Characteristic Impedance of Rectangular Coaxial Transmission Lines

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EXACT mathematical relations between the characteristic impedance and the physical dimensions of circular concentric and eccentric coaxial lines, as well as of coaxial lines having square outer and circular inner conductors have been developed and experimentally verified. Messinger and Higgins were able to develop formulas for the determination of the inductance of coaxial busses composed of square tubular conductors, at power frequencies. Their calculations were based on finding the self and mutual geometric mean distances of the conductor cross sections.

An exact mathematical treatment of coaxial lines composed of rectangular inner and outer conductors is exceedingly difficult, if not impossible, because of the discontinuities at the conductor corners.

The purpose of this research work was to determine experimentally the characteristic impedance of several coaxial rectangular transmission lines, and to establish from the results thus obtained an empirical relation between the line dimensions and its characteristic impedance.

The standing-wave measurement method was found to be most suited for this work. If a line of unknown characteristic impedance is terminated by a dissipative line in free space, the inductance and capacitance of such a line are constant, and that the phase velocity is equal to the velocity of light. Based on these statements, the inductance and capacitance per unit length of a transmission line can be computed from a knowledge of the characteristic impedance and vice versa.

CIRCULAR COAXIAL LINES

The characteristic impedance of circular coaxial lines is given by

$$Z_0 = (L/C)^{1/2}$$

(1)

where $Z_0$ = line characteristic impedance in ohms
$L$ = series inductance per-unit length of the line in henrys
$C$ = shunt capacitance per unit length of the line in farads
$f$ = frequency
$A$ = inside radius of the outer conductor
$A$ = wave length

It is a property of the electromagnetic field that, for a dissipationless line in free space, $L$ and $C$ are so related that their product is constant, and that the phase velocity is equal to the velocity of light. Based on these statements, the inductance and capacitance per unit length of a transmission line can be computed from a knowledge of the characteristic impedance and vice versa.

CIRCULAR CONDUCTORS IN SQUARE TROUGHS

Sidney Frankel, employing the method of conformal transformation and the method of images, was able to deduce a formula for the characteristic impedance of concentric lines having circular inner conductors and square outer ones:

$$Z_0 = 138 \log A/a \text{ ohms}$$

(4)

where $A$ is the radius of the side of the outer conductor, and $a$ is the radius of the inside circular conductor. Curve III of Figure 1 shows a plot of this equation.

CIRCULAR CONFOCAL ELLIPTIC TRANSMISSION LINES

The problem of the capacitance of the confocal elliptical capacitor has been solved, and the capacitance per unit length of such a capacitor is known. Using relations 1 and 2 one can find the characteristic impedance of such configuration to be:

$$Z_0 = 138 \log A/a + \frac{1}{1 + \left(1 - (d/a)^2\right)^{1/2}}$$

(5)

where

$A$ = outside radius of the inner conductor
$A = $inside radius of the outer conductor

Curve I of Figure 1 shows the graphical representation of equation 3.

SQUARE COAXIAL LINES

Messinger and Higgins have derived a formula for the a-c inductance of square tubular lines at frequencies up to several hundred cycles. Figure 2 shows a plot of the equation derived by these authors. This equation was based on the assumption of infinitely thin conductors, and thus it can be applied to very high frequencies where the field is confined between the outer surface of the inner conductor and the inner surface of the outer conductor. The characteristic impedance of such square coaxial lines, shown in curve II of Figure 1, was computed using relations 1 and 2 for values of $L$ taken from Figure 2.

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The characteristic impedance of circular, square, square outer - circular, inner and elliptic coaxial lines can be computed, and these lines are easy to build.

The Design of Circular Coaxial Slotted Transmission Line

A circular coaxial line was built especially for this experimental work. The line itself is 10 feet long and is composed of a copper inner conductor of \( \frac{3}{8} \)-inch outside diameter, and a copper outer conductor having \( \frac{1}{2} \)-inch inside diameter. A 3-foot section of this line was slotted to accept a traveling probe for the measurement of the voltage standing wave ratio. The carriage, which accommodates the probe, is fitted with a gear and pinion, and moves smoothly along two parallel lengths of gear rack, one on each side of the slot. The two conductors, of which the line is composed, were made concentric by using thin circular washers made of Polystyrene and spaced about 1 foot apart.

The Design of Rectangular Coaxial Slotted Transmission Line

Since the problem involves the measurement of the characteristic impedance of several rectangular lines, it was more convenient and economical to construct a built-up line, that is, the outer conductor...
Figure 2. Messenger and Higgins formulas for inductance of square coaxial busses

Figure 3. A built-up rectangular line

of which was built of four flat brass bars joined together to give a hollow rectangular section of the required dimensions. Figure 3 shows a detailed drawing of this line. The inner conductor could be made circular or rectangular, and the outer conductor had upper and lower plates, having a cross section 2 1/2 inches by 1/8 inch. The line was about 9 3/4 feet long. A slot 1/8-inch wide and 3 feet long was cut in the top plate to accommodate a traveling probe. The two vertical sides of the outer conductor were made of flat brass bars 1/4-inch thick and of different heights.

The width of the outer conductor could be changed from about 3/4-inch to about 2 1/4 inches, and the height from about 1/2 inch to any value required. Beside changing the dimensions of the outer conductor, the inner conductor could be changed to any rectangular section, a circular rod, square bar, or any other configuration.

The four plates composing the outer conductor were held together by C-clamps spaced about 4 inches apart to insure good electrical contact. The clamps were applied in such a way that clamping pressure was principally on the vertical sides, and as close as possible to the inner face of these sides. In the slotted part of the line, since only a few clamps could be...
used without interference with the motion of the probe, small brass rivets were used. These were spaced about 4 inches on centers on both sides of the slot. Evidence will be presented later to show that perfect contact is not necessary for this type of work.

The inner and outer conductors of this line were made coaxial, along its entire length, by means of 1/16-inch thick Poly- 
styrene spacers arranged as shown in Figure 3 (C) and (D). The inside and outside dimensions of these spacers corresponded to those of the inner and outer conductors of the line. A different set of spacers was required for every line. Ten spacers, spaced 1 foot apart, were found to be sufficient to insure rigidity of the line. Instead of machining rectangular holes in the spacers to fit the inner conductor, which is a rather difficult and time consuming operation, two spacers, at the point of support, are used as shown in Figure 3(B), (C), and (D). In the slotted part of the line, only spacers of shape A, Figure 3(C), were used to avoid interference with the probe motion.

Figure 4 shows a picture of this type of line together with the tapered section which was used as an electrical shield and impedance matching device between the oscillator output and the transmission line.

**The Probe, Oscillator, and Detector Amplifier**

Figure 5 shows the details of the probe which was designed for use at frequencies in the neighborhood of 400 megacycles. It was built from standard connectors with an arrangement to permit variation of the depth of penetration of the probe in the coaxial line.

The indicator amplifier unit was similar to one described by Kallman and was operated from a regulated power supply.

The oscillator used was a type 757-A manufactured by the General Radio Company. It produces any desired frequency within the range 150 to 600 megacycles. The wave length could be adjusted to within 0.01 centimeter of the desired value by means of a slow-motion drive. For most conditions of loading, the frequency can be relied on to be within the accuracy limit of 2.5 per cent. The power supply is a General Radio type 757-P1, and is provided with a meter to indicate grid current. The oscillator was amplitude modulated by varying the plate voltage in the normal fashion for plate modulation.

**The Design of Loads to Terminate the Coaxial Lines**

When the line spacing is small compared to the operating wave length, that is, at relatively low frequencies, an ordinary 2-terminal lumped resistance can be successfully used. At high frequencies, however, the concept of guided waves becomes necessary and satisfactory results are obtainable only if the terminations are distributed.

Ideally this is done for the coaxial line by closing the end of the line by a disk resistor as shown in Figure 6. In practice these distributed terminations are usually made by spraying some form of carbon on a dielectric support such as bakelized paper. Under certain conditions, a piece of carbon coated paper serves as a perfect resistance termination. Referring to Figure 6, the resistance of the disk between the two conductors is given by

\[
R = \frac{1}{2\pi \sigma h} \log \left( \frac{A}{a} \right)
\]

where \(\sigma\) is the conductivity of the coating material and \(h\) is its thickness.

By the proper choice of \(\sigma\) and \(h\), this resistance can be made equal to any value required, and it can be made equal to the characteristic impedance of the line.

At very high frequencies, experiments with a concentric line terminated by a disk of uniform resistive material (the d-c resistance of which is made equal to the line characteristic impedance) showed a standing wave ratio different from one. By the use of a short-circuiting plate at an effective distance of one quarter of a wave length beyond the graphite film, the standing wave ratio was found to be very close to unity. The explanation is that radiation takes place from the graphite film, and probably from a small length of the inner conductor just inside the graphite film. This effect is only marked when the mean circumference of the outer conductor is comparable with the wave length, which is the condition that the disk shall radiate appreciable energy.
At this wave length, the addition of the quarter-wave short-circuiting plate serves to screen the disk, thus prohibiting the radiation and making the line termination correct.

In the problem at hand, we are not actually interested in terminating a line in its characteristic impedance, but in terminating the line by a resistive disk whose effective resistance, at the operating frequency, is more or less equal to its d-c resistance or bearing a certain relation to it. Since the experimental work involved in this paper was mostly done at one frequency (430 megacycles per second), the line dimensions are small compared to the wave length and negligible energy is radiated from the back of these disks, and no short-circuiting plate is required at an effective distance of one-quarter of a wave length behind these washers. This statement was proved experimentally as will be shown later.

**Choice of the Resistive Coating**

Graphite, carbon, and metallic coating were tried in this work. Washers made of 1/16-inch Polystyrene and other low-loss dielectrics were cut having the shape of the transmission line, as shown in Figure 7. The outer circumference was made to fit the inside of the outer conductor, and the inner to fit the outside of the inner conductor. The electrical contact between the termination and the line was accomplished by using 34 per cent silver (Number 4817), manufactured by the Du Pont Company. The contact surfaces, as well as about 1/16 inch of the surface of these discs were painted with this silver solution and baked for about 10 hours at 100 degrees centigrade.

The particular deposit used to build all of the line terminations employed in this work was type 3051 Rescon made by the General Electric Company. It is a low-resistance surface carbon deposit, which when baked has a resistivity of the order of 1,000 ohms per square. The resistance depends on the baking time which ranges from 15 hours at 100 degrees centigrade to 7 hours at 130 degrees centigrade to 3 hours at 150 degrees centigrade.

Graphite was also used, and experimental results showed that the effective resistance of several samples does not change with frequency up to about 500 megacycles per second, and that it differs from the d-c values by less than about 2 per cent. But, due to the high temperature coefficient of graphite, the resistance was found to be rather unstable and changed pronouncedly with temperature and aging. For this reason deposited carbon resistors were preferred.

Samples were also made by evaporating aluminum, under vacuum, on Polystyrene washers to a thickness of about 10⁻⁴ centimeter. Contact between the aluminum film and the transmission line conductors was accomplished with the silver paint. Experiments with these samples proved them to be much superior to the deposited carbon samples. Their effective resistances were found to be very close to their d-c values (within less than 1 per cent), but due to the different coefficient of expansion of aluminum and silver, it was rather difficult to maintain the electrical contact between the two elements. This method, beside being very costly, does not offer an easy means of controlling the value of the resistances, and for these reasons aluminum coated resistors were not used.

**Experimental Determination of the Relation Between the D-C and Ultra-High-Frequency Effective Resistance of the Disc Resistors**

Tests were made with a circular coaxial line and a line having a square outer and circular inner conductors to measure the impedance of 30 different termination resistances. All tests were made at the same frequency (430 megacycles per second), and the method of measurement was the standing wave method. In this method, the far end of the line is shorted by a solid brass disc, and a point of minimum voltage is located as accurately as possible near the center of the slotted part of the line. This minimum point is taken as the reference point. The load is made to replace the short-circuiting disk and the shift θ in the position of the minimum point is read from a scale, and the ratio of the voltage minimum to voltage maximum is obtained from the probe meter. This ratio is corrected¹¹ and the corrected ratio is called throughout this paper, the Voltage Standing Wave Ratio, or VSWR. Having determined the values of θ and the VSWR it is then convenient to use an impedance chart to determine the magnitude and phase angle of the load. Such charts are given by Meagher and Markly.¹¹

The effective resistance of any load is found from the relation

\[
R_{el} = Z_t \cos \theta
\]  

where

\[
Z_t = \text{terminating load impedance, at the operating frequency}
\]

\[
\theta = \text{load phase angle}
\]

Tests showed that the effective resistance of these terminations, using this particular carbon Rescon, was always less than their direct-current values irrespective of the shape of the line and the ratio of the two resistances was found to be:

\[
R_{el} = 0.793 R_d
\]
Table I. Summary of Experimental Results Obtained with Rectangular Coaxial Lines

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Section Number</th>
<th>Dimensions of Outer Conductor, (2A X 2B) in Inches</th>
<th>Dimensions of Inner Conductor, (2a X 2b) in Inches</th>
<th>A/a</th>
<th>B/b</th>
<th>Z0, Ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.982 by 0.75</td>
<td>1.784</td>
<td>56.704</td>
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<td></td>
<td></td>
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<tr>
<td>2</td>
<td>1.125 by 0.75</td>
<td>2.500</td>
<td>68.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.500 by 0.75</td>
<td>3.500</td>
<td>80.92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.000 by 0.75</td>
<td>4.000</td>
<td>84.56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.110 by 1.00</td>
<td>2.220</td>
<td>70.64</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.250 by 1.00</td>
<td>2.500</td>
<td>74.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.875 by 1.00</td>
<td>3.000</td>
<td>87.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2.000 by 1.00</td>
<td>3.500</td>
<td>87.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1.340 by 1.25</td>
<td>2.680</td>
<td>83.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.175 by 1.25</td>
<td>3.500</td>
<td>91.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2.000 by 1.25</td>
<td>4.000</td>
<td>97.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.175 by 1.50</td>
<td>3.150</td>
<td>92.09</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>13</td>
<td>2.000 by 1.50</td>
<td>4.000</td>
<td>97.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1.250 by 1.50</td>
<td>4.500</td>
<td>106.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The dimensions of these sections were chosen such that the inscribed ellipses, to the inner and outer conductors, are confocal.

All tests were made at the same frequency (430 megacycles per second), same probe insertion, and same silicon crystal detector.

One of the rectangular coaxial lines used in this work was made of a piece of 3-centimeter wave guide, as the outer conductor, while the inner conductor is a length of a solid rectangular bar. This section was chosen as an additional check on the built-up line, as will be discussed later. Table I gives a summary of the data for the line sections studied, as found experimentally.

Analysis and Discussion of Results

General Considerations

One might notice from Table I that throughout every test the dimensions a, b, and B were kept the same while the width of the outer conductor A was the only variable. Thus from each one test it is possible to obtain an idea of the characteristic impedance of a rectangular coaxial line as function of the dimension A. However, the value of B was different in each test thus providing another possible relationship between the line characteristic impedance and the dimension B. In all tests the dimensions a and b were kept the same, that is, only one inside conductor was used, except for the square coaxial line of test E.

Confocal Ellipses

Conductors of rectangular cross section are very difficult to treat mathematically. The exact solution of the potential problem for a rectangular cross section can be solved by making use of the Schwartz Transformation, but this transformation can hardly be used to handle a problem such as a rectangular coaxial line. For this reason we must be satisfied with approximating such configuration with another one which can be handled mathematically. The best approximation of a rectangular cross section is an elliptic section, and the best approximation of a coaxial rectangular section is a pair of coaxial ellipses. But it is very difficult to find, for example, the capacitance of a capacitor composed of two elliptic shells unless these two ellipses are confocal, that is, have common focii.

Keeping this idea in mind, the first
The deviation from the confocal ellipse section

Plotting the experimental results given by Table I, on Figure 8, one can see at once, for every section, that by increasing the dimension $A$ of the outer conductor (while keeping all other dimensions unchanged), the characteristic impedance of the section increases following a semi-

This equation is actually the upper limit of equation 5, one which has the ratio $d/a = 0.968$. Also the experimental result obtained in test $E$ checks very closely the above equation. This justifies our assumption that the percentage decrease in the characteristic impedance of rectangular coaxial lines, whose inscribed ellipses are confocal, is independent of the line dimensions.

The deviation from the confocal ellipse section

Plotting the experimental results given by Table I, on Figure 8, one can see at once, for every section, that by increasing the dimension $A$ of the outer conductor (while keeping all other dimensions unchanged), the characteristic impedance of the section increases following a semi-

where $d^2 = A^2 - B^2 = a^2 - b^2$.

Figure 8 shows a plot of this equation for different values of $d/a$ including the value 0.968 which was fixed by the dimensions of the inside conductor. One can see from Figure 8 that the four first sections of tests $A, B, C$, and $D$ fit the curve $d/a = 0.968$.

To support the statement that equation 13 represents the characteristic impedance of rectangular coaxial lines whose inscribed ellipses are confocal, we may refer to curve II of Figure 1 which gives the characteristic impedance of coaxial square lines. The approximate equation of this curve, as found from curve II, is:

$$Z_0 = 134 \log_{10} \frac{A}{a}$$

This equation is actually the upper limit of equation 5, one which has the ratio $d/a = 0.971$. Also the experimental result obtained in test $E$ checks very closely the above equation. This justifies our assumption that the percentage decrease in the characteristic impedance of rectangular coaxial lines, whose inscribed ellipses are confocal, due to sharp corners is independent of the line dimensions.

The deviation from the confocal ellipse section

Plotting the experimental results given by Table I, on Figure 8, one can see at once, for every section, that by increasing the dimension $A$ of the outer conductor (while keeping all other dimensions unchanged), the characteristic impedance of the section increases following a semi-

Figure 8. Plot of experimental results

Plot of Experimental Results

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logarithmic law. For the four different tests, A, B, C, and D, we get four straight lines on semilogarithmic paper. These lines were found to be very nearly parallel, thus we are in a position to find an empirical formula for the characteristic impedance of any rectangular coaxial line.

To do that we shall define a dimension \( A' \) such as:

\[
A' = (B^2 + a^2 - b^2)^{1/2}
\]  

(15)

and is the width of the outer conductor which is necessary to make the inscribed ellipses (keeping all other dimensions \( B, a \) and \( b \) constant) confocal. Thus for sections whose width \( A \) was taken such that the inscribed ellipses are confocal \( A' \) is equal to \( A \). For such sections then equation 5 gives their characteristic impedance directly. For other sections in which \( A' \) is not equal to \( A \), one must add a correction term to equation 5.

One way to do this is to take any point, say point \( M \), on one of the experimental lines, say line \( LK \), and calculate the value of \( A' \) from Equation as:

\[
Z_0 = 134 \log_{10} \left\{ \frac{(A'/a)^2 - (d/a)^2}{1 + (1 - (d/a)^2)^{1/2}} \right\} + \Delta Z
\]  

(16)

To find the value of \( \Delta Z \), consider the numerical example: At point \( M \)

\[
Z_0 = 100 \text{ ohms (as found from the experimental curve } LK) \\
A'/a = 5.1 \\
A'/a = 2.22 \text{ (as calculated from equation 15)}
\]

In this case the first term of equation 16 contributes an amount of 70.9 ohms. Thus the value of \( \Delta Z \) is 100 – 70.9 = 29.1 ohms. This is due to an increase in the ratio \( A/a \) equal to \( A/a - A'/a \). The slope of the experimental curve \( LK \) is:

\[
m = \frac{\log_{10} (A'/a) - \log_{10} (A/a)}{\Delta Z} = \frac{\log_{10} (A/A')}{\Delta Z}
\]  

(17)

but \( A/A' = 5.1/2.22 = 2.397 \) and \( 1/m \) becomes equal to 80.6 and \( \Delta Z \) becomes:

\[
\Delta Z = 80.6 \log_{10} (A/A')
\]  

(18)

Thus the complete formula for the characteristic impedance of any coaxial rectangular transmission line is:

\[
Z_0 = 134 \log_{10} \left\{ \frac{[(A'/a)^2 - (d/a)^2]^{1/2} + A'/a}{1 + (1 - (d/a)^2)^{1/2}} + \frac{80.6 \log_{10} (A/A')}{\Delta Z} \right\} \text{ ohms (19)}
\]

where

\[
A' \text{ is given by equation 15}
\]

\[
2A = \text{inner width of the outer conductor}
\]

\[
2B = \text{inner depth of the outer conductor}
\]

\[
2a = \text{outer width of the inside conductor}
\]

\[
2b = \text{thickness of the inside conductor}
\]

Equation 19 to equation 15 gives a dimension \( A' \) which upon replacing \( A' \) will give two confocal ellipses inscribed to the coaxial rectangular conductor sections. Knowing the characteristic impedance of such lines, we can, upon the assumption of a air-filled lossless line, find the inductance as well as the capacitance per unit length of the line. In summary, we can write equation 19 as:

The general expression for the characteristic impedance of rectangular coaxial lines:

Here \( A' = (B^2 + a^2 - b^2)^{1/2} \), and

\[
Z_0 = 134 \log_{10} \left\{ \frac{A'/a + [(A'/a)^2 - (d/a)^2]^{1/2}}{1 + (1 - (d/a)^2)^{1/2}} \right\} + \frac{80.6 \log_{10} (A/A')}{\Delta Z} \text{ ohms (20A)}
\]

The characteristic impedance of a rectangular coaxial line whose inscribed ellipses are confocal:

Here \( A'/a = A \), and

\[
Z_0 = 134 \log_{10} \left\{ \frac{A + [(A/a)^2 - (d/a)^2]^{1/2}}{1 + (1 - (d/a)^2)^{1/2}} \right\} + \frac{80.6 \log_{10} (A/A')}{\Delta Z} \text{ ohms (20B)}
\]

The characteristic impedance of a coaxial line:

Here \( d/a = 0 \) and \( A'/a = A \),

\[
Z_0 = 134 \log_{10} \frac{A}{a} \text{ ohms (20C)}
\]

The characteristic impedance of a line having a rectangular outer conductor and a square inner conductor:

Here \( d/a = 0 \) and \( A'/a = B \),

\[
Z_0 = 134 \log_{10} \left\{ \frac{A'/a + [(A'/a)^2 - (d/a)^2]^{1/2}}{1 + (1 - (d/a)^2)^{1/2}} \right\} + \frac{80.6 \log_{10} (A/A')}{\Delta Z} \text{ ohms (20D)}
\]

The characteristic impedance of a line having a rectangular outer conductor and a ribbon inner conductor:

Here \( d/a = 1 \) and \( A'/a = (B^2 + a^2)^{1/2} \),

\[
Z_0 = 134 \log_{10} \frac{A'/a + [(A'/a)^2 - 1]^{1/2}}{1 + (1 - (d/a)^2)^{1/2}} + \frac{80.6 \log_{10} (A/A')}{\Delta Z} \text{ ohms (20E)}
\]

The characteristic impedance of a line having a rectangular outer conductor and a ribbon inner conductor when the inscribed ellipses are confocal:

Here \( d/a = 1 \) and \( A'/a = A \),

\[
Z_0 = 134 \log_{10} \frac{A/a + [(A/a)^2 - 1]^{1/2}}{1 + (1 - (d/a)^2)^{1/2}} \text{ ohms (20F)}
\]

Equations 20 (A-F) will give the characteristic impedance of any rectangular coaxial line, rectangular outer conductor square inner conductor line, square coaxial line, and rectangular outer conductor ribbon inside conductor line.

To test the "fit" of these formulas, we shall go back and apply them to all the different sections used in this experimental work and compare the calculated with the measured values, as shown in Table II. From Table II, one can see that the calculated values of the characteristic impedance of rectangular coaxial lines, using equations 20, differ from the experimentally measured values by less than ±2 per cent.

In test F, as mentioned before, the line has an outer conductor made of a piece of 3-centimeter wave guide while the inner conductor was the same solid rectangular bar used in most of the other tests. The calculated value of the characteristic impedance of this line checks to 1.2 per cent of the measured value. This appears to justify our statement that a very good electrical contact between the four members of the outer conductor of a built-up rectangular line is not very necessary.

In test E, the line is a square coaxial one and as shown in Table II, its characteristic impedance agrees with the measured value to within less than 1 per cent. This proves that the semiempirical formula obtained is valid for any rectangular coaxial line independent of the dimensions of the inside conductor. Moreover, the characteristic impedance was measured as a function of \( (A/a) \) and \( (d/a) \), that is, as ratios of the outer to the inner conductor dimensions independent of any one dimension.

In applying equations 20, it should be remembered that they have not been checked experimentally for the complete range of the dimensional ratios. On the other hand, however, there is no apparent tendency for the experimental values to deviate from the values calculated by the
formula at the extremes of the range of the ratios studied in this work.

Appendix I. Nomenclature

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Section Number</th>
<th>A</th>
<th>B</th>
<th>a</th>
<th>b</th>
<th>d</th>
<th>A'</th>
<th>d/a</th>
<th>A/a</th>
<th>A'/a</th>
<th>Z_0*</th>
<th>Z_0**</th>
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<tbody>
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References


No Discussion