilon\left\{|x_{\text{s}}[i]|^2\right\}$, β denotes the gain of the relay, τ denotes the processing delay.

By recursive substitution of $r_{\text{in}}[i]$ and $r_{\text{out}}[i]$, the signal $r_{\text{out}}[i]$ can be further calculated as $r_{\text{out}}[i] = \beta\sum_{j=1}^{\infty}\left(\beta\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}\right)^{j-1}\left(\mathbf{d}^H\mathbf{h}_{\text{SR}}x_{\text{s}}[i-j\tau] + \mathbf{d}^H\mathbf{n}_{\text{R}}[i-j\tau]\right)$ with the condition $\beta^2 < \frac{1}{\left|\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}\right|^2}$ for preventing oscillation and guaranteeing finite relay transmitted power.

By assuming that all the channels vary slowly and the signals and noise are independent from each other, the transmitted power of the relay $P_{\text{R}} = \varepsilon\left\{\|\mathbf{p}r_{out}[i]\|_2^2\right\} = \varepsilon\left\{|r_{out}[i]|^2\right\}$ can be calculated as:

$$\begin{aligned} P_{\text{R}} &= \beta^2\sum_{j=1}^{\infty}\left(\beta^2\left|\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}\right|^2\right)^{j-1}\left(P_{\text{S}}\left|\mathbf{d}^H\mathbf{h}_{\text{SR}}\right|^2 + \sigma_{\text{R}}^2\right) \\ &= \beta^2\frac{P_{\text{S}}\left|\mathbf{d}^H\mathbf{h}_{\text{SR}}\right|^2 + \sigma_{\text{R}}^2}{1-\beta^2\left|\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}\right|^2}. \end{aligned} \tag{1}$$

The received signal at D is denoted by $y_D = \mathbf{h}_{RD}{}^H\mathbf{p}r_{out}[i] + n_D[i]$ with its power $P_{\text{D}} = \left|\mathbf{h}_{\text{RD}}^H\mathbf{p}\right|^2 P_{\text{R}} + \sigma_{\text{D}}^2$. By substituting (1) into P_{D}, we can further rewrite it as:

$$\begin{aligned} P_{\text{D}} &= \left|\mathbf{h}_{\text{RD}}^H\mathbf{p}\right|^2\beta^2\frac{P_{\text{S}}\left|\mathbf{d}^H\mathbf{h}_{\text{SR}}\right|^2 + \sigma_{\text{R}}^2}{1-\beta^2\left|\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}\right|^2} + \sigma_{\text{D}}^2 \\ &= \underbrace{P_{\text{S}}\beta^2\left|\mathbf{d}^H\mathbf{h}_{\text{SR}}\right|^2\left|\mathbf{h}_{\text{RD}}^H\mathbf{p}\right|^2}_{P_{\text{useful}}} + \underbrace{\beta^2\sigma_{\text{R}}^2\left|\mathbf{h}_{\text{RD}}^H\mathbf{p}\right|^2 + \sigma_{\text{D}}^2}_{P_{\text{noise}}} \\ &\quad + \underbrace{[\beta^2(P_{\text{S}}\left|\mathbf{d}^H\mathbf{h}_{\text{SR}}\right|^2 + \sigma_{\text{R}}^2)\left|\mathbf{h}_{\text{RD}}^H\mathbf{p}\right|^2]\frac{\beta^2\left|\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}\right|^2}{1-\beta^2\left|\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}\right|^2}}_{P_{\text{SI}}}, \end{aligned} \tag{2}$$

where P_{useful}, P_{SI} and P_{noise} denote the power of the useful signal, residual SI and noise, respectively.

Since the achievable rate $R(\mathbf{d}, \mathbf{p}) = \log_2\left[1 + \text{SINR}(\mathbf{d}, \mathbf{p})\right]$ is a monotone increasing function with respect to SINR$(\mathbf{d}, \mathbf{p})$, the achievable rate maximization problem can be equivalently described as:

$$(\text{P1}): \max_{\|\mathbf{d}\|=\|\mathbf{p}\|=1} \text{SINR}(\mathbf{d}, \mathbf{p}) = \frac{P_{\text{useful}}}{P_{\text{SI}} + P_{\text{noise}}}. \tag{3}$$

[3]The results of the proposed scheme will serve as useful theoretical bounds for practical issues.

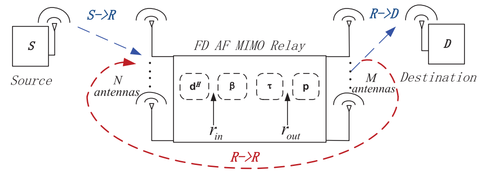

Fig. 1. Block diagram of the FD AF relay system.

It is worth noting that the problem P1 is a challenging convex problem because of the coupled variables and RSI part.

III. e2e Achievable Rate Maximization by Balancing Residual SI Suppression and Useful Signal Improvement

To tackle the complicated problem P1, a joint transceiver beamforming design based on iterative algorithm will be presented in this section. At each iteration, the optimal solution and the closed-form solution are provided for the nonconvex subproblems.

A. Receive Beamforming Vector $\mathbf{d}$ *Optimization With Given* $\mathbf{p}_g$

1) The optimal solution

The problem P1 with a given vector $\mathbf{p}_{\text{g}}$ is transformed into the problem P2.1 with respect to $\mathbf{D} \triangleq \mathbf{d}\mathbf{d}^H$ as follow:

$$\begin{aligned} (\text{P2.1}): &\max_{\mathbf{D}\succ 0\ \text{Trace}(\mathbf{D})=1} \text{SINR} = \frac{f(\mathbf{D})}{g(\mathbf{D})} \\ &\text{s.t. Rank}(\mathbf{D}) = 1, \end{aligned} \tag{4}$$

where $f(\mathbf{D}) = P_{\text{S}}\beta^2\text{Tr}(\mathbf{D}\mathbf{H}_S) - P_{\text{S}}\beta^4\text{Tr}(\mathbf{D}\mathbf{H}_S)\text{Tr}(\mathbf{D}\mathbf{T})$; $g(\mathbf{D}) = P_{\text{S}}\beta^4\text{Tr}(\mathbf{D}\mathbf{H}_S)\text{Tr}(\mathbf{D}\mathbf{T}) + \beta^4\text{Tr}(\mathbf{D}\mathbf{T})\sigma_{\text{R}}^2 + \left(\beta^2\sigma_{\text{R}}^2 + \frac{\sigma_{\text{D}}^2}{|\mathbf{h}_{\text{RD}}^H\mathbf{p}_{\text{g}}|^2}\right)\left(1-\beta^2\text{Tr}(\mathbf{D}\mathbf{T})\right)$, $\mathbf{H}_{\text{S}} \triangleq \mathbf{h}_{\text{SR}}\mathbf{h}_{\text{SR}}{}^H$ and $\mathbf{T} \triangleq (\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}})(\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}})^H$.

The problem P2.1 is difficult to solve due to the nonconvex constraint rank $(\mathbf{D})$=1. By dropping the rank-one constraint, we establish the following result as:

Proposition 1: The optimal solution $\mathbf{D}^*$ for the relaxed problem $\max_{\mathbf{D}\succ 0\ \text{Trace}(\mathbf{D})=1} \text{SINR} = \frac{f(\mathbf{D})}{g(\mathbf{D})}$ can be obtained by leveraging Dinkelbach algorithm.

Proof: Please refer to Appendix A. ■

Though $\mathbf{D}^*$ is obtained without the rank-one constraint, according to [12, Th. 2], it can be proved that the optimal $\mathbf{d}_{\text{opt}}$ for problem P1 with a given vector $\mathbf{p}_{\text{g}}$ can always be recovered from the optimal beamforming matrix solution $\mathbf{D}^*$ when $N > 2$.[4]

2) The closed-form solution

Through the analysis of the structure of P1, we observe that the optimization of $\mathbf{d}$ is a trade-off between maximizing the quality of S-R link and minimizing the quality of R-R link. Therefore, two normalized vectors $\mathbf{d}_{\text{sig}} = \frac{\mathbf{h}_{\text{SR}}}{\|\mathbf{h}_{\text{SR}}\|}$ and

[4]For the special case $N = 2$, the problem P1 can be derived equivalently by finding the normalized vector $\mathbf{d}_{\text{opt}} = [a_1 e^{j\varphi_1^*}, \sqrt{1-a_1^2}e^{j\varphi_2^*}]^H$ with the optimal lengths a_i ($a_i \in [0, 1]$) and angels φ_i ($\varphi_i \in [0, 2\pi)$) where $i = 1, 2$. Then $\mathbf{d}_{\text{opt}}$ can be obtained through searching for the stationary points without using the result of proposition 1. The same applies to the special case $N = 1$.

$\mathbf{d}_{\text{sup}} = \frac{\mathbf{P}^{\perp}\mathbf{v}}{\|\mathbf{P}^{\perp}\mathbf{v}\|}$ are designed for the useful signal improvement and the RSI suppression, respectively. The vector $\mathbf{v}$ denotes $\frac{\mathbf{P}^{\perp}\mathbf{h}_{\text{SR}}}{\|\mathbf{P}^{\perp}\mathbf{h}_{\text{SR}}\|}$ and $\mathbf{P}^{\perp} = \mathbf{I} - \frac{\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}}(\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}})^H}{\|\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}}\|^2}$ denotes the projection matrix onto the null space of $\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}}$. Then, $\mathbf{d}$ can described as a function $f(\eta)$ [13]:

$$\mathbf{d} = f(\eta) = \frac{\eta\mathbf{d}_{\text{sig}} + (1-\eta)\mathbf{d}_{\text{sup}}}{\|\eta\mathbf{d}_{\text{sig}} + (1-\eta)\mathbf{d}_{\text{sup}}\|}. \tag{5}$$

By substituting the equation (5) into problem P1, we can obtain an approximate problem P2.2:

$$\text{(P2.2)}: \max_{0\le\eta\le1} \text{SINR}(\eta) = \frac{P_{\text{S}}\beta^2 b\left(1-\beta^2 a\right)}{P_{\text{S}}\beta^4 ba + \beta^4\sigma_{\text{R}}^2 a + \left(1-\beta^2 a\right)\left(\frac{\sigma_{\text{D}}^2}{|\mathbf{h}_{\text{RD}}^H\mathbf{p}_{\text{g}}|^2} + \beta^2\sigma_{\text{R}}^2\right)}, \tag{6}$$

where $a = \eta^2\left|(\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}})^H\mathbf{h}_{\text{SR}}\right|^2$ and $b = \left[(1-\eta)\left|\mathbf{d}_{\text{sup}}{}^H\mathbf{h}_{\text{SR}}\right| + \eta\right]^2$ are respectively defined as the approximation of $\left|\mathbf{d}^H\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}}\right|^2$ and $\left|\mathbf{d}^H\mathbf{h}_{\text{SR}}\right|^2$ for the sake of obtaining the closed-form solution.

The problem P2.2 is equivalent to minimize $\text{SINR}(\eta)^{-1}$ with respect to η $(0 \le \eta \le 1)$. Meanwhile, $\text{SINR}(\eta)^{-1}$ is a convex function with respect to η $(\eta \ge 0)$ with proven $\frac{d^2\text{SINR}(\eta)^{-1}}{d\eta^2} \ge 0$. Therefore, the optimal η^* can be obtained by solving the equation $\frac{d\text{SINR}(\eta)^{-1}}{d\eta} = 0$. This equation can be reformulated as a cubic equation:

$$a_0\eta^3 + a_1\eta^2 + a_2\eta + a_3 = 0, \tag{7}$$

where $a_0 = 3P_s\beta^4xy(y-1)^2$, $a_1 = -\beta^2x(y-1)\left(3P_{\text{s}}\beta^2y^2 + 2\beta^2\sigma_{\text{R}}^2 + \frac{2\sigma_{\text{D}}^2}{|\mathbf{h}_{\text{RD}}^H\mathbf{p}_{\text{g}}|^2}\right)$ $a_2 = \beta^4xy\left(P_sy^2 + \sigma_{\text{R}}^2\right)$, $a_3 = \left(\beta^2\sigma_{\text{R}}^2 + \frac{\sigma_{\text{D}}^2}{|\mathbf{h}_{\text{RD}}^H\mathbf{p}_{\text{g}}|^2}\right)(y-1)$, $x = \left|(\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}})^H\mathbf{h}_{\text{SR}}\right|^2$ and $y = \left|\mathbf{d}_{\text{sup}}{}^H\mathbf{h}_{\text{SR}}\right|$.

According to *Vietas Theorem*[14], the equation must have only one real nonnegative root $\eta_{\text{real}} = e - \frac{a_2}{3a_3} - \frac{\frac{a_1}{3a_3} - \frac{a_2^2}{9a_3^2}}{e}$ with

$$e = \left(\left(\left(\frac{a_0}{2a_3} + \frac{a_2^3}{27a_3^3} - \frac{a_1a_2}{6a_3^2}\right)^2 + \left(\frac{a_1}{3a_3} - \frac{a_2^2}{9a_3^2}\right)^3\right)^{\frac{1}{2}} - \frac{a_2^3}{27a_3^3} - \frac{a_0}{2a_3} + \frac{a_1a_2}{6a_3^2}\right)^{\frac{1}{3}}.$$

Finally, the closed-form solution $\mathbf{d}_{\text{closed}}$ for problem P1 with a given vector $\mathbf{p}_{\text{g}}$ can be derived by substituting $\eta^* = \min\{\eta_{\text{real}}, 1\}$ into (5).

B. Transmit Beamforming Vector $\mathbf{p}$ Optimization With Given $\mathbf{d}_g$

The solution for the optimal transmit beamforming vector $\mathbf{p}_{\text{opt}}$ or $\mathbf{p}_{\text{closed}}$ can be obtained by the similar process of $\mathbf{d}_{\text{opt}}$ or $\mathbf{d}_{\text{closed}}$, respectively.

Algorithm 1 Joint Design of $\mathbf{d}$ and $\mathbf{p}$

Initialize: $\Delta = 1$, $\varepsilon_{sys} = 0.001$, $i = 0$, $\mathbf{d}_0$, SINR_0;
While not convergence do
  $\mathbf{p}_i = \mathbf{p}_{\text{opt}}$ or $\mathbf{p}_{\text{closed}}$ with given $\mathbf{d}_{\text{g}} = \mathbf{d}_i$;
  $\text{SINR}_p = \text{SINR}(\mathbf{d}_{\text{g}}, \mathbf{p}_i)$;
  $\mathbf{d}_{i+1} = \mathbf{d}_{\text{opt}}$ or $\mathbf{d}_{\text{closed}}$ with given $\mathbf{p}_{\text{g}} = \mathbf{p}_i$;
  $\text{SINR}_d = \text{SINR}(\mathbf{d}_{i+1}, \mathbf{p}_{\text{g}})$;
  $\text{SINR}_{i+1} = \max(\text{SINR}_p, \text{SINR}_d)$;
  If $\text{SINR}_p > \text{SINR}_d$;
    $\mathbf{d}_{i+1} = \mathbf{d}_i$;
  End
  $\Delta = |\text{SINR}_{i+1} - \text{SINR}_i|$, $i = i + 1$;
End while
Output: $\mathbf{d}^* = \mathbf{d}_i$, $\mathbf{p}^* = \mathbf{p}_i$, $\text{SINR}^* = \text{SINR}(\mathbf{d}^*, \mathbf{p}^*)$.

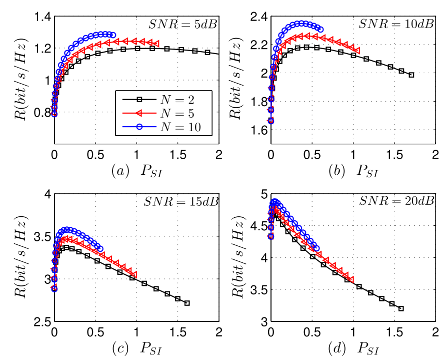

Fig. 2. $\mathbf{P}_{\text{SI}}$ vs. R with different antenna configurations based on different SNRs at S station with given $\mathbf{p}_{\text{g}} = \frac{\mathbf{h}_{\text{RD}}}{\|\mathbf{h}_{\text{RD}}\|}$.

C. The Joint Design of $\mathbf{d}$ and $\mathbf{p}$

An iterative algorithm is adopted as the joint transceiver beamforming design. The details of the joint design are summarized in Algorithm 1. The original vector $\mathbf{d}_0$ is defined as $\frac{\mathbf{h}_{SR}}{\|\mathbf{h}_{SR}\|}$ based on the maximum ratio combining. The convergence condition of Algorithm 1 is $\Delta \le \varepsilon_{sys}$, where Δ denotes $|\text{SINR}_{i+1} - \text{SINR}_i|$ and ε_{sys} denotes the accuracy of Algorithm 1. The i-th iteration result is denoted by SINR_i.

The problem P1 has an upper bound when there is no loopback interference, e.g., the ideal scenario $\mathbf{H}_{\text{RR}} = \mathbf{0}$. Moreover, $\text{SINR}_{i+1} \ge \text{SINR}_i$ due to the fact that the vector $\mathbf{d} = \text{argmax}(\text{SINR}_p, \text{SINR}_d)$ is always chosen as the original vector for the next iteration. Based on the above two aspects, we can guarantee the convergence of Algorithm 1.

IV. Simulation Results

In this section, the numerical results are provided to illustrate the effects of useful signal improvement and SI suppression on the achievable rate and show the advantages of the proposed design. We set $P_{\text{s}} = 1$, $\sigma_{\text{R}}^2 = \sigma_{\text{D}}^2 = 0.1$ and $M = 2$. We set $\beta = 1$, $\rho = 0.5$ and normalize $\mathbf{H}_{\text{loop}}$ to guarantee the condition in Sec.II, which can be relaxed as $\beta^2 < \frac{1}{\rho\|\mathbf{H}_{\text{loop}}\|_F^2}$ according to Cauchy-Schwarz inequality. The achievable rate results based on each static channel realization are averaged over 1000 independent realizations.

Fig. 2 shows $\mathbf{P}_{\text{SI}}$ versus achievable rate R with different antenna configurations. It illustrates the optimal $\mathbf{P}_{\text{SI}}$ for

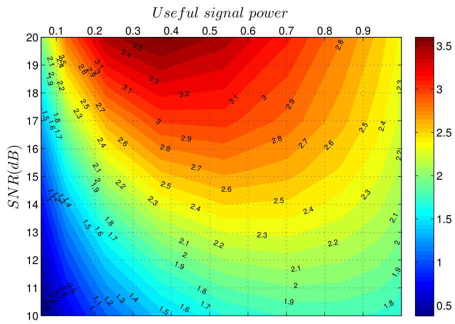

Fig. 3. Contour plot illustrate the effect of P_{useful} on R based on different SNRs with given $\mathbf{p}_{\text{g}} = \frac{\mathbf{h}_{\text{RD}}}{\|\mathbf{h}_{\text{RD}}\|}$. $P_{\text{S}} = 1$. $N = 2$.

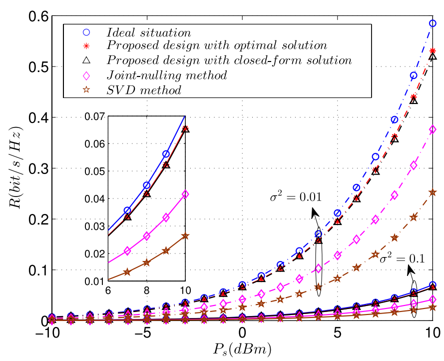

Fig. 4. The achievable rate over different schemes versus with different P_{S}. The solid lines denote $\sigma_{\text{R}}^2 = \sigma_{\text{D}}^2 = 0.1$, the dashed lines denote $\sigma_{\text{R}}^2 = \sigma_{\text{D}}^2 = 0.01$. $N = 3$.

maximizing the achievable rate, with different antenna configurations, is increasing with the decrease of the SNR value. It means that nulling out SI will hurts the achievable rate performance especially in low SNR region. SNR denotes the maximum power of the received useful signal-noise-ratio (SNR) at relay, which represents the channel situation for the useful signal from S to R.

Fig. 3 illustrates the effect of the useful signal improvement on the achievable rate based on different SNRs. The contour plot shows that the P_{useful} for maximal achievable rate is increasing with the decrease of SNR. It implies that a strengthened useful signal is more effective to enhance the achievable rate of the system especially in low SNR regime.

Fig. 4 compares the achievable rates achieved by the proposed design and two typical schemes: joint-nulling method [4] and SVD method. No power control at R is adopted in every scheme. To ensure the fairness of comparison, the maximum achievable rate of every scheme is obtained based on the same transmit power of the relay. The figure shows that the proposed design outperforms the ZF-based schemes and the superiority is more significant in low SNR region. Meanwhile, the effectiveness of the closed-form solution can also be appreciated from Fig. 4 which shows that the closed-form solution can achieve almost the same performance of the optimal solution.

V. Conclusion

In this letter, a joint transceiver beamforming design is proposed for FD AF system with RSI, which enhances the e2e performance by striking the balance between the useful signal improvement and the SI suppression in various channel environments. Simulation results illustrate the improvement of the proposed design in terms of achievable rate.

Appendix A
Proof of Proposition 1

The key points of the proof are the verifications of the convexity of $f(\mathbf{D})$ and $g(\mathbf{D})$. They decide whether or not $\frac{f(\mathbf{D})}{g(\mathbf{D})}$ follows the form of fractional program.

The convexity of $f(\mathbf{D})$ and $g(\mathbf{D})$ are both depend on the property of $h(\mathbf{D}) \triangleq \mathrm{Tr}(\mathbf{D}\mathbf{H}_S)\mathrm{Tr}(\mathbf{D}\mathbf{T})$. The second derivative of $h(\mathbf{D})$ is a rank-one matrix $2\mathbf{H}_S{}^T\mathbf{T}^T$ with $\mathrm{Tr}\left(\mathbf{H}_S{}^T\mathbf{T}^T\right) = \left|(\mathbf{H}_{\text{RR}}\mathbf{p}_{\text{g}})^H\mathbf{h}_{\text{SR}}\right|^2 \geq 0$. Hence, we can prove that $h(\mathbf{D})$ is convex because $\mathbf{H}_S{}^T\mathbf{T}^T$ is a positive semidefinite matrix. Therefore, we can draw the conclusion that the functions $f(\mathbf{D})$ and $g(\mathbf{D})$ are concave and convex, respectively. It means that the relaxed version of problem P2.1 is a fractional program problem. Meanwhile, the constraint $\mathrm{Tr}(\mathbf{D}) = 1$ for the problem is convex. Therefore, $\mathbf{D}^*$ for fractional programming problem can be obtained by leveraging Dinkelbach algorithm with a super-linear convergence rate [15]. ■

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