ERRATA TO

ANALYSIS, MANIFOLDS AND PHYSICS. PART I

Page*	Line	Errata
		Front Matter
xi	8	III. Differentiable Manifolds,
		Chapter I
6	12	Example 1: Let P be the set $\mathbb{N}$ of all
	16	$\mathbb{N}$ has no maximal element.
11	8	(p. 17)
12	−2	is open iff it is a neighborhood
13	3	x: $(N(x) - \{x\}) \cap A \neq$
	12	in A. The set A is dense
14	5	For each $x \in X$ and $U \in \mathscr{U}$ with $x \in U$ there exists $B \in \mathscr{B}$ such that $x \in B$ and $B \subset U$.
	−9	is a filter $\mathscr{F}(x)$.
15	−10	U_j such that $V_i \subset U_j$.
20	12	is simply connected when $n \geq 2$.
21	13	Exercise 2, p. 68.
23	4	vector space (p. 27) is bounded
24	caption	$d_1(P, 0) \leq a$, $d_2(P, 0) \leq a$, $d_3(P, 0) \leq a$
27	−10	continuous (p. 21)
28	3	into the neighborhood $\|\alpha x -$
31	1	p. 56
39	−16	**Haar measure** (p. 180).
46	9	a σ-field of subsets, $\mathscr{A}$
49	−2	$\|m(A)\| < \|m\|(A)$
55	1	$+\|x^2\|^p)^{1/p}$
61	17	Let X and Y be two metric spaces,
62	19	operator T on $L^2(Y)$ by
65	4	$v^2 = (v\|v)$
	22	add C_p in the margin
66	3	Let $P = C(V^3{}_{(3)})$

*Refers to the revised edition.

Page*	Line	Errata
	−8	$\forall \boldsymbol{v},\ \boldsymbol{w} \in V^{n}{}_{(s)}$
67	−12	So we have for k odd
69	3–10	

Answer: We shall construct a covering of the product space $\times_{\alpha\in\mathbb{N}} X_\alpha$ which has no countable subcovering, let alone a finite one. Let $A \subset \mathbb{N}$ and $V_A = \times_{\alpha\in\mathbb{N}} U_\alpha$ be an element of the topology on the product space defined as follows:

$$U_\alpha = \begin{cases} [0, 2/3) & \text{if } \alpha \in A \\ (1/3, 1] & \text{if } \alpha \notin A \end{cases}.$$

Then $\{V_A;\ A \subset \mathbb{N}\}$ is an open covering of the product space; it has no countable subcovering. Indeed let $\{V_{A_i}\}_i$ be any countable subset of $\{V_A;\ A \subset \mathbb{N}\}$. One can always find a point $x = \{x_1, x_2, \ldots\}$ in the product space such that $x \notin \{V_{A_i}\}_i$. For example, let

$$x_i = \begin{cases} 1/6 & \text{if } i \notin A_i \\ 5/6 & \text{if } i \in A_i \end{cases} \quad \text{then } x \notin \{V_{A_i}\}_i.$$

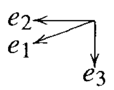
Contributed by J. Labelle.

Chapter II

72	9	mapping: $Dl\|_{x_0} = l$, $D\,\mathrm{Id}\|_{x_0} = \mathrm{Id}$, $\forall x_0$.
	−4	of f. A point where the rank is not maximal is called **critical**. If $n = p$ the determinant of Df is called the **Jacobian**
77	11	for the function q
85	5	A sufficient condition theorem
	6	A sufficient condition for f

Chapter III

111	−12	$x \in U \subset X$.
127	−15	admissible atlas (see p. 543)
129	8	replace “identical” by “isomorphic”
	12	bundle in particular (p. 376)
131	−5	E_1 a subspace of E
	footnote	or [Osborn II 4]
135	−6	if follows that $f^* \circ \boldsymbol{v} = \boldsymbol{v} \circ f^*$.

Page*	Line	Errata
151	−5	$\det\left[\dfrac{\partial x'^i}{\partial x^j}\bigg\vert_{x_0^n=0}\right]$
	footnote	only identity transformations
152	17	$\mathscr{L}_{\mathrm{v}}\mathbf{g}$
	−7	The tensor $\mathscr{L}_{\mathrm{v}}\mathbf{g}$ is the strain tensor generated by the vector field $\boldsymbol{v}$
153	6	$\sigma_g \circ \sigma_h = \sigma_{gh}$ (left action of G on X) or $\sigma_g \circ \sigma_h = \sigma_{hg}$ (right action of G on X)
154	footnote	the group which defines isometries
155	20	effectively, transitively, and freely on G.
156	−1	$[\boldsymbol{v}_\alpha, \boldsymbol{v}_\beta](g)$
158	12, 13	and the n-torus $\mathbb{T}^n = \mathbb{R}^n/\mathbb{Z}^n$
	−7	(p. 209)
162	−3	(or $\sigma_h \circ \sigma_g$)
164	7	$\boldsymbol{v}^k_{(\alpha)}$ as well as the generators $\boldsymbol{v}^L_{(\alpha)}$ and $\boldsymbol{v}^R_{(\alpha)}$ of
165	7	inserted in (5)
165	−3	158 and p. 353
166	2	$[\boldsymbol{v}^k_{(\alpha)}, \boldsymbol{v}^k_{(\beta)}]^j(x)$
	3	$-[D_{,\alpha}, D_{,\beta}]^i_k(e)x^k$
	5	is defined by matrix multiplication, antisymmetrized
176	−11	for every $v \in C_1$
177	9, 12	$\lambda = \frac{1}{4} V^{\alpha\beta} e_\alpha e_\beta$
186	−15	i.e., all rotations
188	−8	be nonsingular. $k_{\alpha\beta}$ defines a metric called **Cartan–Killing**.
190	−6	jacobian matrix of the embedding mapping (p. 240)
192		$\binom{a^1}{a^2} = ((z^1)^2 + (z^2)^2)^{-2} \begin{pmatrix} (z^2)^2 - (z^1)^2 & -2z^1z^2 \\ -2z^1z^2 & (z^1)^2 - (z^2)^2 \end{pmatrix} \binom{b^1}{b^2}$
		Chapter IV
196	footnote	Then if α and β are 1-forms
200	−2	By property 2
203	5	are vectors and $\boldsymbol{W}$ a pseudo-vector
	9	$\operatorname{div} f\boldsymbol{W} = \operatorname{grad} f \cdot \boldsymbol{W} + f \operatorname{div} \boldsymbol{W}$
210	11	$\phi_x\colon (X)^p$
211	16	$[\boldsymbol{\varphi}\,\hat{;}\,\boldsymbol{\psi}]$
216	−12	$\iint_{\text{surface}}$
218	?	

Page*	Line	Errata
223	18	group [See Analysis, Manifolds and Physics, Part II]
	20	Move the sentence "H^p is often called the de Rham group" and the marginal note "de Rham cohomology" back to line 12.
224	1, 2	Delete and replace by: "For the cases of 0-chains see for instance [Patterson]."
	4	Insert the definition of the Euler–Poincaré characteristic, which can be found on p. 293.
225	6	ω depends only on $x^1, \ldots, x^p$.
226	−5	for $C \in H_p$, $\omega \in H^p$.
250	23, 24	delete "A differential ... system."
253	17	vector field. By the theorem on p. 248, C is completely integrable.
254	11	result in a set $\{\bar{\theta}^{(k)}\}$
271	7	equations (see example p. 263)
276	−9	$= \underline{-\mathrm{d}y_\alpha}$

Chapter V

Page*	Line	Errata
285	14	(p. 134)
287	−3	**manifold**. The metric is called **lorentzian**. It is
302	16	$u = C' \dfrac{\mathrm{d}u^i}{\mathrm{d}t} = \dfrac{\mathrm{d}C^i}{\mathrm{d}t}$,
306	5	In a moving frame the
313	18	$(\boldsymbol{\nabla}_{\boldsymbol{v}} \boldsymbol{u})^{\Vert} = (\boldsymbol{\nabla}_{\boldsymbol{u}'} \boldsymbol{v}'$
314	−5	a lorentzian metric it is a maximal
	−4	is a minimal hypersurface; in
316	−8	$\Phi^*(\mathrm{D}f) = \mathrm{D}(\Phi^* f)$
	−4	pp. 482–486.
317	6	$\mathrm{d}(^*\boldsymbol{\omega})$
331	−12	$(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{n/2}, \boldsymbol{J}\boldsymbol{v}_1, \ldots, \boldsymbol{J}\boldsymbol{v}_{n/2})$
	−13	$\mathscr{L}_v \eta = 2\Phi\eta.$ (1)
	−11	such that $\Phi =$ constant is the group of isometries and dilations
	−8	$= \eta_{\lambda\beta}\partial_\alpha v^\lambda + \eta_{\lambda\alpha}\partial_\beta v^\lambda = 2\Phi\eta_{\alpha\beta}$
	−7	where ϕ is obtained by contraction $2\phi = \frac{1}{2}\,\partial_\alpha v^\alpha$
354	12–20	Replace lines 12 to 20 by: This transformation is defined in the case of an euclidean metric if one adds to the space a point at infinity, whose image in an inversion is the origin of this inversion. In the

Page*	Line	Errata
		case of the Lorentz metric the inversion with origin point x_0, for instance $x_0 = 0$, is defined only when $x^2 = \eta_{\alpha\beta}x^\alpha x^\beta \neq 0$, that is when x is not a null (i.e. lightlike) vector. The conformal group is defined in Segal cosmos ($S^3 \times \mathbb{R}$, ref p. 356) universal cover of the compactified Minkowski spacetime (cf. problem V.8). For more on the compactification of Minkowski space see for instance R. Penrose "Conformal Treatment of Infinity" pp. 563–584 in *Relativity, Groups, and Topology* (Les Houches 1963) Eds C. DeWitt and B. DeWitt (Gordon and Breach New York 1964)
356	13	in empty space.
		Chapter V bis
359	15	subspace of $T_p(P)$. Due to
360	16	Add: We also write $\check{\boldsymbol{\gamma}} = \boldsymbol{v}_\gamma(p)$.
362	−14	a 1-form on U with values
363	7	$= \mathrm{Ad}(g^{-1})$
365	2	$= (\phi_j^{-1*}\omega)$
	3	$L_{g_{ji}}(x)$
	9,10	$\begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{w} \end{pmatrix} \mapsto \begin{pmatrix} \partial x/\partial x & \partial x/\partial g \\ \partial L_{g_{ij}(x)}g/\partial x & \partial L_{g_{ij}(x)}g/\partial g \end{pmatrix} \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{w} \end{pmatrix}$
	14	$\boldsymbol{\theta}_{\mathrm{MC}}(g'_{ji}(x)\boldsymbol{v})$
369	8	delete the composition sign
371	10	then (p. 367),
	−13	corresponding to ψ and $\nabla_u \psi$
372	−13	of a covariant vector.
373	−14	Add: "$h \equiv$ hor"
375	4	$= \cdots + \frac{1}{2} \cdots$
377	4	$\mathscr{L}_{\mathrm{ver\,u}}\bar{\theta}$
379	14	differential 2-form $\boldsymbol{\Omega}$ on X
380	−8, −7	Delete: "It is ... p. 359"
	−6	connection as defined on p. 359
381	−13	of the connection, as we have already seen on an
382	15	the injection f in P
	20	of G_1; thus
	−4	if $g_i = g_{ij}(x)g_j$

Page*	Line	Errata
383	13	the realization σ of G
	−4	$\tau_{g_2 g_1} =$
	−3	$P \times_G F$
384	−12	action of G_1, where (P, X, π, G) is a principal fibre bundle
385	17, 18	Delete and insert: 2) Suppose G is reducible to G, and let P_1 denote a reduced bundle with injection $f\colon P_1 \to P$. Let μ be the projection $P \to P/G_1$. The mapping $\mu \circ f$
385	−9, −1	Delete and insert the following

Theorem. Every fibre bundle (E, X, π, F, G) such that the base manifold X is paracompact and the fibre F is diffeomorphic to $\mathbb{R}^n$ admits infinitely many cross sections.

Proof: It is easy to show that when E is a vector bundle, i.e. when F_x is a vector space, it admits infinitely many cross sections. Indeed, let $\{\theta_i\}$ be a partition of unity on X subordinates to a locally finite covering by open sets V_i such that $V_i \subset U_j$ some open set of an atlas of X over which E is trivializable. Let σ_i be an arbitrary cross section over V_i. Then the element of F_x given by the finite sum

$$\sigma(x) = \sum_i \theta_i(x)\sigma_i(x)$$

is a cross section over X. The theorem is stronger because it does not require a canonical identification of a point of F_x with the origin of $\mathbb{R}^m$, nor the group G to be linear. For the proof[1] one uses the property that a differentiable function defined on a closed set of $\mathbb{R}^m$ can be extended to the whole of $\mathbb{R}^m$ together with Zorn's lemma, to show that every cross section defined over a closed set $\overline{Y} \subset X$ can be extended to a cross section over X.

[1]Cf. R. Godement, *Theorie des Faisceaux* (Hermann, Paris, 1958) p. 151 or Kobayashi and Namizu, loc. cit. Vol. **I**, p. 58.

Page*	Line	Errata
386	21	and ρ_x^{-1} the linear
	−15	product in $\mathbb{R}^n$ and ρ_x^{-1} the linear

Page*	Line	Errata
	−9	in part A (pp. 380–381)
387	−12, −13	delete i.e. ... and insert: $[\mathscr{H}_0(p') = g\mathscr{H}_0(p)g^{-1}]$.
389	18	p. 388
	22	and by the subspace
	−7	horizontal field is horizontal (p. 374), and that
	−5	$= -\boldsymbol{\Omega}(\boldsymbol{u}, \boldsymbol{v})$
390	−10	$\cdots = \frac{1}{(2k)!} \cdots (v_{\sigma(1)}, v_{\sigma(2)}), \ldots,$
	−1	$T_x X, x = \Pi(p)$
391	1	and be unique
	9	Add: $\mathrm{d}f(\Omega)(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n) = \pi^* \,\mathrm{d}\bar{f}(\boldsymbol{\Omega})(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n)$ $= \mathrm{d}\bar{f}(\Omega)(\pi' \text{ hor } \boldsymbol{v}_1, \ldots, \pi' \text{ hor } \boldsymbol{v}_n) =$ $\mathrm{d}f(\Omega)(\text{hor } \boldsymbol{v}_1, \ldots, \text{hor } \boldsymbol{v}_n) = \mathrm{D}f(\Omega)(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n)$
	18	$+\frac{1}{2}[\ ]$
	20	$+\frac{1}{2}[\ ] + \frac{1}{2}[\ ]$
392	4	This, together with the fact Φ being invariant
	5	on a vertical vector projects to a form $\bar{\Phi}$ on X,
	7	$= \mathrm{d}\bar{\Phi}$.
394	11	$[f(V)]^2$
395	8	curvature 2-form
	11	complexification
	−1	(p. 224)
397	−8	via a scalar product in the fibres, possibly deduced
398	5	to the scalar product in
399	11	Decomposition theorem.
	17,18	Proof: It follows formally the same lines as the proof given above for the finite dimensional space $E_{p,x}$. Its validity in the new context rests
	−7	therefore $\boldsymbol{\Delta}_p$ is an elliptic
401	4	$))\wedge\rho^*$ ($\wedge$ is exterior product)
	11,13	$\{D_p, \underline{E_p}\}$
402	10	$\mathbf{d}\boldsymbol{x}^{\nu} = \bar{\boldsymbol{\Omega}}_i$,
404	figure	Replace π_i by π_1
405	5	$\mathrm{D}\tilde{\psi}_u(\boldsymbol{v}) =$
	11	$\underset{\triangle}{=} \tilde{\psi}'(\boldsymbol{v}) + \rho_e'(\omega(\boldsymbol{v}))\tilde{\psi}(u)$
406	−13	$\Phi_{i,x}\circ$

Page*	Line	Errata
407	7	$\to (\check{\phi}(s_i(x)))^{-1}$
	−12	when the connections $\boldsymbol{A}$ is flat.
408	−10, −11	bold face dϕ
	−5	$p\colon U(1) \to L(\mathbb{C}, \mathbb{C})$
410	20	, $\gamma_1 = -(2\pi \mathrm{i})^{-1}\, \mathrm{tr}\, \bar{\boldsymbol{\Omega}}$
	−13	$C_1 = \cdots = \dfrac{1}{2\pi} \displaystyle\int_{S^1} \boldsymbol{n}\, \mathbf{d}\boldsymbol{\phi}$
	−10	$\mathrm{i}\boldsymbol{A}_+ = \mathrm{i}\boldsymbol{A}_- \mathrm{i}n\, \mathbf{d}\boldsymbol{\phi}$.
	−9	define the following electromagnetic field
	−6	define the following
411	18	Do F_+ and F_-
415	12	Atiyah
419	−1	$(\tilde{\mathscr{H}} \circ$
420	6	change signs before 1/4
	6,7,14,17	change signs before 1/8
421	6	$-\frac{1}{2}\gamma^\alpha \gamma^\beta (\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)\psi$
	7	if there is no torsion.
	−3, −4	$\times \mathbb{R} \times \mathbb{R}$
422	References	T. Regge, "The group manifold approach to unified gravity", in *Relativité, Groupes et Topologie II*, eds. B.S. DeWitt and R. Stora (North-Holland, Amsterdam, 1984) pp. 933–1006.

Chapter VI

Page*	Line	Errata
435	13	exists C^∞ on U, but cannot
441	−9	in general vanish for
	−8	it vanishes if $\varphi'(a) = 0$.
	−7	Remark. $\varphi \delta'_a =$
455	−10	$T_n = \dfrac{1}{\mathrm{i}} \dfrac{\mathrm{d}}{\mathrm{d}x}$
468	16	Proof: X is solution in $\mathscr{A}$ since,
472	−2	$\mathrm{D}X = B$ *has at most one* … *if* D^* *has an*
477	10	$= (\mathrm{i}y)^\alpha$.
478	5	p. 272
490	−6	$\cdots = \lVert (1 + \lvert x \rvert^2)^{m/2} \mathscr{F} f \rVert_2$
494	13	$\cdots \Box u = (- - -)u = 0$

Page*	Line	Errata
499	−2	$\cdots = \cdots - \int \sum \frac{\partial \varphi}{\partial x^i} \frac{\partial u}{\partial x^i} \, \mathrm{d}x$
512	7	$= \frac{1}{4\pi} \delta_c.$
	9	$= \frac{1}{t} \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$
513	13	$\alpha^1 = \sin\theta \cos\phi$, $\alpha^2 = \sin\theta \sin\phi$, $\alpha^3 = \cos\theta$
522	−9	For a system with an infinite
523	6	$\cdots = \langle T, L^* U \rangle$ when U is C^∞ with compact support
532	8	$Y^-(x) \exp(ax)$
from 538 to 539	−2 23	Replace by:

Answer: a) The elementary kernels $E(t, s)$ are solutions of

$$(-\mathrm{d}^2/\mathrm{d}t^2 - \rho^2) E(t, s) = \delta(t, s). \tag{4}$$

The Fourier transform of this equation is

$$\mathscr{F}((-\mathrm{d}^2/\mathrm{d}t^2 - \rho^2)\delta * E)(\eta) = 1,$$

$$(\eta^2 - \rho^2) \mathscr{F} E = 1,$$

$$(\mathscr{F} E)(\eta) = \frac{1}{2\rho} \operatorname{Pv}\left(\frac{1}{\eta - \rho} - \frac{1}{\eta + \rho} \right) + K_1 \delta(\eta - \rho) + K_2 \delta(\eta + \rho). \tag{5}$$

We can choose K_1 and K_2 such that the elementary kernel is in the convolution algebra $\mathscr{D}'^+$ or $\mathscr{D}'^-$. Recall (Problem VI 7) that

$$(\mathscr{F} Y^\pm)(\eta) = \mp \mathrm{i} \left(\operatorname{Pv} \frac{1}{\eta} \pm \mathrm{i}\pi \delta_\eta \right) = \mp \mathrm{i} (\eta \mp \mathrm{i}0)^{-1}. \tag{6}$$

Hence

$$E^+ \in \mathscr{D}'^+ \qquad \text{if } K_1^+ = -K_2^+ = \mathrm{i}\pi/2\rho \text{ and}$$

$$\mathscr{F} E^+ = \frac{1}{2\rho} \left(\frac{1}{\eta - \rho - \mathrm{i}0} - \frac{1}{\eta + \rho - \mathrm{i}0} \right), \tag{7a}$$

$$E^- \in \mathscr{D}'^- \qquad \text{if } K_1^- = -K_2^- = \mathrm{i}\pi/2\rho \text{ and}$$

$$\mathscr{F} E^- = \frac{1}{2\rho} \left(\frac{1}{\eta - \rho + \mathrm{i}0} - \frac{1}{\eta + \rho - \mathrm{i}0} \right). \tag{7b}$$

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To compute $E^{\pm}$ from (5) using (6) one can translate (see p. 458)

$$\mathrm{Pv}\left(\frac{1}{\eta-\rho} \pm \mathrm{i}\delta(\eta-\rho)\right)$$

by ρ and the other two terms by $-\rho$:

$$E^{\pm}(t,s) = \mp Y^{\pm}(t-s)\rho^{-1}\sin(\rho t - \rho s). \tag{8}$$

The propagator $E = E^{+} - E^{-}$ is

$$E(t,s) = -\rho^{-1}\sin(\rho t - \rho s).$$

We could, of course have obtained (8) by solving (4) according to the method developed on p. 469, which says

$$E^{\pm}(t,s) = Y^{\pm}(t-s)h^{\pm}(t-s)$$

where the C^{∞} functions $h^{\pm}$ satisfy the homogeneous equation and the following boundary conditions:

$$h^{+}(0)=0,\ h^{+\prime}(0)=-1 \quad \text{and} \quad h^{-}(0)=0,\ h^{-\prime}(0)=1.$$

Equation (7) suggests the following integral representation for the ele-

541	−3	equation (9)

Chapter VII

549	6	Hilbert space of norm
571	footnote	The Morse index is the negative of the index defined on p. 287
590	−7	$= -\mathrm{d}(\cdots + \frac{1}{2}m^2u^2)\,\mathrm{d}x)(U,V)$.
593	−4	$\int_A^B \exp(-\lambda t^2/2)h(t)\,\mathrm{d}t = \frac{-1}{\lambda t}\left(h(t) - \frac{h(t)}{\lambda t^2} + \frac{h'(t)}{\lambda t}\right)\exp(-\lambda t^2/2)\Big|_A^B$

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		$$+\frac{1}{\lambda^2 t^2}\int_A^B \exp(-\lambda t^2/2)$$ $$\times\left(3\frac{h(t)}{t^2}-3\frac{h'(t)}{t}+h''(t)\right)\mathrm{d}t$$

References

603	CHERN, S.S., *Selected Papers* (Springer-Verlag, New York, 1978).
606	ITZYKSON, C. and J.B. ZUBER, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
608	SCHUTZ, B., *Geometrical methods of mathematical physics* (Cambridge University Press, Cambridge, 1980).

New and/or Corrected Entries for Index