Theory of discharges at the foil edge in capacitors


Synopsis

There is a high field concentration at the electrode edge in rolled-paper or film capacitors, and if the edge is squarely cut, the field at the corner tends to infinity. Breakdown in the dielectric fluid surrounding the edge constitutes a partial discharge, but a breakdown field, and hence the discharge-inception voltage, cannot be simply derived with an infinite field at the corner and a highly divergent field. It is argued that a discharge occurs when the voltage gradient along a line of maximum field exceeds a certain critical value over a certain length. The applied voltage necessary to reach this condition is calculated from an approximate analytical treatment of the potential configuration. The resulting formula is verified as regards the dependence of discharge-inception voltage on the dielectric and electrode thickness, using castor-oil-impregnated paper capacitors. Capacitors constructed with longitudinally folded foil electrodes show a markedly increased inception voltage. An analysis of the field for a rounded-edge profile is carried out for the first time. The experimental results for the two types of edge profile enable the disposable constants in the theory to be evaluated.

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\lambda$</td>
<td>characteristic length along a field line</td>
</tr>
<tr>
<td>$\Delta V'$</td>
<td>voltage drop over $\lambda$, starting from electrode</td>
</tr>
<tr>
<td>$t_e$</td>
<td>thickness of solid insulation between electrodes</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>relative permittivity of solid insulation</td>
</tr>
<tr>
<td>$t_f$</td>
<td>thickness of fluid insulation between electrode and solid insulation</td>
</tr>
<tr>
<td>$\epsilon_f$</td>
<td>relative permittivity of fluid insulation</td>
</tr>
<tr>
<td>$t'_e$</td>
<td>equivalent thickness of solid insulation</td>
</tr>
<tr>
<td>$a$</td>
<td>half thickness of metal electrode</td>
</tr>
<tr>
<td>$b$</td>
<td>distance from centre of one electrode to surface of other electrode</td>
</tr>
<tr>
<td>$\Delta V_c$</td>
<td>critical voltage drop over $\lambda$ at which a detectable discharge occurs</td>
</tr>
<tr>
<td>$V_i$</td>
<td>discharge-inception voltage</td>
</tr>
<tr>
<td>$X$, $Y$</td>
<td>factors depending on $a$ and $b$ only, defined in Section 3.1</td>
</tr>
<tr>
<td>$z_{\text{plane}}$</td>
<td>normal section through dielectric system before transformation</td>
</tr>
<tr>
<td>$w_{\text{plane}}$</td>
<td>section after one transformation</td>
</tr>
<tr>
<td>$r$, $w_1$, $w_2$</td>
<td>plane sections after various transformations</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>potential</td>
</tr>
<tr>
<td>$\psi$</td>
<td>stream function (field)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>geometrical constant</td>
</tr>
<tr>
<td>$\theta$</td>
<td>parameter describing rounded profile of the electrode in case 2</td>
</tr>
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</table>

1 Introduction

Partial discharges in composite dielectrics, sometimes referred to as internal ionisation or corona, are an important limitation to the working stress that can be applied to insulation. It is well established that liquid-impregnated dielectrics operating at 50 Hz will fail in a relatively short time, of the order of hours or days, if partial discharges are continually present.

The most highly stressed dielectrics are probably to be found in capacitors. Furthermore, in capacitors, the edge of the electrodes causes a field concentration, where discharges are likely to start in the event of a surge or overvoltage. The capacitor designer must therefore know the discharge-inception level accurately, in order to allow a suitable safety factor.

The measurement of discharge-inception voltage in capacitors has been treated in many publications, but few of these discuss the mechanism of discharge inception. In this paper, an attempt is made to relate the onset of discharges to the field configuration at the electrode edge. The treatment differs from previous work in taking account of the highly divergent nature of the field.

No attempt is made to discuss the sequence of discharge pulses observed when a sinusoidal stress is applied to insulation with voids in it. There are well established explanations for these phenomena. It is contended that an initially void-free capacitor dielectric is commonly achieved, and that discharges are started by breakdown of the liquid at the electrode edge. This breakdown may later produce bubbles within which further discharges occur.

2 Theory

2.1 Existing theories

Nye and Wilson\cite{2} report experimental work on discharge-inception measurements with various voltage waveforms, and interpret the results in terms of a resistive extension of the electrode by the oil film at the electrode edge. The persistence and decay of space charges in this oil gap following an initial partial discharge account well for the observed sequences of discharge pulses, but no attempt is made to account for the dependence of the discharge-inception stress on the dielectric thickness. The model does not, in fact, predict any criterion for the initial discharges.

Sträb and Maylandt\cite{3} calculate the field in a uniform oil film between electrode and solid dielectric. If the permittivities of oil and solid (which was impregnated paper) differ, the field in the oil varies as the ratio oil-thickness/solid-dielectric-thickness varies. However, this mechanism cannot account for results obtained on systems where the permittivities of solid and liquid are nearly the same, such as those quoted by Krasucki\cite{4} and in this paper.

Heywang and Preissinger\cite{5} consider capacitors having thin-film electrodes. They postulate a radius of curvature of 25 Å at the edge of the electrode and evaluate the electric field at this edge. For a given uniform field in the plane-parallel region, the edge field varies as the square root of the dielectric thickness, a result in very good agreement with their experiments on nonimpregnated metalised-plastic-film capacitors.

Krasucki's\cite{4} deals with oil-impregnated capacitors having normal solid-foil electrodes. He postulates a square-cut edge, and, after presenting an argument which shows that partial discharges do indeed originate at the edge, he calculates the field at the centre of the edge. Like Heywang and Preissinger, he finds a square-root relationship which fits the experimental results well.

2.2 Statement of new theory

It appears to be necessary to consider the physical concepts more deeply than has been done in References 4 and 5. The maximum field at the edge of the thin-film electrode is derived from Heywang and Preissinger's formula, and their experimental results, is 13 MV/cm. Since the edge is in air, partial breakdown would be expected at far lower

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fields. The explanation is that the development of gaseous discharges involves not merely the field at the electrode, but also the length of path available for ionisation, and this is very short in the present case. It is possible to arrive at a square-root relationship between discharge-inception voltage and thickness of dielectric directly from the Paschen-law curve; such a treatment would be more appropriate. Krasucki calculates the field at the centre of the short face of the squarely cut foil, but largely ignores the fact that the field tends to infinity at the corners of the face. His experimental results lead to 3.72 MV/cm as the critical field for the initiation of discharges, which is rather high for an oil.

The present authors consider it to be essential to take into account the fact that the field at the electrode edge is highly divergent and that a particle, though starting in a very large field, only gains quite a small amount of energy over a finite length of path. It is now postulated that a partial discharge will occur if a charge carrier gains a certain minimum energy over a distance \( A \). This distance will be some multiple of the mean free path of the particle, such that avalanche multiplication is ensured. Therefore, not the maximum field at the edge, but the voltage drop \( \Delta V \) along a distance \( \lambda \) of the line of maximum field, is calculated. Although the term 'avalanche multiplication' has been used to justify the introduction of the distance \( \lambda \), it will be argued in Section 4 that this does not link the present calculations to any particular theory of breakdown in liquids.

Primarily, a slit metal-foil electrode is considered, having an idealised square edge, as assumed by Krasucki (Fig. 1).

![Fig. 1](image1)

Section through electrode system

In practice, the aluminium-foil electrodes do not possess a true, square-cut edge. The foil-slitting operation stretches and thins the foil a little before breaking it, and the edge is of irregular shape, although there are parts having a square contour.

A second calculation is carried out on a rounded profile (Fig. 2), because there are experimental results available for capacitors made with longitudinally folded foil electrodes.\(^5\)

![Fig. 2](image2)

High-potential electrode with rounded edge

2.3 Calculations

Consider a capacitor of wound extended-foil construction, usually with several layers of solid (or semisolid, e.g. paper) insulation between the electrodes, the whole being impregnated with some fluid in which discharges may take place. The practical examples will be concerned with liquids whose permittivity is not very different from that of the solid, and it is assumed that, to a first approximation, the solid of thickness \( t_s \) and relative permittivity \( \epsilon_r \) can be replaced by a layer of dielectric of thickness \( t'_s = t_s \epsilon_r / \epsilon_s \) and relative permittivity \( \epsilon_r \) equal to that of the fluid. Thus, as regards electrostatic fields, the boundary has been eliminated.

The potential distribution may be calculated by applying standard methods of the theory of complex variables; details of this procedure are given in Appendix 8.1.

2.3.1 Case 1: Square-edged electrodes

The calculations of Appendix 8.1.1 yield the result that discharges will be produced, provided that the voltage applied to the capacitor exceeds \( V_i \), given by

\[
V_i = \left( \frac{2}{3} \right)^{2/3} \pi^{1/3} \Delta V \left( \frac{b}{\lambda} \right)^{2/3} \left( 2a^2 - a^2 - \frac{b^2}{\lambda^2} \right) \left( 1 - \frac{a}{b} \right)^{2/3}.
\]

(1)

where \( \Delta V_c \) is the critical potential drop, \( \lambda \) is the distance over which this drop is to be calculated and \( a \) and \( b \) are the dimensions of the capacitor (Fig. 1). In the limit \( a < b \), \( V_i \propto b \).

2.3.2 Case 2: Rounded electrodes

In this case (Appendix 8.1.2), \( V_i \), similarly defined, is given by

\[
V_i = 2^{1/2} \Delta V \left( \frac{b}{\lambda} \right)^{1/2} \left( 2a^2 - a^2 - \frac{b^2}{\lambda^2} \right)^{3/4} \left( 1 + \frac{2a}{b} \right)^{-1}.
\]

(2)

when \( \frac{a}{b} > 0.0898 \)

or \( V_i = 2^{-1/2} \Delta V \left( \frac{b}{\lambda} \right)^{1/2} \left( 2a^2 - a^2 \right)^{1/2} \)

(3)

when \( \frac{a}{b} < 0.0898 \)

The dimensions \( a \) and \( b \) are shown in Fig. 2. In the limit \( a < b \), \( V_i \propto b^{1/2} \), as before.

2.3.3 Case 3: Thin fluid film

The calculations in Appendix 8.1 are based on the assumption that a segment of length \( \lambda \) of the critical-field line is available for the acceleration of charge carriers. This assumption is not always tenable in capacitors, since the surface of the solid insulation acts as a barrier to the motion of charged particles. Fig. 3 shows the situation near one corner of a square-edged electrode; \( t_f \) is the thickness of the fluid layer over the face of the electrode.

![Fig. 3](image3)

Sketch of equipotentials and field lines to illustrate effect of very thin liquid film (thickness \( t_f \))
A study of the arrangement of the equipotentials and field lines, as sketched in Fig. 3, shows that, for \( r_f < \lambda \), the field line originating at the corner of the electrode will intersect the boundary before an arc length \( \lambda \) has been traversed. It is therefore necessary to select a field line originating in a lower field region, but along which an arc length \( \lambda \) can be traversed before the boundary is encountered. The potential drop over \( \lambda \) along such a field line will be less than that along the line originating from the corner, and, therefore, to accelerate a particle to \( \Delta V_e \), a higher voltage has to be applied to the electrodes. Thus, the presence of a solid dielectric close to the electrodes raises the discharge-inception voltage.

A similar effect occurs for electrodes with rounded profiles, except for very thin electrodes \((a/b \ll 0.0089)\), in which case the virtual field line is invariably at the tip of the electrode.

3.1 Geometrical factors

To facilitate calculations based on the experimental results, factors \( X(a, b) \) and \( Y(a, b) \) are defined to include all the numerical and geometrical factors in eqns. 1 and 2, respectively; \( a/b \) was never low enough in the specimens described later for eqn. 3 to apply:

\[
X = \pi^{-1/3} \left[ \frac{2^{1/3}}{3} \right] (2a)\lambda^{-1/3} b^{-2/3} \left( 1 - \frac{a}{2b} \right)^{-1/6} \left( 1 - \frac{a}{b} \right)^{-2/3} \\
Y = 2^{-1/2} (2a)^{-1/2} b^{-1/4} \left( 1 - \frac{a}{2b} \right)^{-1/4} \left( 1 - \frac{a}{b} \right)^{-1} \left( 1 + \frac{2a}{b} \right)^{-1/2} \left( \frac{a}{b} \right)^{-1/2} 
\]

so that

\[
\Delta V_e \lambda^{2/3} = XV_f (\text{square}) \\
\Delta V_e \lambda = YV_f (\text{round})
\]

In the calculations, \( b - a = t_2 + t_e = t_f + t_s / \epsilon_s \), where \( t_s \) and \( \epsilon_s \) are the thickness and relative permittivity of the solid and \( t_f \) and \( \epsilon_r \) are the thickness and relative permittivity of the liquid; in practice, \( t_s \) is difficult to measure, but it is small enough to be neglected in comparison with \( t_f \).

3.2 Effect of thickness

The discharge-inception voltages were measured on a large number of capacitors \((0.1 \mu F)\) with castor-oil impregnant and high-density paper. Various thicknesses of paper and electrode were used; the electrode edges were not folded. Taking values of 4.65 and 5.6 for \( \epsilon_y \) and \( \epsilon_m \), respectively, values of \( a, b \) and \( \lambda \) may be calculated. Table 1 lists the constructional details of the various groups, the inception voltages [mostly for regular, nongrowing discharges, designated type I+II (Appendix 8.2)] and the values of \( \Delta V_e \lambda^{2/3} \) calculated from these voltages. Standard errors are quoted for the means in the last two columns.

The agreement between the first four values of \( \Delta V_e \lambda^{2/3} \) is striking. These values may be compared with Krasucki’s prediction, which, in the present notation, is that \( V_f / \lambda - 1/2(2b - a)^{-1/2} \) is constant. Multiplying the values of this quantity by \( \sqrt{3} \), to bring them into approximate equality with the values of \( \Delta V_e \lambda^{2/3} \), one obtains, respectively, 202.6 ± 5.2, 209.2 ± 3.0, 210.1 ± 2.5, 159.1 ± 2.3 and 219.1 ± 5.1. The variation is rather greater than for \( \Delta V_e \lambda^{2/3} \), and the fourth value (for 12 \( \mu \)m foil) is extremely significant. Krasucki’s theory predicts a much greater variation with electrode thickness than is actually observed in practical capacitors. Held and Kunze also report only a very minor variation of discharge-inception voltage with electrode thickness over the range 6·16 \( \mu \)m.

The value of \( \Delta V_e \lambda^{2/3} \) for tin electrodes is higher than for aluminium, but this difference is not statistically significant. The discharges with these electrodes were type II (growing discharges), which could explain the higher values. There is also the possibility that the shape of the electrode edge differed, since the metals differ in hardness.

Note that, if \( V_f \) is plotted against paper thickness on double logarithmic scales, a reasonably straight line of slope \( 0.56 \pm 0.05 \) is obtained. This compares with an index of 0.58 quoted by Kutchinsky et al.

3.3 Effect of rounded edges

3.3.1 Pentachlor-diphenyl-impregnated specimens

Measurements of discharge-inception voltage were made at intervals during accelerated aging tests on capacitors (about 0.003\( \mu F \)) using this impregnant. These small capacitors were constructed some years ago and certainly do not represent present-day technology. The uniformity of winding, in particular, left much to be desired, and there were wide capacitance variations between samples in the same group, despite the electrode areas being equal. Since the number of samples in each group was quite high, however, the average discharge-inception voltages should be representative of the construction, even though the imperfect techniques introduce a high scatter.

Various thicknesses of dielectric were used. The electrodes were 6 \( \mu \)m aluminium foil, with edges either folded over twice, as shown in Fig. 4, or not folded at all. Taking values of 5·0 and 5·7 for \( \epsilon_y \) and \( \epsilon_m \), respectively, values of \( a, b \) and \( \lambda \) may be calculated for each geometry; these are listed in Table 2.

Table 3 gives the measured inception voltages and the derived values \( XV_f \) and \( YV_f \) for the two types of capacitor. From these \( \lambda \) and \( \Delta V_e \) have been calculated. The inequality signs in the Table are due to deliberate restriction of the test voltage to avoid breakdown of the samples. The scatter in the \( V_f \) measurements is indicated by the standard errors of the mean shown in the Table. The corresponding standard errors in the derived quantities are also listed. There will, however, be further errors in the geometrical factors, because

Table 1

<table>
<thead>
<tr>
<th>Group</th>
<th>Foil</th>
<th>Thickness</th>
<th>Paper thickness</th>
<th>( a )</th>
<th>( b )</th>
<th>( X )</th>
<th>Number</th>
<th>( V )</th>
<th>m.s.</th>
</tr>
</thead>
</table>
| A     | Al   | 6         | 4 x 10         | 3    | 21   | 0·117| 5      | 1690 ± 43
| B     | Al   | 6         | 4 x 15         | 3    | 52   | 0·9548| 60     | 2120 ± 30
| C     | Al   | 6         | 4 x 25         | 3    | 86   | 0·730 | 28     | 2730 ± 32
| D     | Al   | 12        | 4 x 15         | 6    | 55   | 0·8616| 6      | 2310 ± 33
| E     | Sn   | 6         | 4 x 15         | 3    | 52   | 0·9548| 2      | 2220 ± 52

\( X \) and \( Y \) are defined to include all the numerical and geometrical factors in eqns. 1 and 2, respectively; \( a/b \) was never low enough in the specimens described later for eqn. 3 to apply:

\[
X = \pi^{-1/3} \left[ \frac{2^{1/3}}{3} \right] (2a)\lambda^{-1/3} b^{-2/3} \left( 1 - \frac{a}{2b} \right)^{-1/6} \left( 1 - \frac{a}{b} \right)^{-2/3} \\
Y = 2^{-1/2} (2a)^{-1/2} b^{-1/4} \left( 1 - \frac{a}{2b} \right)^{-1/4} \left( 1 - \frac{a}{b} \right)^{-1} \left( 1 + \frac{2a}{b} \right)^{-1/2} \left( \frac{a}{b} \right)^{-1/2} 
\]
the dielectric thickness was not accurately known and because the electrode profile assumed in deriving \( Y \) is unlikely to be an accurate representation of the true profile. Any errors due to the slow solution of residual gas bubbles in the liquid.

Errors cancel. The increase in inception voltage over this time is approximately the same for all groups of samples. In such comparisons, the geometrical mean of the standard error is used. The magnitudes of \( A \) and \( \Delta V_c \) will be discussed in Section 4.

Note: All inception voltages are for random pulses (type I) except where starred, in which cases at least one capacitor showed regular groups of discharges (type I-II).

Table 2

<table>
<thead>
<tr>
<th>CAPACITOR GEOMETRIES—PENTACHLOR-DIPHENYL-IMPREGNATED SAMPLES</th>
</tr>
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<tbody>
<tr>
<td>Group</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
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</tbody>
</table>

![Fig. 4](image)

Section through twice-folded electrode

Table 3

<table>
<thead>
<tr>
<th>RESULTS FOR CAPACITORS IN TABLE 2</th>
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<tbody>
<tr>
<td>Group</td>
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<tr>
<td>-------</td>
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<td>A</td>
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</table>

Note: All inception voltages are for random pulses (type I) except where starred, in which cases at least one capacitor showed regular groups of discharges (type I-II).

It will be seen that \( V_i \) rises with time under stress, possibly due to the slow solution of residual gas bubbles in the liquid. Less weight should therefore be attached to the earlier readings. The values of \( XV_i \) and of \( YV_i \), should remain constant for different thicknesses of the dielectric, at a given time of aging, and this is roughly true. The agreement is not as good as for the castor-oil impregnated samples described. The magnitudes of \( \lambda \) and \( \Delta V_c \) will be discussed in Section 4.

It is interesting to note that \( \Delta V_i \) is unchanged (to a small fraction of the standard error) between the second and the third sets of measurements (about 4000 and 7000h) within each group of samples. In such comparisons, the geometrical errors cancel. The increase in inception voltage over this time interval therefore seems to be due to a decrease in \( Y \), while the energy requirements remain constant.

3.3.2 Castor-oil specimens

Some measurements were also made on groups of capacitors (about 0-1 \( \mu F \)) impregnated (all at the same time) with castor oil. Once more, high-density paper and 6 \( \mu m \) aluminium extended foil electrodes were used; the paper was emulsion in contact with these electrodes were stressed until internal discharges were detected electrically. After dis-
mantling the capacitors and developing the emulsions, the raw edges were clearly recognisable by the point-like exposures, while the outlines of the rounded edges were hardly visible.

The expression for the discharge-inception voltage derived in the present treatment resembles previous theories in predicting an approximate square-root relation between \( V_d \) and dielectric thickness. This is not surprising, since, in spite of the refinements introduced, the field at a sharp edge is involved as a basic feature. However, as seen in Section 3.2, the present treatment leads to a distinctive dependence of \( V_d \) on electrode thickness, which is borne out by experiment.

The concept of a unique discharge-inception voltage must be modified by the experiment described in Section 3.4. The finding that the initiating voltage depends on the magnitude of the discharges being detected is by no means new. Over the range of magnitudes involved, it appears that the inception voltage is a continuous and reproducible function of discharge magnitude. This is characteristic of type I–II discharges (Appendix 8.2), with which the theory of this paper is primarily concerned.

The results obtained for groups of capacitors for which the detection sensitivity was constant support the hypothesis that, when the potential drop over a certain length \( \lambda \) of a line of maximum field reaches a given value, an ‘event’ occurs which is detected as a discharge.

This event may be thought of as a prebreakdown current pulse, or as a partial breakdown. There are various theories of breakdown in use, mostly formulated to explain the phenomena in a uniform-field gap between metal electrodes. Naturally, any detailed application to the present case, where the field is highly divergent and the path between the two electrodes is interrupted by solid insulation, is ruled out. A short discussion of the relation between the discharge criterion postulated and certain possible breakdown mechanisms is, however, in order.

A simple collision-ionisation process, resulting in an electron avalanche, is the most suitable mechanism. A simple theory, not going much deeper than the present calculations, may well be adequate, as the field is so nonuniform that the geometrical factors outweigh the statistical ones. For instance, there is no necessity to take the average potential drop over all possible paths for the charge carriers. The ionisation coefficient is known to be low for liquids, and so a large number of collisions are required before a critical avalanche size is reached. This is consistent with the rather large value of \( \lambda \) suggested by the present experimental results.

Another class of breakdown mechanism is based on the emergence of a bubble of gas or vapour. This involves the formation of a bubble and its growth. While the formation could be entirely dependent on the field at the electrode surface,\(^1\) the growth must depend on the field in the liquid. In a highly divergent field, the growth of a bubble could cease before it was large enough to support a discharge, and so, once again, it appears necessary to postulate a minimum energy gained over a certain distance. Kao\(^1\) states that the condition is that the voltage across the bubble should exceed the minimum on the Paschen curve and, obviously, that the product \( pd \) (pressure \( \times \) gap length) must also exceed a certain value. These conditions fit the approximate \( \Delta V \), values of 300V rather well, but details of a criterion for the growth of a bubble are lacking.

Note that the average field over \( \lambda \), i.e. \( \Delta V/\lambda \), calculated from the results is about 2 MV/cm (peak), which is a reasonable value.

If accurate values of \( \lambda \) and \( \Delta V \), were available, insight would be gained. It is possible that, if measurements were made on a point-plane gap (the point being considered as a hyperboloid of revolution) immersed in the impregnant, values of \( \lambda \) and \( \Delta V \), could be obtained, by comparison with results on capacitors with square-edged electrodes, which would be more accurate than those in Tables 2, 3 and 4.

5 Conclusions

The formula derived here for the discharge-inception voltage in terms of a minimum energy over a certain distance is in good agreement with measured values, as regards variance with dielectric and electrode thickness. In a separate experiment, the marked effect of having a rounded electrode-edge contour was demonstrated. This experiment should enable a quantitative estimate of the disposable parameters in the formula to be made, but the scatter in the experimental results, as well as the mathematical approximations made, only allow an order of magnitude for these quantities to be determined.

6 Acknowledgments

The authors wish to thank A. L. Williams, Director of Research & Engineering, BICC Ltd., for permission to publish this paper and gratefully acknowledge a useful discussion with Z. Krasucki on the subject matter.

7 References

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6. UK Patent 911777

8 Appendixes

8.1 Calculation of potential distributions

8.1.1 Case 1: Square-edged electrode

A Schwarz–Christoffel transformation is used, in order to map the model capacitor into an arrangement of electrodes for which the potential distribution is known. The model is shown (\( z \) plane) on the left of Fig. 5; the transformed problem is shown on the right of the Figure (\( w \) plane).

The Schwarz-Christoffel formula for the requisite mapping \( z = f(w) \) is

\[
\frac{dz}{dw} = C \left( \frac{w^2 - \alpha^2}{w^2 - 1} \right)^{1/2}
\]

It is found that \( C = 2ib/\pi \) and that \( 1 - \alpha^2 = (1 - a/b)^2 \) and, on integration,

\[
z = a + 2b \left\{ \frac{2}{\pi} \log \left( \frac{w - \alpha}{w + \alpha} \right)^{1/2} - \frac{2}{\pi} \log \left( \frac{w - \alpha}{w + \alpha} \right)^{1/2} \right\} \]

where \( \log w = \ln |w| + i \arg w \), \( 0 < \arg w < 2\pi \).

It is now required to locate the image in the \( w \) plane of a segment of the field line of steepest gradient, i.e. that through \( C \) or \( D \), of length \( \lambda \) measured from the electrode. An integral expression for \( \lambda \) in terms of arc length in the \( w \) plane can be found: if \( \lambda \) is small compared with \( b \), as is expected, the integral can be evaluated by elementary means, and the arc length in the \( w \) plane is hence known. Thus the location of the image segment is known, and the potential drop \( \Delta \) across the segment can be calculated.

This is given by

\[
\Delta V = \left( \frac{2}{b} \right)^{1/2} \alpha \left( \frac{\lambda}{b} \right)^{1/2} \left( \frac{\lambda}{b} \right)^{1/2} (1 - \alpha^2)^{-1/2}
\]

The present theory states that a partial discharge will occur if \( \Delta V \) exceeds some critical value \( \Delta V_c \). Substituting for \( \alpha \) in...
terms of $a$ and $b$ and solving for the discharge-inception voltage $V_h$, eqn. 1 is obtained.

8.1.2 Case 2: Rounded electrode

This case is rather more straightforward than case 1, since there is no sharp corner, and hence the potential drop in distance $\lambda$ may be calculated (to first order in $\lambda$) as

$$-\lambda (d\phi/ds),$$

where $s$ is the arc length along the line of maximum field. The transformations to determine the potential distribution are, however, more complicated; it proves to be easier to assume that the electrode profile generated by a plausible set of transformations gives a reasonable approximation to the true profile, instead of endeavouring to find a transformation based on an assumed profile.

Fig. 6 shows the various stages in the transformation. In the $w_1$ plane (top left), the potential distribution is similar to that in the $w_1$ plane of Appendix 8.1.1. The first transformation, to the $r$ plane (top right) is a change of scale by a factor $(1 + a^2)/2$; the second transformation, to the $w_2$ plane (bottom right), generates a semicircular boss:

$$w_2 = 1 + (r^2 - a^2)^{1/2}.$$  

The third transformation, to the $z$ plane (bottom left), is similar to that given by eqn. 4, with the same constants.

The equation of the curve $CD$ in the $z$ plane may be expressed parametrically by substituting $w = \alpha \exp i\theta$, $0 < \theta < \pi$, in eqn. 5. At the point on $CD$ represented by the parameter $\theta$, the magnitude of the field is $E = d\phi/dz = |d\phi/dw|$; and it may be calculated in terms of $|d\phi/dw|$ and scale factors.

For $a > 2^{1/2} - 1$ ($a/b > 0.089$), $E$ is a maximum for $\sin \theta = (1 - a^2)/2a$, and is given by

$$E_1 = \frac{V}{2^{1/2}a} \frac{1 + a^2}{a^3(1 - a^2)^{1/2}}.$$  

For $a < 2^{1/2} - 1$, $E$ is a maximum for $\cos \theta = 0$, i.e. at the tip of the electrode, and is given by

$$E_2 = \frac{2^{1/2}V}{b\alpha}.$$  

The argument at the end of Appendix 8.1.1. may now be applied to obtain equations for the discharge-inception voltage. Eqns. 8 and 9 lead to eqns. 2 and 3, respectively.

8.2 Notes on types and nature of discharges

When carrying out internal discharge measurements on impregnated capacitors, using high-sensitivity equipment and a gradually rising 50 Hz alternating test voltage, it is usual to observe the following types of discharges:

Type I: sporadic single pulses of small magnitude, not confined to any particular phase of the test voltage.

Type I-II: groups of pulses confined to the rising portion of each halfcycle of the voltage wave, steady in magnitude. The extinction voltage is only a little below the inception voltage.

Type II: groups of pulses at the same portion of the wave as type I-II, but growing rapidly in magnitude. The extinction voltage is much lower than the inception voltage, but depends on the length of time for which the discharges are allowed to continue.

The growing, type II, discharges are associated with bubbles.
of gas in the impregnant. Breakdown in the bubble generates more gas by ionic bombardment.

In type I–II discharges, the initiating event is thought to be a breakdown of the liquid, as discussed in the paper. It is likely that the pulses observed still correspond to discharges in bubbles; the energy per pulse is enough to vaporise a 2μm-diameter sphere of liquid, but any breakdown in a liquid will produce a bubble if the energy is available. The fact that the discharges do not grow is characteristic of certain impregnants; it is, in fact, cited as one of the advantages of a chlorinated diphenyl as compared with mineral oil. One possibility is that such impregnants carbonise easily and the breakdown track then remains short-circuited. Provided that the solid insulation can withstand the field concentration at the tip of such a carbonised path, the situation could remain stable for a time.

Type I discharges are more difficult to interpret, in view of their randomness. It is possible that they are the peaks of a type I–II display which is mostly obscured by noise. Further elucidation would be aided by using low-noise detection equipment. This would differ from the standard types of detector by being of narrow bandwidth, and hence sacrificing resolution and flexibility. The use of specimens of small capacitance would also be helpful.