A reduced-form intensity-based model under fuzzy environments

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The external shocks and internal contagion are the important sources of default events. However, the external shocks and internal contagion effect on the company is not observed, we cannot get the accurate size of the shocks. The information of investors relative to the default process exhibits a certain fuzziness. Therefore, using randomness and fuzziness to study such problems as derivative pricing or default probability has practical needs. But the idea of fuzzifying credit risk models is little exploited, especially in a reduced-form model. This paper proposes a new default intensity model with fuzziness and presents a fuzzy default probability and default loss rate, and puts them into default debt and credit derivative pricing. Finally, the simulation analysis verifies the rationality of the model. Using fuzzy numbers and random analysis one can consider more uncertain sources in the default process of default and investors’ subjective judgment on the financial markets in a variety of fuzzy reliability so as to broaden the scope of possible credit spreads.

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1. Introduction

Default probability and default recovery (default loss rate) are the key factors of default bonds, and credit derivative pricing. Consequently, how to measure default probability and default recovery becomes a hotspot in the field of financial pricing. The existing large amount of literature can be divided into two main categories: structural models and reduced-form models. Structural models regard defaulting as an endogenous variable, which is related with the company's assets value and uses company information to set up a model on assets value and liabilities. Merton\textsuperscript{[1]} puts forward structural models firstly. His main idea is to put option pricing theory to credit derivative pricing. The deficiency of structural models is the default unpredictability. Compared with the structure models, reduced-form models are more flexible in reality. Reduced-form models put forward by Fons\textsuperscript{[2]} for the first time finds its credit depending on the issuer's credit rating changes. The reduced-form models are simple, but its result is very close to real market data. From then on, all kinds of reduced-form models appear constantly. Such as Lando\textsuperscript{[3]}, Duffle et al.\textsuperscript{[4]}, etc. Reduced-form model’s disadvantage is the lack of explanatory power to exogenous default mechanism, and demands more historical data.

Reduced-form models are divided into intensity-based models and not-intensity-based models. Intensity-based models are based on Doob Meyer's martingale decomposition theory, mainly for modeling default intensity. Its existing research achievements include Jarrow and Turnbull\textsuperscript{[5]}, Duffle and Singleton\textsuperscript{[6]}, etc. They introduce the mechanism of exogenous
default regarding default decided by the strength of random events. Lando [3] uses those models to Cox process, in which credit rating transition intensity is established by using Markov transition matrix model. In addition, Duffle et al. [4] establish jump diffusion intensity of affine model. Not-intensity-based models are set up by getting accurate data and information about the company credit, related studies may refer to Brody et al. [7]. However, in the study of credit risk and derivative pricing, intensity-based models are more commonly used. This article, based on reduced-form models, studies the probability of default and default loss rate.

As Lando [3] points out, the default intensity of company is a function of the market state variables, including interest rates, stock prices and market, credit rating, and other variables that cause corporate defaults, that is, the external shocks and internal contagion are the important sources of default events. However, the external shocks and internal contagion effect on the company is not observed, we can’t get the accurate size of the shocks. In volatile financial markets, financial data may can not be recorded timely or accurately because of unpredictable situations or human error. For example, interest rates in different commercial Banks and financial institutions may be different. Although the difference is very small, the assumed constant is unreasonable. At the same time, due to lacking cognitive of things’ inherent ambiguity, most stochastic process has certain fuzziness. That is, the uncertainty in reality contain both randomness and fuzziness. Therefore, using randomness and fuzziness to study such problems as derivative pricing or default probability has practical needs. There exists some literature about option pricing in fuzzy random environments. Simonelli [8] is the first to introduce fuzziness into option pricing, and Yoshida [9,10], Wu [11–13], Xu et al. [14] also study option pricing in fuzzy environments.

The idea of fuzzifying credit risk models is little exploited, especially in a reduced-form model. Agliardi and Agliardi [15] propose a fuzzy pattern of structural credit risk model, similar literature still can refer to Agliardi and Agliardi [16]. However, this article will puts forward a brand-new default intensity model with randomness and fuzziness, and fuzzifies the market risk-free interest rate and default recovery rate at the same time. Then it presents a fuzzy default probability and default loss rate, and puts them into default debt and credit derivative pricing. Using fuzzy numbers and random analysis one can consider more uncertain sources in the default process of default and investors’ subjective judgment on the financial markets in a variety of fuzzy reliability so as to broaden the scope of possible credit. At the same time, the fuzzy analysis alleviates the problem of the reduced-form model’s lack of explanatory power and high dependence on the historical data of defects.

The structure of this paper is as follows: In Section 2, we review some basic knowledge for the fuzzy sets and give the fuzzy pattern of the reduced-form intensity-based credit risk model. Then, in Section 3, we present some applications of the methodology for pricing defaultable bonds and CDS. In Section 4 a numerical analysis is performed to illustrate our results. Finally, Section 5 concludes.

2. The reduced-form intensity-based model under fuzzy environments

2.1. Stochastic preliminaries

We recall some basic facts about stochastic filtration method. Let \((\Omega, F, (F_t)_{0 \leq t \leq T}, P)\) be a complete probability space with filtration satisfying standard assumptions. Here, \(P\) is a given real-world probability measure and \((F_t)_{0 \leq t \leq T}\) is a càdlàg (i.e., right continuous with left limits). To describe the evolution of the state of financial environments over time, we define \((X_t)_{t \geq 0}\) to be a state process, and suppose that \((X_t)_{t \geq 0}\) is a càdlàg process on \((\Omega, F, (F_t)_{0 \leq t \leq T}, P)\) real-valued process.

We build on a reduced-form intensity-based credit risk model which was proposed by Lando [3], where the default events were described by the first jump of a Cox process with stochastic intensity process \((\lambda_t)_{t \geq 0}\) depending on an underlying state process \((X_t)_{t \geq 0}\). Let \((N_t)_{t \geq 0}\) be a Poisson process, which has a bounded, non-negative stochastic intensity process \((\lambda(X_t))_{t \geq 0}\). \((N_t)_{t \geq 0}\) and \((\lambda(X_t))_{t \geq 0}\) are independent under \(P\). Following Lando [3], we define the arrival time of default \(\tau\) as: \(\tau := \inf\{t \geq 0 : \int_0^t \lambda(X_s)ds \geq E_1\}\). Where \(E_1\) is a unit exponential random variable. Then we may express the information at time \(t\) as:

\[ G_t = \sigma\{X_s : 0 \leq s \leq t\}; \quad H_t = \sigma\{1_{\{(\tau \leq s)\}} : 0 \leq s \leq t\}; \quad F_t = G_t \lor H_t; \quad G_t \subset G_t \lor H_t. \]

Here \(1_{\{(\tau \leq s)\}}\) is an indicator function of set \(\{\tau \leq s\}\). \(F_t\) corresponds to monitoring the evolution of state variables to time \(t\) and whether default has occurred or not.

From the definition we get the following famous relationships:

\[ P(\tau > t| (F_t)_{0 \leq t \leq T}) = \exp\left(-\int_0^t \lambda(X_s)ds\right) \quad t \in [0, T]; \]

\[ P(\tau > t) = E\left[\exp\left(-\int_0^t \lambda(X_s)ds\right)\right] \quad t \in [0, T]. \]

Here, if the probability distribution of default time \(\tau\) admits a probability density function \(f(t)\), then the expected value can be computed by \(E[\tau] = \int_0^\infty t \cdot f(t)dt\).

2.2. The fuzzy pattern of the reduced-form intensity-based model

Now we remind some facts about fuzzy sets and numbers.
Definition 2.1 (Zadeh [17]). A fuzzy set $\tilde{A}$ in $\mathbb{R}$ is a set of ordered pairs $\tilde{A} = \{(x, \mu(x)) : x \in X\}$, where $\mu(x)$ is the membership function or grade of membership, or degree of compatibility or degree of truth of $x \in X$ which maps $x \in X$ on the real interval $[0, 1]$.

The $\gamma$-cut of a fuzzy set $\tilde{A}$ is defined by $\tilde{A}_{\gamma} = \{x \in \mathbb{R} : \mu(x) \geq \gamma\}$ for all $\gamma \in [0, 1]$ and $\tilde{A}_0 = cl\{x \in \mathbb{R} : \mu(x) \geq 0\}$, where $cl$ denotes the closure of a set. Now, $\tilde{A}$ is called a normal fuzzy set if there exists an $x$, such that $\mu_{\tilde{A}}(x) = 1$, and $\tilde{A}$ is called a convex fuzzy set if $\mu_{\tilde{A}}(\lambda x + (1 - \lambda)y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$ for all $\lambda \in [0, 1]$.

Definition 2.2 (Wu [11]). Let $\tilde{a}$ be a fuzzy subset of $\mathbb{R}$. Then $\tilde{a}$ is called as a fuzzy number, if the following conditions are satisfied:

(i) $\tilde{a}$ is a normal and convex fuzzy set;
(ii) Its membership function $\mu_{\tilde{a}}(x)$ is upper semi-continuous;
(iii) The $\gamma$-cut set $\tilde{a}_{\gamma}$ is bounded for each $\gamma \in [0, 1]$.

From Zadeh [17], if $\tilde{a}$ is a fuzzy number, then its $\gamma$-cut set $\tilde{a}_{\gamma}$ is a compact and convex set, that is, $\tilde{a}_{\gamma}$ is a closed interval in $\mathbb{R}$, then the $\gamma$-cut set of $\tilde{a}$ can be denoted by $\tilde{a}_{\gamma} = [\tilde{a}_{\gamma}^-, \tilde{a}_{\gamma}^+]$. Hence, we introduce a partial order ‘$>$’, the so called fuzzy max order: Let $\tilde{a}$, $\tilde{b}$ be fuzzy numbers, then $\tilde{a} > \tilde{b}$ means that $\tilde{a}_{\gamma}^- > \tilde{b}_{\gamma}^-$ and $\tilde{a}_{\gamma}^+ > \tilde{b}_{\gamma}^+$ for all $\gamma \in [0, 1]$.

A triangular fuzzy number $\tilde{a}$ with membership function $\mu_{\tilde{a}}(x)$ is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - \frac{x - x_2}{a_2 - a_1}, & \text{if } a_1 \leq x \leq a_2, \\ 1 - \frac{x - x_3}{a_3 - a_2}, & \text{if } a_2 \leq x \leq a_3, \\ 0, & \text{otherwise}. \end{cases}$$

Here $[a_1, a_2]$ is the supporting interval and the membership function has peak in $a_2$. A triangular fuzzy number $\tilde{a}$ is denoted by $\tilde{a} = (a_1, a_2, a_3)$. The $\gamma$-cut set of $\tilde{a}$ is described as $\tilde{a}_{\gamma} = [a_1, (1 - \gamma)a_2 + \gamma a_3]$. Kaufmann and Gupta [18] discussed the arithmetic of any two fuzzy numbers. Let “$+$” be a binary operator $+, - , \times, /$, between two fuzzy numbers $\tilde{a}$ and $\tilde{b}$. The membership function of $\tilde{a} + \tilde{b}$ is given by $\mu_{\tilde{a} + \tilde{b}} = \sup\{(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y))\}$.

Proposition 2.1. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers, and $\tilde{a}_{\gamma} = [a_{\gamma}^-, a_{\gamma}^+]$, $\tilde{b}_{\gamma} = [b_{\gamma}^-, b_{\gamma}^+]$. Then $\tilde{a} + \tilde{b}$, $\tilde{a} - \tilde{b}$, $\tilde{a} \times \tilde{b}$ are also the fuzzy numbers and their $\gamma$-cut sets are given by:

$$\tilde{a} + \tilde{b}_{\gamma} = [a_{\gamma}^-, b_{\gamma}^+] + [a_{\gamma}^+, b_{\gamma}^+] = [a_{\gamma}^- + a_{\gamma}^+, b_{\gamma}^- + b_{\gamma}^+],$$

$$\tilde{a} - \tilde{b}_{\gamma} = [a_{\gamma}^-, b_{\gamma}^+] - [a_{\gamma}^+, b_{\gamma}^+] = [a_{\gamma}^- - b_{\gamma}^+, a_{\gamma}^+ - b_{\gamma}^-],$$

$$\tilde{a} \times \tilde{b}_{\gamma} = \min\{[a_{\gamma}^- b_{\gamma}^-, a_{\gamma}^- b_{\gamma}^+, a_{\gamma}^+ b_{\gamma}^-, a_{\gamma}^+ b_{\gamma}^+]\} \max\{[a_{\gamma}^- b_{\gamma}^-, a_{\gamma}^- b_{\gamma}^+, a_{\gamma}^+ b_{\gamma}^-, a_{\gamma}^+ b_{\gamma}^+]\}$$

for all $\gamma \in [0, 1]$.

If the $\gamma$-cut set $\tilde{a}_{\gamma}$ does not contain zero $\forall \gamma \in [0, 1]$, then $\tilde{a}/\tilde{b}$ is also a fuzzy number and its $\gamma$-cut set is:

$$\tilde{a}/\tilde{b}_{\gamma} = \min\{[a_{\gamma}^- / b_{\gamma}^-, a_{\gamma}^- / b_{\gamma}^+, a_{\gamma}^+ / b_{\gamma}^-, a_{\gamma}^+ / b_{\gamma}^+]\} \max\{[a_{\gamma}^- / b_{\gamma}^-, a_{\gamma}^- / b_{\gamma}^+, a_{\gamma}^+ / b_{\gamma}^-, a_{\gamma}^+ / b_{\gamma}^+]\}.$$
evolution of the state of financial environments over time: they tend to shift the firm’s default probability to the right, because the published financial situation of firm is more likely to be overstated than understated as misreporting or deceiving behavior is a possibility, especially in a financial crisis. We can see that the fuzziness in the process increases as $\sigma^+$ become larger. The $\gamma$-cut of $\lambda(X_t)$ is

$$\tilde{\lambda}_{\gamma} = \left\{ x | \mu_x(x) \geq \gamma \right\} = [\lambda(X_t) - (1 - \gamma)\alpha^-(X_t), \lambda(X_t) + (1 - \gamma)\alpha^+(X_t)]$$

Meanwhile, we denote $r$ as the short-term interest rate and $\delta$ as the fractional recovery rate of defaultable bonds at default. In what follows, $r$ and $\delta$ are assumed to be fuzzy numbers. Specifically, $r(x) = \max\{1 - |x - r|/\beta, 0\}$, $\delta(x) = \max\{1 - |x - \delta|/\kappa, 0\}$, that is, the fuzzy number $\tilde{r}$ and $\tilde{\delta}$ have a symmetric triangle-type shape, respectively, with center $r$ and $\delta$. The rationale behind a fuzzy interest rate and fractional recovery rate lies in the difficulty of getting a precise estimate of the actual interest rate and recovery rate of defaultable bonds. By modeling the interest rate and recovery rate as a fuzzy number one can take into account the investors’ subjective judgement.

Following Yoshida [9] we introduce a reasonable Assumption 2.1, which is used to simplify the discussing.

**Assumption 2.1.** The stochastic process $\sigma^+(X_t)$ is specified by $\sigma^+(X_t) = c^+\lambda(X_t)$, where $0 < c^- < c^+ < 1$. The value $\beta$ is specified by $\beta = ar$, where $0 < a < 1$. The value $\kappa$ is specified by $\kappa = bc$, where $0 < \kappa < 1$.

Assumption 2.1 is reasonable since $\sigma^+(X_t)$ is related to the fuzziness of the financial environment’s volatility. Thus,

$$\tilde{\lambda}_{\gamma} = [(1 - (1 - \gamma)c^-)\lambda(X_t), (1 + (1 - \gamma)c^+)\lambda(X_t)]$$

Similarly, $F_r = [(1 - (1 - \gamma)a)r, (1 + (1 - \gamma)a)r]$, $F_\delta = [(1 - (1 - \gamma)b)\delta, (1 + (1 - \gamma)b)\delta]$.

According to the setup above, we have a novel reduced-form intensity-based model. In the fuzzy environments, the default time is defined as:

$$\tilde{\tau} = \inf\left\{ t : \int_0^t \lambda(X_s)ds > E_1 \right\}.$$

**Proposition 2.3.** For any $t \geq s > 0$, the conditional distribution of the fuzzy default time $\tilde{\tau}$ is given as: $\tilde{S}(t) = P(\tilde{\tau} > t | (F_s)_{0 \leq s < t}) = \exp\left( -\int_0^t \tilde{\lambda}(X_s)ds \right)$, $t \in [0, T]$.

The unconditional distribution of the fuzzy default time $\tilde{\tau}$ is given as:

$$P(\tilde{\tau} > t) = E\left[ \exp\left( -\int_0^t \tilde{\lambda}(X_s)ds \right) \right], \quad t \in [0, T].$$

Notation: Based on this proposition, the survival (or default) probability is going to be an interval number, rather than a constant. Here, we provide a sketch of this proposition, see Figs. 2 and 3.

3. Some applications of the model

In this section, the novel model is implemented to pricing defaultable bonds and credit default swap (CDS).
3.1. Pricing defaultable bonds

Let the face value of zero-coupon defaultable bond be 1 units and the maturity date be $T$. The interest rate and recovery rate are defined in Section 2.2. Here, the defaultable bond is a contract that pays full face value at maturity date, as long as the bond’s issuer as not default. If a default occurs during the validity period of the contract, then the recovery is paid to the bond’s holder, and the contract is ended. Taking advantage of the model, we have the following theorem.

**Theorem 3.1.** Under an equivalent martingale measure $Q$, the present fuzzy value of a defaultable bond $V_t$ is given as:

$$
\tilde{V}_t = E^Q \left[ \exp \left( - \int_t^T \tilde{r}_s ds \right) 1_{\{T > t\}} + \exp \left( - \int_t^T \tilde{r}_s ds \right) \tilde{\delta} 1_{\{T > t\}} \bigg| F_t \right]
$$

$$
= 1_{\{T > t\}} E^Q \left[ \left( \exp \left( - \int_t^T \tilde{r}_s + \tilde{\lambda}(X_s) ds \right) + \int_t^T \tilde{\delta}_s \tilde{\lambda}(X_s) \exp \left( - \int_t^s \tilde{r}_u + \tilde{\lambda}(X_u) du \right) ds \right| G_t \right]
$$

(1)

Hence, the $\gamma$-cut of $\tilde{V}_t$ is computed as follows:

$$
(\tilde{V}_t)_\gamma = \left[ \tilde{V}_{t,\gamma^-}, \tilde{V}_{t,\gamma^+} \right].
$$

Fig. 2. A sketch of the survival probability of triangular type.

Fig. 3. A sketch of the survival probability of triangular type.
\[ V_{t,y}^- = 1_{\{t \geq t^\gamma\}} \times E \left[ \exp \left( - \int_t^T (1 + (1 - \gamma)r(t) + (1 + (1 - \gamma)c)\hat{X}(u)du \right) ds \right] \]
\[ + \int_t^T (1 - (1 - \gamma)b)(1 - (1 - \gamma)c)\delta(\hat{X}(u)) \exp \left( - \int_t^u (1 + (1 - \gamma)r(t) + (1 + (1 - \gamma)c)\hat{X}(u)du \right) ds \right] G_t \]
\[ V_{t,y}^+ = 1_{\{t \geq t^\gamma\}} \times E \left[ \exp \left( - \int_t^T (1 + (1 - \gamma)r(t) + (1 - (1 - \gamma)c^-)\hat{X}(u)du \right) \right] - \int_t^T (1 + (1 - \gamma)b)(1 + (1 - \gamma)c^-)\delta(\hat{X}(u)) \exp \left( - \int_t^u (1 + (1 - \gamma)r(t) + (1 - (1 - \gamma)c^-)\hat{X}(u)du \right) ds \right] G_t \]

Proof. The proof of Eq. (1) is similar to Lando, the details can refer to [3]. Meanwhile, based on Proposition 2.1 and Assumption 2.1, as well as the monotony of the functions, we can derive the \( \gamma \)-cut of \( \hat{V}_t \) as \( \hat{V}_t = [V_{t,y}^-, V_{t,y}^+] \).

For a given fuzzy number \( \hat{A} \) we denote by \( \hat{A}^\gamma \) the lower and upper bound of its \( \gamma \)-cut set, and \( B \) has a similar assumption. Obviously, \( (\hat{A} \times B)_\gamma = (\hat{A}_\gamma \times B)_{\gamma,\gamma} \). Since the exponential function \( \exp(-f(x)) \) is a decreasing function, the \( \gamma \)-cut set of \( \exp(-f(x)) \) is given by
\[ \left( \exp \left( - \int_t^T r_s + \hat{X}(u)ds \right) \right) \gamma = \left[ \exp \left( - \int_t^T r_s + \hat{X}(u)ds \right), \exp \left( - \int_t^T r_s + \hat{X}(u)ds \right) \right] \]

Then, according to the above description and \( \hat{\tau}, \hat{r}, \hat{\delta}, \hat{\lambda} \) are all non-negative, we derive the left-end and right-end points of the \( \gamma \)-cut set of the fuzzy price \( \hat{V}_t \).

\[ \hat{V}_{t,y}^- = 1_{\{t \geq t^\gamma\}} \times E \left[ \exp \left( - \int_t^T r_s + \hat{X}(u)du \right) \right] - \int_t^T (1 + (1 - \gamma)b)(1 + (1 - \gamma)c^-)\delta(\hat{X}(u)) \exp \left( - \int_t^u (1 + (1 - \gamma)r(t) + (1 - (1 - \gamma)c^-)\hat{X}(u)du \right) ds \right] G_t \]
\[ + \int_t^T (1 + (1 - \gamma)b)(1 + (1 - \gamma)c^-)\delta(\hat{X}(u)) \exp \left( - \int_t^u (1 + (1 - \gamma)r(t) + (1 - (1 - \gamma)c^-)\hat{X}(u)du \right) ds \right] G_t \]
\[ \hat{V}_{t,y}^+ = 1_{\{t \geq t^\gamma\}} \times E \left[ \exp \left( - \int_t^T r_s + \hat{X}(u)du \right) + \int_t^T \delta(\hat{X}(u)) \exp \left( - \int_t^u r_s + \hat{X}(u)du \right) ds \right] G_t \]

So, the proof of theorem is completed. \( \square \)

3.2. CDS valuation

A credit default swap (CDS) is a financial swap agreement that the seller of the CDS will compensate the buyer in the event of a loan default or other credit events. The buyer of the CDS makes a series of payments (the CDS "fee" or "spread") to the seller and, in exchange, receives a payoff if the loan defaults. Let \( N \) be the notional amount; \( \delta(x) \) be the recovery rate; \( \hat{s} \) be the credit spreads; \( D(t, T) = \exp \left( - \int_t^T r_s ds \right) \) be the discount factor between \( t \) and \( T \); \( (T_i)_{i \in \mathbb{N}} \) be the sequence payment dates, with \( T_0 = t \).

The present fuzzy value of the expected future flows from the seller to the buyer of protection is:
\[ PV(\text{default leg}) = E \left[ (1 - \hat{s})N \exp \left( - \int_t^T r_s ds \right) 1_{\{t \leq T_i \}} | F_t \right] = (1 - \hat{s})N \int_t^T \exp \left( - \int_t^u r_s ds \right) d(1 - \hat{s}(u)) \]
\[ = (1 - \hat{s})N \int_t^T \hat{X}(u) \exp \left( - \int_t^u r_s + \hat{X}(u)ds \right) du. \]
The present fuzzy running spread gets smaller and smaller, until the believe degree is zero, which is the final result of the fuzzy running spread. Theorem 3.2.

\[ \sum_{i=1}^{n} \Delta T_i \exp \left( -\int_t^{T_i} \tilde{r} ds + \tilde{\lambda}(X_i) ds \right) + \sum_{i=1}^{n} \Delta T_i \exp \left( -\int_t^{T_i} \tilde{r} ds + \tilde{\lambda}(X_i) ds \right) du. \]

The results of the simulations may be found in Table 1.

**Theorem 3.2.** The present fuzzy running spread \( \tilde{s} \) is given as:

\[ \tilde{s} = \frac{(1 - \tilde{\delta}) \int_t^T 1 \tilde{\lambda}(X_i) \exp \left( -\int_t^{u} (1 + (1 - \gamma)a) r + (1 + (1 - \gamma)c^{-}) \tilde{\lambda}(X_i) ds \right) du, \]

The \( \gamma \)-cut of \( \tilde{s} \) is calculated as: \( \tilde{s}_\gamma = \left[ \tilde{s}_\gamma^-, \tilde{s}_\gamma^+ \right] \), where the left and right endpoints are:

\[ \tilde{s}_\gamma^- = l_1/l_2 + l_3, \quad \tilde{s}_\gamma^+ = l_4 + l_5 + l_6. \]

**4. Numerical analysis**

In this section, we present some numerical results for CDS pricing, under the fuzzy pattern of the reduced-form intensity-based model. For simplicity, we use the following triangular fuzzy numbers as parameters of the formulas in Theorem 3.2.

\[ \tilde{r} = ((1 - 0.1)0.08, 0.08, (1 + 0.1)0.08), \quad \tilde{\delta} = ((1 - 0.07)0.85, 0.85, (1 + 0.07)0.85), \]

\[ \tilde{\lambda} = ((1 - 0.1)0.3, 0.3, (1 + 0.3)0.3). \]

The results of the simulations may be found in Table 1.
range will provide investors more flexibility in making a choice. Therefore, the model can be used as a reasonable pricing tool for financial participants.

5. Conclusions

We have designed a novel fuzzy version of a reduced-form model for credit risk, which is used to price defaultable bonds and credit default swap. However, different from the known models, the credit spreads calculated by the proposed model are interval numbers instead of a constant. An appropriate price range will give investors more flexibility in making a choice, make the model results are more fit the actual market environments. Similar to the proposed model, basket credit derivative can be priced under fuzzy environments. For example, collateralized debt obligations (CDO), index default swaps etc, these are our future research content.

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Fig. 4. The dynamic between fuzzy credit spreads and subjective reliability (T = 1).
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