Yield and Closure of Directional and Horizontal Wells

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Directional and horizontal wells in weak rocks were modeled using a nonlinear strain-hardening elastoplastic finite element model. The distribution of yielding and plastic straining around the hole depends strongly on wellbore deviation and in situ stresses. Deformations around deviated wells are three-dimensional. The support pressure required to prevent failure of the wellbore wall is less than that predicted by a linear elastic analysis. This helps explain field observations of stable wells and laboratory observations of apparent high strengths next to model holes. In some cases wellbore closure may be as much of a problem for drilling as wellbore collapse.

INTRODUCTION

Wellbore stability is a key concern in the drilling and completion of oil and gas wells. Many costly problems are caused by wellbore instability, such as increased drilling time, stuck drillpipe or logging tools, poor log quality, inadequate cement placement around well casing, and increased fluid and solid volumes requiring disposal. Instability is more likely in directional and horizontal wells, due to the increased stresses acting on the well, especially in formations with low compressive strengths such as shales and weak sandstones.

Wellbore instability can be prevented by applying a combined rock mechanics/chemical approach [1]. The mechanical approach is to increase the support pressure in the hole by using a higher density for the drilling fluid. If the in situ stresses and the rock strength are known, then the required support pressure can be calculated by employing the equations for three dimensional linear elastic stress concentration around a hole [e.g. 2]. This method has been applied successfully in many field situations [e.g. 1,3-5]. The linear elastic analysis tends to be conservative, however, because various nonlinear and inelastic rock behaviors result in lower stresses at the wellbore than predicted by the linear elastic equations.

Conservatism is not in itself undesired, but there are frequently limits to the drilling fluid density that can be feasibly used, due to unintentional hydraulic fracturing or differential pressure sticking, for example. The goal is to use as low a fluid density as possible, without resulting in hole instability. This less conservative solution can be determined quantitatively if one incorporates a more realistic description of rock behavior in the wellbore stability model [e.g. 6,7].

This paper presents results obtained through nonlinear finite element analyses of directional and horizontal wells. The rock deformation and strength are described by a strain-hardening elastoplastic constitutive model, which was fitted to the behaviors of weak sedimentary rocks that were observed in the laboratory. A modeling technique was developed to simulate the three-dimensional response of deviated wells efficiently using a single "slice" of elements.

MODELING APPROACH

3-D finite element slice model

Finite element models capable of simulating a well at any chosen orientation were developed using the ABAQUS [8] general purpose finite element program. Figure 1 shows a portion of the typical element mesh, which defines a slice taken orthogonal to the well axis. The two planar surfaces of the slice are constrained to move only identically or in a parallel manner, but displacements in all three dimensional directions are allowed (subject to appropriate boundary conditions). This ensures that the slice maintains constant thickness, yet allows it to warp, or become non-planar. It also ensures that any line parallel to the hole axis (for example, the hole wall) always remains parallel to the hole axis. These models correctly calculate all three-
dimensional stresses and displacements in the material surrounding the hole, including the concentration of out-of-plane shear stresses near the hole wall.

The creation of the well is simulated by removing material from the pre-stressed model and replacing it with a uniform support pressure on the hole wall. This support pressure is then incrementally reduced to obtain solutions for lower support pressures. While this does not simulate the exact stress path imposed when the face of the hole (and the drill bit) passes by, it is considered to be a good approximation.

**Constitutive model**

A nonlinear strain-hardening constitutive model was chosen to represent the behaviors of weak sedimentary rocks observed in the laboratory. This model has an initial yield surface which expands as plastic strain accumulates until it becomes coincident with the failure surface, which defines the ultimate strength. The yield surface is a Drucker-Prager type, given by

\[ q - p \tan \beta = \sigma_y(e^{pl}) \]  

where

\[ q = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} \]  

\[ = \sigma_1 - \sigma_3 \text{ when } \sigma_2 = \sigma_3 \]  

\[ p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} - \text{ pore pressure} \]  

\[ \sigma_y(e^{pl}) \] defines the size of the yield surface as a function of the work equivalent plastic strain \[ e^{pl} \], defined as

\[ e^{pl} = \int \sqrt{\frac{2}{3} (de_1^{pl} d e_1^{pl} + de_2^{pl} d e_2^{pl} + de_3^{pl} d e_3^{pl})} \]  

where \[ de_1^{pl}, de_2^{pl}, de_3^{pl} = \text{ incremental principal plastic strains} \].

The yield surface and the failure surface are illustrated for the condition \( \sigma_2 = \sigma_3 \) in Figure 2. For these models, the initial value of \( \sigma_y \) has been set to zero as shown. The ultimate value of \( \sigma_y \) represents the cohesion of the rock, while \( \tan \beta \) defines the internal friction.

![Fig. 2. Relationship between initial yield surface and failure surface in the triaxial plane.](image)

The value of \( \beta \) and the relationship \( \sigma_y(e^{pl}) \) depend on the rock being modeled. Figure 3 illustrates these for one of the weak rocks studied, showing axial stress-strain curves at different constant confining pressures. Once the yield surface reaches the failure surface, plastic strains continue at constant stress. The three principal plastic strains are related using a non-associated flow rule. For the rocks modeled, isovolumetric flow was found to closely approximate the deformations during strain hardening.

This constitutive model is one possible way of capturing important aspects of nonlinear strain hardening that can influence weak rock behavior around directional and horizontal wells. The model has no ability to simulate plastic strains associated with increasing hydrostatic stress, but the creation of a well subjects the rock mostly to deviatoric loading. Loading into tension is not permitted in the analysis. The failure surface may overestimate strength for triaxial extension type conditions (shown in Figure 2 as \( \sigma_2 = \sigma_3 = \sigma_1 \)), especially for sandstones or for the stronger shales. However, the focus of these analyses is to compare nonlinear strain-hardening wellbore stability predictions with linear elastic predictions obtained using the same failure criterion.

This constitutive model can be used to represent rocks that undergo strain softening following attainment of peak strength, as long as the wellbore simulation is not carried beyond the peak strength point.

Constitutive properties of the rocks modeled in this paper are given in Table 1. The properties were determined from triaxial compression tests on weak shales [9], although similar behaviors have been observed on weak sandstones as well. The cohesion (given by ultimate \( \sigma_y \)) and friction angle, \( \beta \), can be related to the Mohr-Coulomb cohesion, \( c \), and friction angle, \( \phi \), under triaxial compression (\( \sigma_2 = \sigma_3 = \sigma_1 \)) conditions [10]. These values are also listed in Table 1.

<table>
<thead>
<tr>
<th>Rock</th>
<th>( E ) (GPa)</th>
<th>( \beta )</th>
<th>( \sigma_y ) (MPa)</th>
<th>( \phi )</th>
<th>( c ) (MPa)</th>
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<tr>
<td>Rock 1</td>
<td>0.97</td>
<td>30°</td>
<td>15.3°</td>
<td>1.96</td>
<td></td>
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<td>Rock 2</td>
<td>2.48</td>
<td>40°</td>
<td>21.6°</td>
<td>5.86</td>
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</table>

\( E \) is Young modulus. Poisson's ratio=0.3 for both rocks.
YIELD AND CLOSURE PATTERNS

Using the 3D finite element slice model with the nonlinear strain-hardening constitutive model, wells at different orientations in different stress regimes, and in different rocks, were simulated. Table 2 summarizes the rocks (see Table 1), in situ stresses and approximate depths for the results presented in the remaining figures.

Table 2. Effective stresses, approximate depth and rock for Figures 4-11

<table>
<thead>
<tr>
<th>Rock</th>
<th>( \sigma_v )</th>
<th>( \sigma_h )</th>
<th>( \sigma_{ht} )</th>
<th>Depth (m)</th>
<th>Figure #</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock 1</td>
<td>13.79</td>
<td>8.28</td>
<td>8.28</td>
<td>1200</td>
<td>4,7,8,10</td>
</tr>
<tr>
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<td>8.28</td>
<td>11.03</td>
<td>1200</td>
<td>5,6,9</td>
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<tr>
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<td>20.69</td>
<td>20.69</td>
<td>3000</td>
<td>11</td>
</tr>
</tbody>
</table>

Yield distribution around wells

For a vertical well subject to equal horizontal stresses, any yielding that occurs theoretically will be distributed axisymmetrically around the hole. As the hole deviation from vertical increases, however, yielding becomes more concentrated at the "sides" of the hole. This is illustrated in Figure 4 for wells deviated 30° and 60° from vertical. The view in these figures is looking directly down the axis of the well, with the X-axis of the figure corresponding to a horizontal line in the earth. The plane of the figure is orthogonal to the well axis, and the solution in the remaining 3/4 plane is implied by symmetry.

For the illustrations in Figures 4 through 6, the effective support pressure in the well (called "overbalance") has been reduced to zero, and the region which has reached ultimate strength \( (\varepsilon > 0.02) \) has been contoured. While the amount of plastic strain can be somewhat high, these figures are useful for comparing the distribution of yielding around wells at different deviations subject to the same support pressure. Note that greater plastic strains occur as hole deviation increases. The most extreme concentration of yielding usually occurs for a horizontal (90°) well, and the severity increases with increasing difference between the vertical and horizontal stress.

![Fig. 4](image1)

![Fig. 5](image2)

Fig. 4. Fully yielded zone around deviated wells with zero effective support pressure, equal horizontal stresses (see Table 2). (a) 30° inclination. (b) 60° inclination.

Fig. 5. Fully yielded zone around 45° wells with zero effective support pressure, unequal horizontal stresses (see Table 2). Well azimuth parallel to (a) max. horizontal stress, (b) min. horizontal stress.
If the horizontal stresses are not equal then the azimuth (projection of the well axis onto a horizontal plane) of a non-vertical well becomes important. Figure 5a shows the yielding pattern around a 45° well whose azimuth is parallel to the direction of the maximum horizontal stress, \( \sigma_h \). In this case the far-field stresses orthogonal to the well are significantly unequal. If the well is instead oriented parallel to the direction of the minimum horizontal stress, \( \sigma_h \), then yielding is closer to axisymmetric (Figure 5b) because the stresses orthogonal to the well are closer to isotropic. A low-angle (near vertical) well with this azimuth would have yielding more concentrated at the "top" and "bottom" of the hole, while one at a high angle (near horizontal) would have yielding more concentrated at the "sides".

One cannot interpolate between the two solutions shown in Figure 5 to obtain approximate solutions for other well azimuths. If a deviated well's azimuth is not parallel to one of the principal horizontal stresses then stress concentrations around the well become much more complex. Figure 6 illustrates yielding around a 45° well whose azimuth is 45° clockwise to \( \sigma_h \). Although the yielding appears to be mirror-symmetric about a set of rotated axes in the model plane, it is not. The only symmetry in this more general case is between points located exactly across the hole from each other (180° apart).

In the situations illustrated in Figures 4 and 5 the positions of the points of maximum plastic strain are independent of the support pressure in the hole. In the situation of Figure 6, however, the positions of maximum plastic strain rotate (in this case, clockwise) around the hole as the support pressure is reduced.

### Hole closures

Hole closures are determined mainly by the far-field stresses acting orthogonally to the hole, in conjunction with the rock deformation properties. Figure 7 plots radial hole closures for wells at 0° (vertical), 45° and 90° (horizontal) inclinations subject to equal horizontal stresses (as listed in Table 2). X-direction closures (Urx) represent the "sides" of the hole (X-axis direction of Figures 4-6), and Y-direction closures (Ury) represent the "top" and "bottom". Closures are plotted against two scales: one is the effective support pressure (overbalance) in the hole (Pw), and the other is the drilling fluid density (DFD) that would give this overbalance for a formation with normal pore pressure (1.0 g/cm³ gradient) at the depth indicated in Table 2. The closure curves are valid only for decreasing support pressure.

![Fig. 7. Maximum and minimum diametral hole closures for different hole inclinations, under equal horizontal stresses (conditions listed in Table 2). Pw = effective support pressure, DFD = drilling fluid density in specific gravity.](image)

As a well is increasingly deviated from vertical it deforms more strongly into an elliptical type shape, as shown by the difference between the X and Y direction closures in Figure 7. Hole closures in all directions are greater with lower support pressure in the hole. As yielding progresses, the diameter connecting the points of maximum plastic strain (X direction in Figure 7) increases the most due to the volume dilation that occurs in the yielding material (see Figure 4). If enough yielding and dilation occur then this diameter can become the smallest diameter of the hole, as shown by Detournay & Fairhurst [11].

For a well that is not parallel to a principal in situ stress, the displacements around the hole are actually three-dimensional. Figure 8 shows displacements (magnified by a factor of 10) for the 45° well in Figure 7 with 8.28 MPa overbalance pressure. By comparing the deformed mesh to the original mesh position, one can see that the top of the wellbore has warped "down" and the bottom of the wellbore has warped "up". This is due to the release of pre-existing out-of-plane shear stresses. Most of this deformation parallel to the wellbore axis is elastic and takes place as soon as the hole is created, although it increases somewhat as yielding occurs.

When the horizontal stresses are unequal, hole closures are influenced by both hole inclination and hole azimuth, as suggested by the yielding patterns in Figures 5 and 6. For a well with its azimuth parallel to \( \sigma_h \), the direction of maximum hole closure can switch once the inclination exceeds a certain amount, and a deviated well
can have radial closures that are more uniform (axisymmetric) than for a vertical well. This is illustrated in Figure 9 for the conditions listed in Table 2.

![Diagram](image)

Fig. 8. Deformations (solid lines) near the 45° hole of Fig. 7 with 8.28 MPa support pressure. Displaced positions have been magnified by 10 relative to original positions (dashed lines).

![Diagram](image)

Fig. 9. As in Fig. 7 but for unequal horizontal stresses with well azimuth parallel to min. horizontal stress (see Table 2).

Rock movements around the hole become quite complex for a deviated well whose azimuth is not aligned with a principal horizontal stress (as in Figure 6, for example). The directions of maximum and minimum radial closure rotate as the support pressure is reduced, and they may not remain perpendicular to each other. They also may not align with the locations of maximum plastic strain.

**APPLICATION TO DRILLING AND WELL COMPLETIONS**

Models such as these can be used to predict the stability of directional and horizontal wells and to help plan drilling and completion programs. The benefit of using nonlinear finite element analysis is that the rock behavior can be closely approximated, removing conservatism that can result from linear elastic models.

The top line in Figure 10 shows the effective support (overbalance) pressure, and corresponding drilling fluid density, that is predicted to ensure stability of a well as a function of inclination, based on a linear elastic analysis. The conditions are as listed in Table 2. This linear elastic solution gives the overbalance that would allow the three-dimensional stresses right at the wellbore to just equal the strength criterion, where this criterion is obtained from equation (1) by inserting the ultimate \( \sigma_y \) value from Table 1.

![Diagram](image)

Fig. 10. Support requirements as a function of hole inclination, based on different criteria, for the conditions listed in Table 2.

The linear elastic support requirement increases nonlinearly with increasing hole inclination from vertical, rising most strongly between 10° and 60°. This pattern of increase is typical for areas where the horizontal stress magnitudes are less than the vertical stress magnitude and are not too unequal. Similar patterns, although different values, are obtained if a different strength criterion, such as Mohr-Coulomb, is used.

Shown for comparison in Figure 10 is the overbalance for which the rock at the wellbore wall just reaches its ultimate strength, as computed by the nonlinear finite element analysis. Pictorially, comparing to Figure 3, the rock at the point of highest stress on the wellbore wall has moved all the way up the strain hardening curve and has just reached the position where the stress-strain curve flattens. Because of the rock's nonlinear behavior, the stress concentration at the wall is not as high as predicted by the linear elastic analysis, so a lower support pressure is required for the same strength criterion.

If the rock around the wellbore behaves as described by the constitutive model, then realistic, yet safe, support requirements can be taken from the nonlinear analysis instead of the more conservative linear elastic analysis. The nonlinear analysis result in Figure 10 could be applied even if the rock strain-softens after peak strength, because none of the rock is being allowed to go beyond the peak strength point. Note that the required support does not increase as much with increasing hole inclination as it does according to the linear elastic analysis.

Another important consideration for drilling operations is the amount by which the hole closes after it is created, since this determines clearances around the drilling tools. Also shown in Figure 10 is the overbalance...
at which the smallest hole diameter is 2% less than gauge hole diameter. The amount of closure that will result in drilling difficulties depends on the sizes of the drilling tools being used, on whether the drill string is rotating or is sliding with a downhole motor, and on the ease with which the rock can be scraped away as drilling tools are pushed or pulled past. Drill string stabilizers typically have blades that are between 0 and 3 mm (0.125 inch) undergauge relative to nominal hole diameter, which is commonly 311 mm (12.25 inches) or 216 mm (8.5 inches).

If 2% closure were allowable for the situation in Figure 10, and if the rock behaved in a ductile manner after attaining ultimate strength (no strain softening), then support pressures even lower than given by the ultimate strength limit could be used. If one planned to allow continued deformation at ultimate strength, however, it would be wise to also put limits on the total allowable plastic strains to reduce the risk of the rock breaking apart.

Whether using an ultimate strength limit or a closure limit, the amount by which the support can be reduced below the linear elastic solution is not constant with the wellbore inclination. In this example, the allowable reduction is greater for higher angle holes. The reduction depends also on the in situ stresses and, of course, on the rock strength and deformation properties.

Figure 11 shows results for a deeper well in a formation which is stronger and stiffer, and which also undergoes less plastic strain before ultimate strength is reached (see Tables 1 and 2). In this case the relationship among the support curves is somewhat different. Wells up to 15° inclination in this case could be drilled with zero overbalance pressure without exceeding the rock's ultimate strength. If the rock did not strain soften after attaining ultimate strength, then wells could perhaps be drilled without overbalance to even higher angles, depending on the hole closure limit chosen.

Figure 11 also illustrates that support requirements based on a hole closure limit can be either greater or less than support requirements based on the ultimate strength limit. In materials that undergo a large amount of plastic strain before reaching ultimate strength (loose, weakly consolidated sands, for example), hole closure can be more of a concern for drilling than instability.

The results from a nonlinear finite element wellbore analysis also provide valuable information for planning completion operations, by characterizing the state of the wellbore after drilling. The amount of yielding that has occurred may influence the type of completion required. The orientation of plastically-strained zones, as well as the distribution of stresses around the well, may help determine an orientation for perforations that will reduce the risk of perforation collapse. The actual stresses and elastic and plastic strains in the rock must be used as the starting point for accurate modeling of completion and post-completion events, such as prediction of weak sand failure during production. The finite element analysis can also help predict if a completion fluid that provides no overbalance pressure will result in instability.

**COMPARISON WITH LABORATORY AND FIELD OBSERVATIONS**

Addis et al. [12] modeled holes at different deviation angles, and holes at different azimuths relative to unequal horizontal stresses, in blocks of an artificial weak rock material. Although the blocks were loaded only externally to cause failure, and although the material was characterized by strain softening as well as strain hardening, the resulting shapes of the failed zones around the holes are very similar to those in Figures 4-6.

Many researchers using small-scale model holes have found that the external loads required to fail the rock next to the hole are significantly higher than predicted by a linear elastic analysis in conjunction with the Mohr-Coulomb failure criterion (for example [12-15] plus the list in [16]). Nonlinear deformation at stress levels below ultimate strength is a very real mechanism that can explain part, or all, of this apparent strength enhancement. The higher external loads required to cause failure are somewhat analogous to the lower support pressures, compared to the linear elastic solution, plotted in Figures 10 and 11. However, there are additional factors that could contribute to an apparent strength enhancement, such as a reduction in near-wellbore stress caused by non-constant or non-isotropic moduli [13,14], strengthening due to the intermediate principal stress [10,17], and strength increase with decreasing hole size [15].

Field cases are difficult to interpret because factors other than the drilling fluid density, such as chemical hydration of shales and mechanical erosion, can influence wellbore failure. However, Exxon and other operators have noted many cases where deviated and horizontal wells remained stable with fluid densities less than predicted by a linear elastic analysis [e.g. 17,18]. This is often most apparent when a balanced-activity oil-base fluid [19] is used, because this eliminates any effects on shales due to chemical hydration or to possible invasion of wellbore fluid pressure. In addition, wells that have followed properly calculated linear elastic guidelines have never been unstable, indicating that the linear elastic approach is certainly not unconservative.

Often the only indicators of wellbore failure are
CONCLUSIONS AND FUTURE WORK

Yield and closure of directional and horizontal wells have been simulated using a strain-hardening elastoplastic constitutive model in an efficient 3D finite element "slice" model. The wellbore pressure, or drilling fluid density, required to prevent failure of the wellbore wall is less than that predicted by a linear elastic analysis. The allowable safe reduction below the linear-elastically computed value cannot be predicted by a simple rule. It depends not only on the rock strength and deformation characteristics but also on the hole orientation and the in situ stress environment. In some cases wellbore closure may be as much or more of a problem for drilling than wellbore collapse.

Deformations around deviated wells are three-dimensional. The distribution of yielding and plastic straining around the hole depends strongly on wellbore deviation and in situ stresses. Predicted failure zones agree quite well with results from laboratory simulations. The distribution of yielding around the hole is just one result which has significant implications for well completions.

Nonlinear strain hardening is an important aspect of weak rock behavior that helps explain both field observations of stable wells and laboratory observations of apparent high strengths next to model holes. More advanced constitutive models are being developed to incorporate more aspects of sedimentary rock behavior. Weak shales, for example, might be best described by a critical-state type model [7,21]. Such a nonlinear model can include plastic strains due to both hydrostatic and deviatoric loading, non-constant dilation, modulus dependence on pressure, and influences of void ratio. A reliable three-dimensional constitutive model also requires careful incorporation of the intermediate principal stress.

Other important considerations for accurate wellbore stability modeling are the effects of local pore pressure changes, due either to "undrained" loading of low-permeability shales or to pressure invasion from the drilling fluid, and chemical hydration effects in shales.

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REFERENCES
