Position tracking control of electro-hydraulic single-rod actuator based on an extended disturbance observer

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A B S T R A C T

This paper presents a nonlinear cascade controller based on an extended disturbance observer to track desired position trajectory for electro-hydraulic single-rod actuators in the presence of both external disturbances and parameter uncertainties. The proposed extended disturbance observer accounts for external perturbations and parameter uncertainties separately. In addition, the outer position tracking loop uses sliding mode control to compensate for disturbance estimation error with desired cylinder load pressure as control output; the inner pressure control loop is designed using the backstepping technique. The stability of the overall closed-loop system is proved based on Lyapunov theory. The controller performance is verified through simulations and experiments. The results show that the proposed nonlinear cascade controller, together with the extended disturbance observer, provide excellent tracking performance in the presence of parameter uncertainties and external disturbances such as hysteresis and friction.

1. Introduction

Electro-hydraulic systems are widely used in many industrial and mobile applications, e.g., robot manipulators [1], hydraulic excavators [2], and tunnel boring machines [3] because of their high power-to-weight ratio compared with electric drives [4,5]. However, the dynamic behaviors of electro-hydraulic servo systems suffer from strong nonlinearities, such as square-root relationship between pressure and flow, temperature and pressure dependent oil properties and friction. Furthermore, industrial applications are likely to be affected by external disturbances and system parameter variations, such as the damping coefficient, the time-varying internal leakage coefficient, or supply pressure drops. These are great challenges for controller design of electro-hydraulic servo systems.

To obtain better dynamic performance, various control methods have been used. Local linearization of the nonlinear dynamics about a nominal operating condition allows the use of techniques such as pole placement [6] and adaptive control [7]. However, these controllers cannot guarantee satisfactory performance in all working points and are likely to fail if plant properties change drastically. The feedback linearization method was used in [8–10]. However, this method is based on cancelling nonlinear terms and does not account for system uncertainties. So, another control approach, sliding mode control (SMC) was applied to electro-hydraulic control systems in [11–14]. In SMC, trajectories are forced to reach a desired sliding manifold in finite time and then stay on this manifold for all future time, and dynamics on the sliding surface are independent of matched uncertainties and disturbances. However, chattering in the control signal, which is inherent in SMC, can easily excite high frequency modes and degrade system performance. Adaptive control has been proven to be a valid method to system uncertainties, therefore several nonlinear adaptive controllers were proposed for electro-hydraulic control systems in the literature. In [15,16], a nonlinear adaptive control scheme based on the backstepping method was proposed to the force control of electro-hydraulic systems. Yao et al. [17–20] proposed the nonlinear adaptive robust controller (ARC) for trajectory tracking of hydraulic actuators in the presence of uncertain nonlinearities and parameter uncertainties. A sliding mode adaptive controller was proposed in [21,22] to compensate for nonlinear uncertain parameters due to variations of the original control volumes. A novel adaptive controller based on the backstepping technique was proposed in [23].

Load disturbance or unmodeled load force would significantly degrade position tracking performance because force available to the system is diminished [24]. To obtain better tracking performance, disturbance compensation is needed. However, direct measurement of disturbances is not always possible in practice,
therefore disturbance observers for disturbance rejection is critical and several disturbance observers have been adopted to solve this problem so far. In [25,26], disturbance was estimated and compensated according to the observer proposed in [27] for position tracking. A high-pass disturbance observer was designed for position tracking of electro-hydraulic actuators in [28] and the disturbances within the observer bandwidth can be cancelled. In [29,30], a disturbance observer was used to reject low-frequency disturbances and high-frequency noises in an electro-hydraulic servo system. An integral sliding mode disturbance compensator was proposed in [31] for load pressure control of hydraulic drives.

Motivated by [20,30], an extended disturbance observer is proposed to estimate uncertain parameters and external disturbances simultaneously. Unlike the disturbance observer used in [30], the proposed extended disturbance observer deals with parameter uncertainties and external disturbances separately. Compared with [20], parameter and disturbance updates proposed in this paper are driven by the state estimation error while that proposed in [20] are driven by the tracking error, in addition, alternative disturbance observers proposed in [28,32] could be used in this control scheme. Based on the proposed extended disturbance observer, a nonlinear cascade controller is developed for position tracking of an electro-hydraulic single-rod actuator. It comprises a position tracking outer loop and a load pressure control inner loop which provides the hydraulic actuator the characteristic of a force generator. SMC is also used to compensate for disturbance estimation errors. Stability of the closed-loop system consisting of the extended disturbance observer, the nonlinear controller and the plant model is proved by means of Lyapunov theory.

To verify the performance of the proposed controller, it is applied to an electro-hydraulic test bench. The test bench, as shown in Fig. 1, consists of a single-rod hydraulic actuator as a driving cylinder and a twin-tube shock absorber as a load force generator. The damping valves are integrated into the shock absorber. For a specific speed of the shock absorber cylinder rod, fluid is displaced through the damping valves at a specific flow rate. The damping valve flow resistance produces the pressure difference across the damping orifice which generates a damping force resisting the shock absorber rod motion. Therefore, the damping force–velocity characteristic of the shock absorber is closely related to the pressure-flow characteristic of the damping valve, besides, the damping coefficients in the compression stroke and the rebound stroke are different because of asymmetric orifice configurations. When the solenoid valve in the shock absorber switches on/off, the damping orifice equivalent flow area also changes, which varies the shock absorber damping coefficients. What’s even worse is that the realistic force–velocity characteristic of a shock absorber exhibits huge hysteresis because of many factors [34–37] such as effective compliance of the damping fluid, the tube, and the entrained air; significant friction force also exists in the driving cylinder and the shock absorber. The performance of the proposed controller is verified through simulations and experiments.

The rest of the paper is organized as follows. Detailed nonlinear mathematical model is presented in Section 2. In Section 3, the extended disturbance observer and the nonlinear cascade controller are given. In Section 4, simulation results are presented. Then, experimental setup and results are discussed in Section 5. Finally, conclusions are shown in Section 6.

2. Mathematical modeling

The electro-hydraulic single-rod driving cylinder is shown in Fig. 2. The goal is to have the cylinder rod track any smooth motion trajectory as closely as possible. In the following, the nonlinear mathematical model will be derived.

The dynamics of the driving cylinder can be given by

\[ m\ddot{x}_p = P_1 A_1 - P_2 A_2 + F_i \]  

(1)

where \( m \) is the mass of the load, \( x_p \) is the displacement of the cylinder rod, \( P_1 \) and \( P_2 \) are the pressures in the cylinder forward and return chamber, respectively, \( A_1 \) and \( A_2 \) are the ram areas of the forward and return chamber, respectively, \( F_i \) is the total load force of the driving cylinder.

Neglecting the external leakage flow, the pressure dynamics of the two actuator chambers can be written as [4]

\[ \frac{V_{01} + A_1 x_p}{\beta_1} \dot{P}_1 = Q_1 - A_1 \dot{x}_p - C_t (P_1 - P_2) \]  

(2)

\[ \frac{V_{02} - A_2 x_p}{\beta_2} \dot{P}_2 = -Q_2 + A_2 \dot{x}_p + C_t (P_1 - P_2) \]  

(3)

Define function

\[ s_g(\bullet) = \begin{cases} 1, & \text{if } \bullet \geq 0 \\ 0, & \text{if } \bullet < 0 \end{cases} \]  

(4)

where \( k_i \) is the servo valve flow gain coefficient, \( x_s \) is the servo valve spool displacement, \( P_i \) is the pump supply pressure, and \( P_t \) is the tank pressure.

Because the desired closed loop dynamics is significantly slower than the servo valve dynamics, the servo valve dynamics can be neglected without a significant reduction on the model accuracy. For simplicity, we use the following approximation:

\[ x_s = k_i u \]  

(5)

where \( k_i \) is a positive constant and \( u \) is the control input voltage. Hence, combining (3)–(5), we get:
$Q_1 = k_1 x u [g(x) \sqrt{P_1 - P_1} + s_1(-u) \sqrt{P_1 - P_1}]$

$Q_2 = k_2 x u [g_2(x) \sqrt{P_2 - P_1} + s_2(-u) \sqrt{P_1 - P_2}]$. 

Load force modeling accuracy is a key issue for accurate position tracking. There are two ways to model shock absorbers: first-principle dynamic modeling based on internal structures and nonparametric modeling based on experimental data. The physical model can capture shock absorber behaviors in a wide range of operating conditions very accurately; however, it is usually computationally complex and is not suitable for control-oriented applications. In contrast to physical models, nonparametric model is computationally efficient and is able to capture shock absorber dynamic behaviors for the tested operating conditions [38]. Therefore, nonparametric model is used in this paper to model the shock absorber damping force. Unlike the shock absorbers commonly used in vehicle suspensions [33], the damping orifice flow area of the shock absorber shown in Fig. 1 is independent of the shock absorber piston rod and the cylinder tube. A second order polynomial model is used, as shown in (7)

$$F_i = b_1 x_2 x_1^2 - b_2 x_2(-x_2) x_1^2 + d$$

where $b_1$ and $b_2$ are the damping coefficients during the forward and return strokes, respectively, $d$ is a lumped disturbance due to hysteretic force–velocity characteristics of the shock absorber, frictional forces, and other unmodeled external disturbances. Using experimental data, the damping coefficients are developed using least squares regression. The force–velocity plot of the considered shock absorber when solenoid valve switches on/off is shown in Fig. 3. The close match between the mathematical model and the experimental data demonstrates the effectiveness of the model.

Define the state variables as $x = [x_1, x_2, x_3, x_4]^T = [x_0, x_1, P_1, P_2]^T$. The entire system can be expressed in a state space form as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m} \left[ A_1 x_1 - A_2 x_3 - b_1 x_2 x_1^2 + b_2 x_2(-x_2) x_1^2 - d \right]$$

$$\dot{x}_3 = h_1(x_1) |A_1 x_1 - C_1(x_3 - x_4) + k_1 x u g(x _1, u)|$$

$$\dot{x}_4 = h_2(x_1) |A_2 x_2 + C_1(x_3 - x_4) - k_2 x u g_2(x_4, u)|$$

where

$$h_1(x_1) = \frac{b_1}{V_{x1} + A_1 x_1}$$

$$h_2(x_1) = \frac{b_2}{V_{x2} - A_2 x_1}$$

$$g_1(x_1, u) = s_1(u) \sqrt{P_1 - x_1} + s_2(-u) \sqrt{x_1 - P_2}$$

$$g_2(x_4, u) = s_3(u) \sqrt{P_2 - x_4} + s_4(-u) \sqrt{P_1 - x_4}$$

The control task can now be summarized as follows: Given the desired motion trajectory $x_i(t)$, the objective is to synthesize a bounded control input $u$ such that the output $x_i$ tracks $x_d(t)$ as closely as possible in spite of various model uncertainties and disturbances. For a practical electro-hydraulic servo system, the following assumption is made.

Assumption 1. The desired trajectory $x_d(t)$, its velocity $x_v(t)$ and acceleration $x_a(t)$ and $x_{ad}(t)$ are all bounded; under normal working conditions, $P_1$ and $P_2$ are bounded by $P_i$ and $P_t$, i.e., $0 < P_i < P_t < P_i$, $0 < P_t < P_t < P_t$.

3. Controller design

3.1. Design model and issues to be addressed

To make system dynamics (8) fall into the well-known strict feedback form to use the backstepping method, a new state variable is defined as $x = x_3 - x_4$, where $x = A_2 x_1 - A_1 x_1$ denotes the piston area ratio; this variable corresponds to the driving force of the cylinder. In addition, following the coordinates change proposed in [39], a new variable representing the sum pressure, $x_4 = x_3 + x_4$, is used. This new variable reflects the internal dynamics of the system, which arises from the physical phenomenon that there are more than one pair of $(P_1, P_2)$ which can produce the desired driving force [20,39]. Therefore, the stability of the internal dynamics is needed and it is shown in the simulation and experimental results.

Thus system dynamics (8) can be rewritten as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m} \left[ A_1 x_1 - b_1 x_2 x_1^2 + b_2 x_2(-x_2) x_1^2 - d \right]$$

$$\dot{x}_3 = -f_1 x_2 - f_2 C_1(x_3 - x_4) + f_3 u$$

$$\dot{x}_4 = -f_4 x_2 - f_5 C_1(x_3 - x_4) + f_6 u$$

where

$$f_1(x_1) = h_1(x_1) A_1 + 2 h_2(x_1) A_2$$

$$f_2(x_1) = h_1(x_1) + 2 h_2(x_1)$$

$$f_3(x_1, x_3, x_4, u) = k_1 [k_1 h_1(x_1) g_1(x, u, x_1) + 2 k_2 h_2(x_1) g_2(x, u, x_1)]$$

$$f_4(x_1) = h_1(x_1) A_1 + h_2(x_1) A_2$$

$$f_5(x_1) = h_1(x_1) - h_2(x_1)$$

$$f_6(x_1, x_3, x_4, u) = k_3 [k_1 h_1(x_1) g_1(x, u, x_1) - k_2 h_2(x_1) g_2(x, u, x_1)]$$

For the system considered here, load damping coefficients $b_1$ and $b_2$ vary with oil temperature and pressure, they also change when solenoid valve switches on/off, and therefore the two parameters are uncertain. Besides, $d$ is a lumped disturbance due to hysteretic force–velocity characteristic of the shock absorber, frictional forces, and other unmodeled external disturbances.

Assumption 2. The unknown disturbance $d$ is bounded within a known limit, i.e., $d_{min} \leq d \leq d_{max}$, where $d_{min}$ and $d_{max}$ are the known lower and upper bounds of $d$.
3.2. Extended disturbance observer

In previous studies [24–31], estimated disturbance included not only external load disturbances but also perturbations arising from parameter uncertainties. If the dynamics of disturbances caused by parameter uncertainties exceed the disturbance observer bandwidth, excessive disturbance estimation error will occur which deteriorate control performance. However, if partial information about the load force is known, i.e., the structure of the load force is known, but parameter uncertainties exist in the model, extended disturbance observer which combines disturbance observer and parameter adaptation is a better choice. For this reason, an extended disturbance observer will be presented where parameter uncertainties will be dealt with while estimating disturbance.

The extended disturbance observer is based on an extended state observer of \( x_2 \) although this quantity is available by measurement, by doing this, additional degrees-of-freedom for the controller design is provided, in addition, actuator acceleration signal is not needed. It is given by

\[
\dot{x}_{2p} = \frac{1}{m} [A_1 x_3 - b_1 s_2 (x_2) x_2^2 + b_2 s_1 (-x_3) x_2^2 - \ddot{d} + w_1 \dot{x}_{2p}] \\
\ddot{F}_l = b_1 s_2 (x_2) x_2^2 - b_2 s_1 (-x_3) x_2^2 + \ddot{d}
\]

where \( x_{2p}, \ b_1, \ b_2, \ \ddot{d}, \) and \( F_l \) are the estimations of \( x_2, \ b_1, \ b_2, \ \ddot{d}, \) and \( F_l, \ \dot{x}_{2p} \) is the estimation error of \( x_2 \) and is defined as \( \dot{x}_{2p} = x_2 - x_{2p}, \) and \( w_1 \) is a positive feedback gain. Introducing estimation errors \( \dot{b}_1 = \ddot{b}_1, \ \dot{b}_2 = b_2 - b_{2p}, \ \ddot{d} = \ddot{d} - \ddot{d}_{2p}, \) the dynamics of \( \dot{x}_{2p} \) can be obtained as follows:

\[
\dot{x}_{2p} = \frac{1}{m} [b_1 s_2 (x_2) x_2^2 - b_2 s_1 (-x_3) x_2^2 + \ddot{d} + w_1 \dot{x}_{2p}] \\
\ddot{F}_l = b_1 s_2 (x_2) x_2^2 - b_2 s_1 (-x_3) x_2^2 + \ddot{d}
\]

The adaptation law is chosen as

\[
\dot{b}_1 = -k_{11} \frac{1}{m} s_2 (x_2) x_2^2 + \dot{x}_{3b} \\
\dot{b}_2 = k_{12} \frac{1}{m} s_2 (x_2) x_2^2 + \dot{x}_{3b} \\
\dot{\ddot{d}} = \text{Proj}_d (-k_{13} \frac{1}{m} \dot{x}_{2p})
\]

where \( k_{11}, \ k_{12}, \ k_{13} \) are positive constants, \( \dot{x}_{3b}, \ \dot{x}_{3b} \) are extra corrector terms designed to ensure the closed-loop system stability and will be introduced later, the projection mapping \( \text{Proj}_d(\bullet) \) is defined in [20] as

\[
\text{Proj}_d(\bullet) = \begin{cases} 
0, & \text{if } \ddot{d} = \ddot{d}_{\text{max}} \text{ and } \bullet > 0 \\
0, & \text{if } \ddot{d} = \ddot{d}_{\text{min}} \text{ and } \bullet < 0 \\
\bullet, & \text{otherwise}
\end{cases}
\]

It can be shown that by using (15), we can ensure that

\[
(P1) \ d_{\text{min}} \leq \ddot{d} \leq d_{\text{max}} \\
(P2) \ -\frac{1}{m} \dot{x}_{2p} - k_{13} \ddot{d} \leq 0
\]

**Proof.** From (15), we can see that whenever \( \ddot{d} \) reaches its lower and upper limit, the projection guarantees that \( \ddot{d} \) never exceeds the known limit, thus (P1) of (16) is proved.

In addition, if \( \ddot{d} = \ddot{d}_{\text{max}} \) and \(-k_{13} \frac{1}{m} \dot{x}_{2p} > 0\), we can get \( \dot{d} = 0 \) and \( \ddot{d} = \ddot{d} - \ddot{d}_{\text{max}} \leq 0, \) thus, \( \ddot{d} - \frac{1}{m} \dot{x}_{2p} - k_{13} \ddot{d} \leq 0 \). If \( \ddot{d} = \ddot{d}_{\text{min}} \) and \(-k_{13} \frac{1}{m} \dot{x}_{2p} < 0\), we can get \( \dot{d} = 0 \) and \( \ddot{d} - \ddot{d} = \ddot{d} - \ddot{d}_{\text{min}} \geq 0, \) so we can get \( \ddot{d} - \frac{1}{m} \dot{x}_{2p} - k_{13} \ddot{d} \leq 0 \). In another case, we have \( \ddot{d} = -k_{13} \frac{1}{m} \dot{x}_{2p}, \) so \( \ddot{d} - \frac{1}{m} \dot{x}_{2p} - k_{13} \ddot{d} = 0 \). Therefore, we can always get \( \ddot{d} - \frac{1}{m} \dot{x}_{2p} - k_{13} \ddot{d} \leq 0 \). Hence, (16) is proved.

Define the following Lyapunov function

\[
V_1 = \frac{1}{2} x_{2p}^2 + \frac{1}{2} k_{13}^{-1} s_1^2 + \frac{1}{2} k_{12}^{-1} s_2^2 + \frac{1}{2} k_{11}^{-1} d^2.
\]

Taking into account (13) and (14), the time derivative of \( V_1 \) is given by

\[
\dot{V}_1 = -\frac{1}{m} w_1 x_{2p}^2 + d \left( -\frac{1}{m} \dot{x}_{2p} - k_{13} \ddot{d} \right) - k_{11} b_{13} x_{3b} - k_{12} b_{22} z_{2b}.
\]

**Remark 1.** In this step, the disturbance \( d \) is assumed to be a constant, however, it is indeed time-varying. Although the case of time-varying disturbance \( d \) is not covered by the stability proof, the following simulation and experimental results show that the proposed controller is also reliable in this situation.

3.3. Nonlinear cascade tracking control

The cascade nonlinear controller comprised of a position tracking outer loop and a load pressure control inner loop which provides the hydraulic actuator the characteristic of a force generator, besides, SMC is adopted to compensate for disturbance estimation errors [40]. By using recursive backstepping design procedure, the controller is shown as follows.

**Step 1:** The aim of this step is to design a desired load force for outer position tracking loop, and this controller must be robust to disturbance estimation errors. Define the tracking error as \( \tilde{x}_1 = x_1 - x_{\text{opt}} \), then a sliding surface \( s \) is defined in the form

\[
s = \tilde{x}_1 + \lambda \tilde{x}_1
\]

where \( \lambda \) is positive constant. Because making \( \tilde{x}_1 \) small or converging to zero is equivalent to making \( s \) small or converging to zero, the rest of the work is to make \( s \) as small as possible. The time derivative of \( s \) along the system (10) is obtained as

\[
\dot{s} = \frac{1}{m} [A_1 \dot{x}_3 - b_1 s_2 (x_2) x_2^2 + b_2 s_1 (-x_3) x_2^2 - d] - \ddot{x}_d + \lambda (x_2 - x_d).
\]

A virtual controller \( \tilde{s} \) for \( x_3 \) is designed as

\[
\tilde{s} = \frac{1}{A_1} \left[b_1 s_2 (x_2) x_2^2 - b_2 s_1 (-x_3) x_2^2 + d + m(x_2 - x_\text{desired}) - c_1 s - c_2 sgn(s)\right]
\]

where \( c_1 \) and \( c_2 \) are both positive constants. Note that the term \(-c_2 sgn(s)\) is designed to compensate for disturbance estimation error and \( c_2 \) is chosen to satisfy

\[
c_2 > |d|/m.
\]

Let \( \tilde{z}_3 = x_3 - \tilde{x}_3 \) denote the tracking error of inner load pressure control loop, we can get

\[
\tilde{z}_3 = \frac{1}{A_1} [b_1 s_2 (x_2) x_2^2 - b_2 s_1 (-x_3) x_2^2 + d + m(x_2 - x_\text{desired}) - c_1 s - c_2 sgn(s)]
\]

\[
- \frac{1}{m} \dot{d}.
\]

Define the Lyapunov function as

\[
V_2 = V_1 + \frac{1}{2} k_2^{-1} s^2
\]
where \( k_2 \) is a positive constant. Combining (18) and (23), the time derivative of \( V_2 \) is given by

\[
V_2 = -\frac{1}{m}w_i x_2^2 + \dot{\theta} \left( -\frac{1}{m} x_{2p} - k_{13} \dot{d} \right) - k_{11} b_1 x_{2b} - k_{12} b_2 x_{2s} + k_2^2 \left( -c_2 |s| - \frac{1}{m} \ddot{x}_2 - k_{13} \ddot{d} \right) + k_2^2 \left( -c_2 |s| - \frac{1}{m} \ddot{x}_2 \right).
\]

Then the extra corrector terms \( \lambda_{d1}, \lambda_{d2} \) are chosen as

\[
\lambda_{d1} = -k_{11} k_2^2 \frac{1}{m} \ddot{x}_2 \dddot{x}_2,
\]

\[
\lambda_{d2} = k_{12} k_2^2 \frac{1}{m} \ddot{x}_2. \tag{26}
\]

Substituting (26) into (25) yields

\[
V_2 = -\frac{1}{m} w_i x_2^2 - c_2 k_2^2 |s|^2 + \ddot{\lambda} - \frac{1}{m} \ddot{x}_{2p} - k_{13} \ddot{d} + k_2^2 \left( -c_2 |s| - \frac{1}{m} \ddot{x}_2 \right) + \frac{1}{m} \ddot{s}_2. \tag{27}
\]

**Step 2:** In step 1, we have designed a virtual control law \( \lambda_2 \), which is the command input of inner load pressure control loop. In this step, an actual control law for \( u \) is determined. The time derivative of \( \lambda_2 \) is given by

\[
\dot{\lambda}_2 = -f_3 x_2 - f_2 C_3 (x_3 - x_4) + f_3 u - \dot{x}_3. \tag{28}
\]

The actual control \( u \) is designed as

\[
u = \frac{1}{f_3} \left[ f_1 x_2 + f_2 C_2 (x_3 - x_4) + \dot{x}_3 - c_3 z_3 - k_{11} \frac{A_1}{m} \right]. \tag{29}
\]

where \( c_3 \) and \( k_{11} \) are positive constants. Substituting (29) into (28) yields the dynamics of \( \dot{z}_3 \)

\[
\dot{z}_3 = -c_3 z_3 - k_{11} \frac{A_1}{m} z_3. \tag{30}
\]

Define the Lyapunov function as

\[
V = V_2 + \frac{1}{2} k_{31} x_3^2 \tag{31}
\]

where \( k_{31} \) is a positive constant, combining (27) and (30), the time derivative of \( V \) is given by

\[
\dot{V} = -\frac{1}{m} w_i x_2^2 - c_2 k_{21}^2 s^2 - c_2 k_{31} x_2^2 + \ddot{\lambda} - \frac{1}{m} \ddot{x}_{2p} - k_{13} \ddot{d} + k_2^2 \left( -c_2 |s| - \frac{1}{m} \ddot{x}_2 \right) + \frac{1}{m} \ddot{s}_2. \tag{32}
\]

From (16), we can get \( \ddot{d} - \frac{1}{m} \ddot{x}_{2p} - k_{13} \ddot{d} \leq 0 \). In addition, from (22), we can also obtain \( k_2^2 \left( -c_2 |s| - \frac{1}{m} \ddot{x}_2 \right) \leq 0 \), therefore we get

\[
\dot{V} \leq -\frac{1}{m} w_i x_2^2 - c_2 k_2^2 s^2 - c_2 k_{31} x_2^2 < 0. \tag{33}
\]

Hence, the stability of the closed-loop system consisting of the nonlinear controller and the extended disturbance observer is guaranteed and all system signals are bounded under closed-loop operation.

**Remark 3.** Note that (29) includes the control input \( u \) on both sides of the equation, so (29) cannot be calculated directly [28]. However, the control input \( u \) on the right side is only used for \( s_2 \) function in \( f_5 \), since \( f_3 \) is always greater than zero, the sign of \( u \) is determined by \( f_1 x_2 + f_2 C_2 (x_3 - x_4) + \dot{x}_3 - c_3 z_3 - k_{11} \frac{A_1}{m} \). Therefore, we use the modified control law

\[
u = \frac{u_0}{f_3 (x_3 - x_4)} \tag{34}
\]

\[
u_0 = f_1 x_2 + f_2 C_2 (x_3 - x_4) + \dot{x}_3 - c_3 z_3 - k_{11} \frac{A_1}{m} z_3.
\]

**Fig. 4.** Block diagram of the controller structure.

**Fig. 5.** Force–velocity dynamics of shock absorber.

### Table 1 Parameters of the test rig.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>Load mass</td>
<td>200 kg</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>Driving cylinder piston side chamber area</td>
<td>5.027 x 10⁻³ m²</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>Driving cylinder rod side chamber area</td>
<td>2.564 x 10⁻³ m²</td>
</tr>
<tr>
<td>( P_r )</td>
<td>Pump supply pressure</td>
<td>2.5 x 10⁵ Pa</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>Driving cylinder leakage coefficient</td>
<td>0 m³/s Pa</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>Oil bulk modulus</td>
<td>3 x 10⁸ Pa</td>
</tr>
<tr>
<td>( k_{o1} \times k_o )</td>
<td>Servo valve flow gain coefficient</td>
<td>8.91 x 10⁻⁹ m³/s V√Pa</td>
</tr>
<tr>
<td>( k_{o2} \times k_o )</td>
<td>Servo valve flow gain coefficient</td>
<td>8.91 x 10⁻⁹ m³/s V√Pa</td>
</tr>
</tbody>
</table>
chosen as least-squares approximations of the shock absorber force–velocity characteristics when solenoid valve was switched off. The friction force, \(2500\text{sgn}(x^2)\) and the desired profile shown in Fig. 6, \(x_d = 0.175 + 0.125 \sin (0.2\pi t + 1.5\pi)\), were used.

Three controllers were used and compared; among which the first was the proposed extended disturbance observer based nonlinear cascade controller (EDOBNC), and the second was a disturbance observer based controller (DOBNC) without using parameter adaptation which treated external disturbances and parameter uncertainties as lumped perturbations, i.e., \(k_{11} = 0, k_{12} = 0\). The third controller (NC) performed position tracking without considering parameter variations and external disturbances, i.e., \(k_{11} = 0, k_{12} = 0, k_{13} = 0\).

The three controllers were first tested when solenoid valve in shock absorber was switched off. Fig. 7 shows the tracking errors, and the parameter estimations of EDOBNC are shown in Fig. 8. It shows that the proposed EDOBNC controller and the DOBNC controller performed better than the NC controller; it is because the EDOBNC and the DOBNC employ disturbance observation laws to compensate for unmodeled uncertainties such as hysteresis and friction in hydraulic systems, while the NC controller just has some robustness with respect to uncertainties. Furthermore, because the initial parameter estimates of \(b_1\) and \(b_2\) were the least-squares fits of the shock absorber force–velocity dynamics, tracking performance of the EDOBNC and the DOBNC are similar which shows the effectiveness of disturbance estimation when parameter uncertainties are comparatively small.

To test the influence of variations of parameters \(b_1\) and \(b_2\) on the control performance, the solenoid valve in the shock absorber was switched on which increased the equivalent orifice diameter and increased the damping coefficients \(b_1\) and \(b_2\) almost twofold, besides, the hysteresis coming from shock absorber force–velocity characteristics was bigger as shown in Fig. 5. The tracking performance is shown in Fig. 9 and the parameter estimations of the EDOBNC are shown in Fig. 10. As seen, the tracking error improves with the adaptation of the parameters during the first two cycles; even in the face of dramatic variations in damping coefficients \(b_1\) and \(b_2\), the EDOBNC could still attenuate the unexpected effects and achieve better performance than the DOBNC and the NC which illustrates the effectiveness of the proposed EDOBNC controller. The relatively large tracking error during the transition from positive to negative speed was mainly due to friction effects which can be tackled by incorporating detailed mathematical load force models. It can also be seen that the estimated parameters do not converge to constant values, however, they are bounded and approach stable limit cycles; such effects may be due to friction force, hysteresis and other model uncertainties. The control input \(u\) is shown in Fig. 11.

5. Experimental results

The experimental installation is presented in Fig. 12; a shock absorber was used as a load force generator. In the test bench, the driving cylinder of which the dimensions were 80 mm/56 mm/700 mm was controlled by a servo valve (Rexroth 4WRPH10C3B100L), its bandwidth was above 80 Hz with a 10% control signal. The system states used in the controller, including cylinder displacement, the pressures in the two cylinder chambers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_1)</td>
<td>5000</td>
<td>(c_1)</td>
<td>10</td>
</tr>
<tr>
<td>(k_{11})</td>
<td>(4 \times 10^{11})</td>
<td>(c_3)</td>
<td>160</td>
</tr>
<tr>
<td>(k_{12})</td>
<td>(1.25 \times 10^{11})</td>
<td>(\dot{c})</td>
<td>0.05</td>
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<tr>
<td>(k_{13})</td>
<td>(6 \times 10^6)</td>
<td>(\dot{\lambda})</td>
<td>320</td>
</tr>
<tr>
<td>(k_2)</td>
<td>10</td>
<td>(b_1(0))</td>
<td>(3.54 \times 10^6)</td>
</tr>
<tr>
<td>(k_{31})</td>
<td>1</td>
<td>(b_2(0))</td>
<td>(2.45 \times 10^6)</td>
</tr>
<tr>
<td>(c_1)</td>
<td>380</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6. Desired profile.
and \( P_2 \), supply pressure \( P_s \) were directly measured by transducers, and the cylinder velocity was obtained by backward difference of the position signal. All the measurement signals were fed back to the controller through 16 bit A/D and 16 bit D/A converters. The controller sampling time was 1 ms.

For the verification of the proposed controller, it was compared with the proportional-integral (PI) controller with velocity feedforward controller which is commonly used in industrial applications [28,34], it is given as

\[
 u = 600(x_d - x_1) + 30 \int_0^t (x_d - x_1)dt + 12x_d.
\]  

Fig. 14. Parameter estimation of EDOBNC with solenoid valve switched off. (a) \( b_1 \), (b) \( b_2 \), (c) \( d \).

The PI controller parameters were well tuned for position tracking performance of the desired sinusoidal signal.
Fig. 13 shows tracking errors of the three controllers when the solenoid valve in the shock absorber was switched off. As seen, the EDOBNC performed better than the PI controller in terms of tracking error. This indicates that the extended disturbance observer can effectively attenuate parameter variations and external disturbances. Due to accurate estimate of damping coefficients, the tracking performances of the EDOBNC and the DOBNC were similar to each other. Parameter estimations of the EDOBNC are shown in Fig. 14. In addition, big deviations exist between the simulation and the experimental results; this is mainly due to inaccurate load force in the simulation environment.

The pressure $P_1$ and $P_2$ of the EDOBNC are shown in Fig. 15, and the control input $u$ is shown in Fig. 16. It can be seen that they are all bounded as assumed. The cylinder driving force and the estimated load force are shown in Fig. 17. The driving force means $(P_1 - 2P_2) A_1$, and the estimated load force includes the shock absorber damping force, friction force, and other external disturbances. It can be seen that a large jump exists at velocity reversal which was mainly caused by friction. Because the estimated load force was similar to cylinder driving force, it can be obtained that the load force was well estimated.

Fig. 18 shows the tracking performance of the three controllers when solenoid valve in shock absorber was switched on. The tracking errors of the PI controller and the DOBNC were greater than that of the case when solenoid valve was switched off, whereas the tracking performance of EDOBNC remained almost unchanged. The parameter estimations of the EDOBNC are shown in Fig. 19. The cylinder driving force and estimated load force are shown in Fig. 20. It can be seen that the estimated parameters do not converge to constant values, which is the same as the simulation results. The tracking error improvement is reached within the first cycle and the remaining parameter updates have little effect on the tracking error because of the almost unchanged estimated load force during each cycle. In addition, despite twofold increase in the shock absorber damping force compared with the case when solenoid valve was switched off, the load force was also well estimated.

The three controllers were then run for a fast motion trajectory given by $x_0 = 0.175 + 0.125 \sin (0.31 \pi t + 1.5\pi)$. The tracking performance is shown in Fig. 21. The PI controller and the DOBNC controller exhibited large tracking errors under such an aggressive...
movement. In contrast, the tracking error of the EDOBNC was smaller, which shows the effectiveness of the proposed EDOBNC control strategy.

In the end, the proposed control scheme was compared with two alternative control solutions: ARC control as described in [20] and observer-based cascade control (OBCC) as described in [30]. The trajectory shown in Fig. 6 was used and the solenoid valve in the shock absorber was switched on. Fig. 22 shows the tracking performance. The largest tracking errors occur with the OBCC as a result of parameter uncertainties, whereas the ARC leads to similar tracking performance compared with the proposed control scheme.

6. Conclusion

In this paper, a nonlinear cascade trajectory tracking controller based on an extended disturbance observer was proposed for an electro-hydraulic system driven by a single-rod actuator. The inner control loop involved a load pressure control, and the outer loop achieved position tracking using sliding mode method. In addition, external disturbances and parameter uncertainties were taken into account through the extended disturbance observer. This novel control scheme paralleled the backstepping method and its stability was proved through Lyapunov method. The effectiveness of the control scheme was verified by the simulation and experimental results.

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References


