The high temperature peak of the yield strength of γ'-strengthened superalloys

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Abstract
The yield strength and the critical resolved shear stress of many γ'-strengthened nickel-base superalloys exhibit an anomalous temperature dependence: they peak at about 1000 K. Since this peak occurs at the highest temperatures at which these materials are used, it is of great technical importance. A frequently referred to interpretation of the peak is that it is governed by the anomalies of the strength of single-phase L12-long range ordered γ'-intermetallics. In this paper it is, however, shown that there are at least five alternative dislocation mechanisms, which may bring about the peak of the strength of superalloys. These mechanisms are described and their potentials discussed.

Keywords: Nickel alloys; γ'-Precipitates; Nanostructure; Anomalies of the yield strength; Intermetallic phases

1. Introduction
Nickel-base superalloys are structural materials meant for applications at elevated temperatures. Their high strength derives from nano-scale coherent precipitates of the L12-long range ordered γ'-phase. Their approximate composition is Ni₃(Al,Ti). They may be considered as derivatives of the best known L12-long range ordered intermetallic phase Ni₃Al [1,2].

The yield strength σₚ and the critical resolved shear stress (CRSS) τₚ of many γ'-strengthened nickel-base superalloys do not decrease monotonically as the temperature T is raised: the functions σₚ(T) and τₚ(T) peak at Tₘₚeak ≈ 1000 K [e.g. 3–28]. This is demonstrated in Fig. 1. Also the flow stress after appreciable plastic deformation (see Fig. 1(b)) [20] and the strain to fracture [14,24] of γ'-strengthened materials show unusual temperature variations. From a technical point of view it is very fortunate that σₚ(T) and τₚ(T) peak at the highest temperatures at which these materials are used.

Since it is possible that in different ranges of temperature different slip systems are activated (see Section 2.1) there may be some doubt as to which system the normal stress applied to a single crystal specimen has to be resolved. Therefore even if the following single crystals are dealt with, often their critical normal stress σₚ(T) (=critical load divided by the specimen’s cross-sectional area) will be considered instead of their critical resolved shear stress τₚ(T). Unless the slip geometry changes with T, σₚ(T) and τₚ(T) of single crystals differ only by the constant Schmid factor. In the following σₚ may refer to the yield strength of polycrystalline superalloy specimens and to the unresolved critical normal stress of single crystals.

Peaks of the functions σₚ(T) and τₚ(T) have been reported for γ'-volume fractions f as low as 0.06 [23] and for f around or even exceeding 0.6 [e.g. 3,5,7,8,15,17,18,20,21] and for small average diameter less than or around 50 nm [16,23,25,26] as well as for large γ'-precipitates (average diameter 200 nm or above [7,8,12,14,15,17,18,20,24,27]). Many of the investigated γ'-dispersions were bimodal (see Section 1.6).

Let T₀ ≈ 800 K be the temperature at which ∂σₚ/∂T, respectively ∂τₚ/∂T, start to become positive. The difference between σₚ(Tₘₚeak) and σₚ(T₀), respectively between τₚ(Tₘₚeak) and τₚ(T₀), ranges from hardly outside the experimental limits of error [e.g. 6.9–11,14,19,23,25,26,28] to about 50% or more of σₚ(T₀) [5,12]. σₚ(Tₘₚeak) of polycrystals and τₚ(Tₘₚeak) of single crystals of the commercial γ'-strengthened superalloy NIMONIC PE16 (see Section 1.5) exceed σₚ(T₀), respectively τₚ(T₀), by 13% (see Fig. 1(a)). Evidently the peaks may be very strong. Copley et al. [8] reported that σₚ(T) of single crystals of the commercial γ'-strengthened superalloy MAR-M200 (composition in at%: 10 Co, 10 Cr, 4 W, 1 Nb, 11.0 Al, 2.5 Ti, 61 Ni, f = 0.6, edge length of
the cuboidal γ′-precipitates \$\approx 250\,\text{nm}\$ with the two orientations \(\langle 001 \rangle\) and \(\langle 112 \rangle\) exhibited two peaks: one at \(T_{\text{peak1}} \approx 700\,\text{K}\) and one at \(T_{\text{peak2}} \approx 1030\,\text{K}\).

The standard interpretation of the peaks of the functions \(\tau_c(T)\) and \(\sigma_y(T)\) of \(\gamma'\)-strengthened nickel-base superalloys is that they reflect the anomalies of the yield strength \(\sigma_y(T)\) and the CRSS \(\tau_c(T)\) of the strengthening \(L_12\)-long range ordered \(\gamma'\)-material. This will be discussed in Section 2.1. There are, however, alternative explanations: in Sections 2.2–2.6, five peak producing dislocation mechanisms will be described.

The low temperature decrease of the functions \(\sigma_y(T < T_0)\) and \(\tau_c(T < T_0)\) is governed by the decrease of the strength of the disordered or perhaps short range ordered solid solution strengthened γ-matrix. This is demonstrated in Fig. 1(a). Another interpretation of this decrease will be given in Section 1.7.1. The precipitous decrease of \(\sigma_y(T)\) and \(\tau_c(T)\) above about 1000 K has been attributed to the unresolved critical normal stress of single crystals. Due to this anisotropy, deviations of the specimens’ actual orientations from the nominal one may cause an appreciable scatter of the data and may even mask or produce peaks of \(\sigma_y(T)\), respectively of \(\tau_c(T)\).

### 1.1. Orientation dependence

The height of the peaks of the functions \(\sigma_y(T)\) and \(\tau_c(T)\) depends on the orientation of single crystalline specimens \([4,12,15,17,18,27]\). Examples are given in Sections 1.2 and 1.3. It is reiterated that \(\sigma_y\) may refer to the unresolved critical normal stress of single crystals. Due to this anisotropy, deviations of the specimens’ actual orientations from the nominal one may cause an appreciable scatter of the data and may even mask or produce peaks of \(\sigma_y(T)\), respectively of \(\tau_c(T)\).

### 1.2. Tension/compression asymmetry

The peaks measured in tension and in compression are different. In the following this will be referred to as "tension/compression asymmetry"; the expression "anisotropy" will only be used in connection with orientation dependences (see Section 1.1). Shah and Duhl \([12]\) found pronounced peaks of \(\sigma_y(T)\) in tension tests of single crystals of the commercial \(\gamma'\)-strengthened nickel-base superalloy PWA 1480 (composition in at.%: 5 Co, 12 Cr, 1 W, 4 Ta, 11.3 Al, 1.9 Ti, 65 Ni). The orientations were \(\langle 001 \rangle\), \(\langle 011 \rangle\), and \(\langle 111 \rangle\). The peaks were highest/lowest for the \(\langle 001 \rangle\) and \(\langle 111 \rangle\)-orientation, respectively.

In the case of compression tests well-defined peaks were only
found for (001)-crystals. Heredia and Pope [15] investigated single crystals of the same material and found rather weak peaks of \( \sigma_c(T) \), the relatively strongest ones for compression tested (001)-crystals and for tension tested (011)-crystals. Österle et al. [27] reported peaks of \( \sigma_c(T) \) of single crystals of the superalloy SC 16 (composition in at.\%: 17 Cr, 1 Ta, 2 Mo, 7.4 Al, 4.2 Ti, 69 Ni; bimodal: cuboids with \( f = 0.35 \) and edge length \( = 450 \text{ nm} \) plus spheres with \( f = 0.05 \) and diameter \( = 80 \text{ nm} \) for all three quoted orientations in tension as well as in compression.

1.3. Dependence on the size of the \( \gamma' \)-precipitates

Shah and Duhl [12] reported a drastic increase of the height of the peak of \( \sigma_c(T) \) with the size of the \( \gamma' \)-precipitates in compression tested PWA 1480-single crystals (see Section 1.2) orientated along (001). For \( \gamma' \)-precipitates of 3 \( \mu \text{m} \) size the peak was extremely high: \( \sigma_c(T_{\text{peak}}) > \sigma_c(T_0) \) by more than 50\%. (1 1 1)-single crystals with 3 \( \mu \text{m} \)-\( \gamma' \)-precipitates, however, showed no peaks in compression tests.

1.4. Strain rate sensitivity

The temperature at which the function \( \sigma_y(T) \) peaks increases with strain rate [7,14,24].

1.5. Serrated yielding

Several groups [5,8,14,23,26,28] reported serrated, discontinuous yielding of homogenized, quenched as well as of aged specimens in the temperature range of the peak. Beardmore et al. [5] found that an experimental superalloy (approximate composition in at.\%: 18 Cr, 7 Al, 75 Ni) with very fine \( \gamma' \)-precipitates showed strain aging effects and that the function \( \sigma_c(T) \) of this material had a “hump” at about 900 K. Copley et al. [8] interpreted the peak of \( \sigma_c(T) \) of MAR-M200 at \( T_{\text{peak}} = 700 \text{ K} \) (see above) as being associated with a dislocation pinning effect because there was pronounced discontinuous flow. Between 683 and 1123 K, the plastic deformation of single crystals of the commercial \( \gamma' \)-strengthened superalloy NIMONIC PE16 (composition in at.\%: 17 Cr, 34 Fe, 2 Mo, 2.6 Al, 1.5 Ti, 41 Ni) [9,10] quenched from above the solvus temperature of the \( \gamma' \)-forming elements was discontinuous [23,28]. During heating the specimens to deformation temperatures above 980 K and deforming them, very fine \( \gamma' \)-precipitates formed, which raised the CRSS.

1.6. Bimodal \( \gamma' \)-precipitate dispersions

It has been mentioned that peaks of \( \sigma_y(T) \) have been reported for small as well as for large \( \gamma' \)-precipitates and for low as well as for high \( \gamma' \)-volume fractions \( f \). Often the \( \gamma' \)-precipitate dispersions were bimodal [e.g. 5,14,24,27] with large cuboidal \( \gamma' \)-precipitates with \( f \) exceeding 0.3 and edge lengths exceeding 300 \text{ nm} plus a low volume fraction of much smaller, approximately spherical \( \gamma' \)-precipitates in between the larger cuboidal ones. Very fine \( \gamma' \)-precipitates formed during cooling the specimens from the aging temperature. It is indeed possible that some groups were unaware of the presence of such very fine \( \gamma' \)-precipitates in their specimens. Even if the specimen is quenched very rapidly from the aging temperature, additional fine \( \gamma' \)-precipitates may form during heating it to the deformation temperature and deforming it elastically and plastically (see Section 1.5). Three prerequisites for this to happen are: (i) that the deformation temperature is markedly lower than the aging temperature, (ii) that the solubility of the \( \gamma' \)-forming elements varies strongly with temperature, and (iii) that their diffusion is sufficiently fast at the deformation temperature. There may also be some further Ostwald ripening [30,33,34].

1.7. Dislocation configurations in plastically deformed specimens

Many groups [e.g. 3,5,14,20,21,24,26,29,30] studied the configurations of dislocations in slightly deformed, unloaded specimens by transmission electron microscopy. Since the experimental variables such as material, \( \gamma' \)-precipitate dispersion, deformation temperature and mode (tension versus compression tests) varied widely, comparisons of the results published by different groups are difficult. Moreover most authors did not pay special attention to the peaks of \( \sigma_y(T) \) and \( \tau_c(T) \). Therefore here only some relevant results are shortly summarized. The results of recent computer simulations of the low-temperature glide of dislocations in \( \gamma' \)-strengthened superalloys are in agreement with the respective microscopic observations [35,36].

1.7.1. Low temperatures

At relatively low temperatures, two dislocations with identical Burgers vectors of the type \( \frac{1}{2}(0 \, 1 \, 1) \) form a pair and glide together [3,5,20,24,26,29,30]. They shear \( \gamma' \)-precipitates. Slip is planar and confined to rather few \( \{1 \, 1 \, 1\} \) planes. Most dislocations lie in the \( \gamma \)-matrix and there are hardly any in the \( \gamma' \)-precipitates. The reason for pairing of \( \frac{1}{2}(0 \, 1 \, 1) \)-dislocations is that due to the \( L_1_2 \)-long range order of the \( \gamma' \)-phase its perfect \( \{1 \, 1 \, 1\} \)-plane is planar and confined to rather few \( \{1 \, 1 \, 1\} \)-planes. Most dislocations lie in the \( \gamma \)-matrix and there are hardly any in the \( \gamma' \)-precipitates. Since pairing of \( \frac{1}{2}(0 \, 1 \, 1) \)-dislocations is coupled to shearing of the \( \gamma' \)-precipitates, there is no pairing if all dislocation motion is confined to the \( \gamma \)-matrix.

After monotonic or cyclic deformation at 293 K, Milligan and Antolovich [21] found a high density of superlattice-intrinsic stacking faults (S-ISFs) within the \( \gamma' \)-precipitates in the superalloy PWA 1480 (see Section 1.2). These faults were often present as \( \frac{1}{2}(1 \, 1 \, 2) \)-faulted loops. The density of the S-ISFs decreased as the deformation temperature was raised to 673 K, above which temperature no such faults were seen. The authors stated that the decrease in fault density corresponded to the decrease of the yield strength \( \sigma_y(T) \) between 293 and 673 K. Above 673 K, \( \sigma_y(T) \) started to increase with \( T \). The faults were interpreted to reduce the mobility of dislocations either by directly acting as barriers to their glide or by the decomposition of \( (0 \, 1 \, 1) \)-superlattice Shockley partial dislocations. Bette et al. [24] reported similar observations of faulted dislocation loops within the \( \gamma' \)-precipitates of the superalloy IN 738 LC (see Section 1.7.3). After slight plastic deformation at 293 K,
but not at 673 K, Dollar and Bernstein [20] found a high density of dislocations in the γ'-matrix of PWA 1480 (see Section 1.2). They were interpreted to give rise to the relatively high yield strength measured at 293 K (see Fig. 1(b)). The authors gave no explanation for the decrease of the γ'-dislocation density between 293 and 673 K.

The three just mentioned groups – Milligan and Antolovich [21], Bettge et al. [24], and Dollar and Bernstein [20] (see Fig. 1(b)) – reported distinct minima of the experimental data \(\sigma(T)\) at \(T_0 = 700\) K. The authors gave the following interpretations for the minima of \(\sigma(T)\). The decrease of the density of S-ISFs in the γ'-precipitates [21,24], respectively of the dislocations in the γ'-matrix [20], in the temperature range 293–673 K enhanced the decrease of \(\sigma(T)\) beyond the usual thermally activated decrease of solid solution strengthening of the γ'-matrix. Hence the total decrease of \(\sigma(T)\) in the range 293–673 K was so strong that a distinct minimum of \(\sigma(T)\) appeared at \(T_0\).

1.7.2. High temperatures

At high temperatures \(T\), i.e. in the range where \(\sigma(T)\) and \(\tau_e(T)\) decrease precipitously as \(T\) is raised, single \(\frac{1}{2}[011]\) dislocations predominate [5,24,29,30]. They tend to bypass the γ'-precipitates. Thermally activated processes operate: local climb and/or cross-slip. Slip is rather homogeneously distributed throughout the specimen.

1.7.3. Peak temperature

Bettge et al. [24] tension tested polycrystals of the commercial γ'-strengthened superalloy IN 738 LC (composition in at.%: 8 Co, 18 Cr, 1 W, 1 Ta, 1 Mo, 7.2 Al, 4.1 Ti, 59 Ni, bimodal: cuboids with 450 nm edge length plus spheres with 80 nm diameter, total γ'-volume fraction = 0.43) between 293 and 1223 K. The dislocation configurations observed at low and high temperatures agreed with those described above. At \(T_{\text{peak}} = 1023\) K, no striking features were found after 0.002 plastic strain. The dislocations were still concentrated in slip bands, which, however, were no longer planar. The number of slip bands had increased beyond that found at lower temperatures. Straight dislocations in the γ'-precipitates were rare; most dislocations were seen to bow out between the γ'-precipitates.

2. Interpretations of the peaks of the yield strength and of the CRSS

In the subsequent sections six different mechanisms will be discussed which may give rise to the peaks of the yield strength \(\sigma(T)\) and of the CRSS \(\tau_e(T)\) of γ'-strengthened nickel-base superalloys. The basic idea of all interpretations is that at elevated temperatures a strengthening mechanism starts to operate which above \(T_0\) (see Section 1) overcompensates the negative slopes \(\partial\sigma/\partial T\) and \(\partial\tau_e/\partial T\) caused by the decrease of solid solution hardening of the γ'-matrix and perhaps by the dislocation processes reported by Dollar and Bernstein [20], Milligan and Antolovich [21], and Bettge et al. [24] (see Section 1.7.1). The strengthening mechanism, which gives rise to the positive slopes \(\partial\sigma/\partial T\) and \(\partial\tau_e/\partial T\) above \(T_0\), will be referred to as "peak producing mechanism". The right descending flank of the peak may be governed by two entirely different dislocation processes.

(a) The peak producing mechanism ceases to operate above \(T_{\text{peak}}\) examples will be presented in Sections 2.2 and 2.3.

(b) The two softening mechanisms: (i) [coarsening and/or dissolution of γ'-precipitates] and (ii) [thermal activation helps dislocations to overcome the γ'-precipitates by local climb and/or cross-slip], which bring about the precipitous decrease of the functions \(\sigma(T)\) and \(\tau_e(T)\) at very high temperatures (see Section 1 and Fig. 1), outweigh the effects of the peak producing mechanism.

If a potential peak producing mechanism operates in one of the two temperature regions where the general slopes \(\partial\sigma/\partial T\) and \(\partial\tau_e/\partial T\) are strongly negative (below ca. 400 K and above ca. 1000 K in Fig. 1), it is unlikely that a peak becomes noticeable. This applies primarily to very high temperatures at which \(\sigma(T)\) and \(\tau_e(T)\) decrease precipitously. If a peak producing dislocation mechanism involves thermal activation, it is strain rate sensitive: \(T_{\text{peak}}\) increases with strain rate (see Section 1.4).

Though the following analyses refer to \(\tau_e\) as well as to \(\tau_e\), for the sake of simplicity often only either \(\sigma(T)\) or \(\tau_e\) will be quoted: e.g. in Section 2.1 mainly \(\tau_e\) will be referred to.

2.1. Abnormal plastic behavior of the γ'-phase

The peak of the CRSS \(\tau_e(T)\) of γ'-strengthened superalloys is interpreted to be caused by the anomalies of the CRSS \(\gamma(T)\) of the single-phase L1₂-long range ordered γ'-phase. As examples for the latter anomalies, experimental \(\gamma(T)\)-data taken for the single-phase γ'-intermetallic (composition in at.%: 8 Co, 2 Cr, 1 Mo, 19.0 Al, 4.2 Ti, 66 Ni) which strengthens the commercial nickel-base superalloy NIMONIC 105 (see below) [9,11,25,26], are presented in Fig. 2. In these diagrams all stresses were resolved to the primary \{1 1 1\} \{0 1 1\}-slip system, even though at temperatures above the maximum of \(\gamma(T)\) slip on \{0 1 1\} planes dominates-unless the specimen axis is orientated along \(\langle 0 0 1\rangle\). The yield strength \(\sigma_{\gamma}^p(T)\) is obtained by dividing the CRSS \(\gamma(T)\) by the Schmid factor, which is close to 0.43 for \(\langle 0 0 1\rangle\), \{0 1 1\}, and \{1 2 3\}-single crystals, but only 0.27 for \{1 1 1\}-crystals. Hence if \(\sigma_{\gamma}^p(T)\) is plotted instead of \(\gamma(T)\), the diagrams look similar to those shown in Fig. 2 except that \(\sigma_{\gamma}^p(T)\) of \{1 1 1\}-crystals is raised by about 50% relative to the other curves. Here a short summary of the anomalies of \(\gamma(T)\) and of their interpretations is given [2,13,26,37].

(i) \(\gamma(T)\) is anisotropic, i.e. it varies with the orientation of the specimen. This means that Schmid’s law is violated.

(ii) \(\gamma(T)\) measured in tension and compression tests are different, i.e. there is a tension/compression asymmetry.

(iii) The function \(\gamma(T)\) has a maximum at \(T = T_{\text{max}}\), \(T_{\text{max}}\) is different for tension and compression tests.
In order to avoid confusions with the peaks of $\tau_{c}(T)$ and $\tau_{\tau}(T)$ of $\gamma'$-strengthened superalloys, the expression "peak" is only used in connection with the latter two-phase materials; this applies also to $T_{\text{peak}}$. If $\sigma_{c}^{\prime}(T)$ or $\tau_{c}^{\prime}(T)$ of single-phase $\gamma'$-intermetallics are dealt with, the expression "maximum" is used. The maxima of $\tau_{c}^{\prime}(T)$ at $T_{\max}$ may be very high: in Fig. 2(a) $\tau_{c}^{\prime}$ ($T_{\max} = 800$ K) measured in compression tests of $\{011\}$-single crystals is seen to equal 2.4 times $\tau_{c}^{\prime}$ ($T = 400$ K).

In L1$_2$-long range ordered intermetallic phases two dislocations with identical $\frac{1}{2}[011]$-Burgers vectors form a pair and glide together (see Section 1.7.1). Current models [2,13,37] explain the above listed anomalies on the basis of thermally activated short distance cross-slip of a short segment of the leading $\frac{1}{2}[011]$-dislocation of a screw pair from its original $\{111\}$-glide plane onto a $\{001\}$-plane. Since below $T_{\max}$ no glide over long distances is possible on $\{001\}$-planes, the cross-slipped segment gets stuck in the $\{001\}$-cross-slip plane and thus acts as a pinning center for the non-cross-slipped parts of the leading $\frac{1}{2}[011]$-screw dislocation. This type of short distance cross-slip raises the CRSS of single-phase L1$_2$-materials – in contrast to single-phase disordered fcc materials, in which cross-slip lowers the CRSS. The probability of short distance cross-slip from $\{111\}$-onto $\{001\}$-planes depends on [2,13,26,37]:

(a) temperature;
(b) the resolved shear stress acting in the $\{001\}$-cross-slip plane;
(c) the anisotropy of the specific energy of antiphase boundaries. It is lower on $\{001\}$- than on $\{111\}$-planes [1,2,38];
(d) the anisotropy of the shear modulus;
(e) the resolved shear stress acting on the edge components of the two Shockley partial dislocations into which the leading $\frac{1}{2}[011]$-screw dislocation of a pair dissociates in its primary $\{111\}$-plane. Depending on the orientation of the specimen and on the sign of the external resolved shear stress, the forces experienced by the two Shockley partials press them together or drive them apart. This gives rise to the tension/compression asymmetry (see above (ii)).

Since cross-slip is a thermally activated process, its probability and consequently pinning of dislocations increase with temperature. This governs the ascending flank of the maximum of $\tau_{c}^{\prime}(T)$. Except for crystals orientated along $\{001\}$, the decrease of the function $\tau_{c}^{\prime}(T)$ above $T_{\max}$ is caused by long distance glide on $\{001\}$-planes.

Following Lall et al. [37], Miner et al. [18] divided the CRSS $\tau_{c}(T)$ of $\gamma'$-strengthened superalloys into two terms, which were supposed to add up linearly: one term is analogous to the normal CRSS for slip on a $\{111\}$-plane and the other term represents the effects of short distance $\{111\}$ $\rightarrow$ $\{001\}$-cross-slip, which brings about the anomalies of $\tau_{c}^{\prime}$ and hence of $\tau_{\tau}(T)$. The authors applied their model to the commercial $\gamma'$-strengthened superalloy Rene N4 (composition in at. %: 8 Co, 10 Cr, 1 Ta, 2 W, 1 Mo, 8.1 Al, 5.2 Ti, 64 Ni, $\gamma'$ volume fraction = 0.65, size of the $\gamma'$-precipitates $\approx 250$ nm). The model involves three adjustable, temperature dependent parameters. At given temperature, it described the anisotropy and asymmetry of $\tau_{c}$ satisfactorily. No special attention was paid to the peak of $\tau_{c}(T)$. Österle et al. [27] published a similar study for SC 16 (see Section 1.2). The latter authors stated that the influence of the specific alloy microstructure on the three adjustable parameters is quite pronounced. Both groups found that the sign of that adjustable parameter which governs the effects of the shear stress acting in the $\{001\}$-cross-slip plane (see (b) above), was the opposite of the expected one.

Copley and Kear [3] related the CRSS $\tau_{c}$ of $\gamma'$-strengthened superalloys to the CRSSs of their two constituent phases: to $\tau_{c}^{\prime}$
of the respective single-phase γ-matrix material and to \( \tau_c^\prime \) of the respective single-phase γ'-intermetallic. A detailed dynamic model yielded Eq. (1):

\[
\tau_c = \frac{\Gamma}{2b} \frac{S}{\bar{r}} + \frac{1}{2}K(\tau_c^2 + \tau_c^\prime),
\]

with \( \Gamma \) specific energy of antiphase boundaries on \{111\} planes in the γ-phase, \( b \) the length of the \{011\}-Burgers vector, \( S \) the dislocation line tension, \( \bar{r} \) the average radius of the γ'-precipitates, they are supposed to be of spherical shape. The parameter \( K \) is approximately constant and close to unity. The first term on the right side of Eq. (1) is the dominant one; it is independent of temperature \( T \). Due to the term in brackets, \( \tau_c(T) \) reflects the anomalies of \( \tau_c^2(T) \) listed above: in the temperature range in which \( \tau_c^2(T) / \partial T \) is strongly positive, \( \tau_c / \partial T \) too is positive and there is a peak of \( \tau_c(T) \). The derivation of Eq. (1) was based on the assumption that the dislocations shear the γ'-precipitates. Evidently a simple average over \( \tau_c^2 \) and \( \tau_c^\prime \) enters Eq (1). This implies that the dislocation processes which operate in each of the two constituent phases of the γ'-strengthened superalloy, are supposed to be identical to those operating in the respective compact single-phase materials, e.g. no allowance is made for possible effects of the small size of the γ'-precipitates on the above described \{111\} \rightarrow \{011\} cross-slip mechanism. Experimentally, however, the height of the peak of \( \sigma_c(T) \) of PWA 1480-singles crystals was found to increase with the size of the γ'-precipitates (see Section 1.3).

Several groups [e.g. 3, 7, 8, 20, 29] discussed their experimental data on the basis of Eq. (1) and found qualitative agreement. Nitz et al. [25, 26] amended Eq. (1): these authors allowed for the γ'-volume fraction \( \bar{r} \) to be an adjustable parameter, the latter authors aimed at an approximate spherical shape. Their average radius was 17 nm. Indeed Nitz et al. [25] found a positive slope \( \partial \tau_c / \partial T \) for \( \tau_c(T) \) calculated from Eq. (1), but this slope was much too strongly positive and the anisotropy too low. The authors gave the following interpretation for these discrepancies. \( \tau_c(T) < \tau_c(\text{Max}) \) i.e. at all relevant temperatures the shear stresses acting in the γ'-precipitates in two-phase NIMONIC 105 are higher than those in the respective single-phase γ-material at any temperature. Therefore Nitz et al. inserted \( \tau_c^\prime(\text{Max}) \) instead of \( \tau_c^\prime(\text{Max}) \) into Eq. (1).

Below \( \text{T}_{\text{Max}} \), the strain rate sensitivity of \( \tau_c^\prime(\text{T}) \) is positive, but only very slightly; above \( \text{T}_{\text{Max}} \), it is strongly positive [2]. According to Eq. (1), the ascending flank of the maximum of \( \tau_c(T) \) generates the ascending flank of the peak of \( \tau_c(T) \). Evidently the experimentally observed pronounced strain rate sensitivity of the peak of \( \tau_c(T) \) (see Section 1.4) is hard to reconcile with the weak strain rate sensitivity of \( \tau_c^\prime(\text{T} < \text{T}_{\text{Max}}) \). To the author’s knowledge no other quantitative correlations of experimental \( \tau_c(T) \)-data with experimental \( \tau_c^\prime(T) \), and \( \tau_c(\text{T}) \)-data have been published. There is a shortage of the respective single-phase data; \( \tau_c^\prime(T) \) [1, 2, 13] and \( \tau_c(T) \) vary with the composition of the alloy.

In summary: Eq. (1) does not describe all three anomalies of the function \( \tau_c(T) \) of NIMONIC 105 – anisotropy, tension/compression asymmetry, and peak – consistently. The above described inconsistencies to which the application of Eq. (1) to NIMONIC 105 leads, shed doubt on the widely spread belief (see Section 1) that the peak of the CRSS \( \tau_c(T) \) of γ'-strengthened superalloys is exclusively governed by the anomalies of the CRSS \( \tau_c^\prime(T) \) of the strengthening γ'-phase and that Eq. (1) can be used to describe the peak of \( \tau_c(T) \). These findings necessitate the search for alternative mechanisms which may produce the peak of \( \tau_c(T) \).

2.2. Changes of the γ'-precipitate dispersion

In Section 1.6 it has already been mentioned that during heating a quenched γ'-strengthened superalloy specimen to the deformation temperature and during its elastic and plastic deformation new very fine γ'-precipitates may form in addition to those grown during the aging treatment. Prerequisite for this to happen have been listed in Section 1.6. This process renders the slopes \( \partial \sigma / \partial T \) and \( \partial \tau_c / \partial T \) positive. Moreover there may be some further Ostwald ripening [30, 33, 34] of the original γ'-precipitates.

An extreme example has been referenced in Section 1.5. During heating up and deforming NIMONIC PE16-singles crystal specimens, which had been quenched from above the solvus temperature of the γ'-forming elements to ambient temperature, very fine γ'-particles precipitated. They raised the CRSS at inter-
mediate temperatures, but dissolved again at very high ones. Thus a peak appeared [23].

2.3. Ternary phases

In principle this peak producing mechanism is similar to the one described at the end of the preceding section: in a limited range of temperature particles of a ternary phase form. Nathal et al. [39] measured the lattice constant \(a\) of the \(\gamma\)-phase of the experimental \(\gamma^\prime\)-strengthened nickel-base Alloy 143 (composition in at.\%: 2 Ta, 9 Mo, 13 Al, 76 Ni). The authors found an anomalous variation of \(a\) with temperature between 923 and 1073 K. The reason was the precipitation of Ni3Mo-particles. The final heat treatment of the alloy was at 1143 K. Below 1073 K, it was supersaturated with Mo. These Ni3Mo-particles were found only in the quoted range of temperature, because at lower temperatures diffusion is so slow that they do not form sufficiently fast and at higher temperatures all Mo is solved. Regrettably the authors did not investigate the effects of the Ni3Mo precipitates on the yield strength \(\sigma_y(T)\) and on the CRSS \(\tau_y\) of Alloy 143. It goes without saying that precipitates of ternary phases can raise \(\sigma_y(T)\) and \(\tau_y\) only if they do not disturb the formation of the \(\gamma^\prime\)-dispersion.

2.4. Dynamic strain aging effects

Dynamic strain aging (DSA) effects are brought about by the interaction of moving dislocations with mobile solute atoms: they diffuse towards the dislocations and pin them until the latter ones are released from the solute cloud by thermal activation. Since both processes, diffusion as well as breaking free, are thermally activated, DSA effects can only be observed in limited ranges of temperature and strain rate. DSA effects give rise to serrated, discontinuous stress-versus-strain curves and to maxima of the CRSS-versus-temperature curves [40].

DSA effects have been found for single-phase fcc solid solutions [40] as well as for single-phase long-range ordered intermetallic phases [2]. Such effects may be the cause of the peaks of the functions \(\sigma_y(T)\) and \(\tau_y(T)\) of \(\gamma^\prime\)-strengthened superalloys, no matter whether the DSA effects occur in the solid solution \(\gamma\)-matrix or in the L12-long-range ordered \(\gamma^\prime\)-precipitates. It is reiterated from Section 1.5 that several groups reported discontinuous yielding in the temperature range of the peaks of \(\sigma_y(T)\) and \(\tau_y(T)\) of superalloys. Moreover DSA effects render the peaks strain rate sensitive (see Section 1.4).

2.5. Lattice misfit of the \(\gamma^\prime\)-precipitates

Though the \(\gamma\)-matrix as well as the \(\gamma^\prime\)-precipitates in superalloys have the fcc crystal structure, their respective lattice constants \(a^\gamma\) and \(a^\gamma^\prime\) differ slightly. This misfit is quantified by the lattice misfit parameter \(\delta\):

\[
\delta = \frac{a^\gamma - a^\gamma^\prime}{0.5(a^\gamma + a^\gamma^\prime)}
\]

Depending on whether \(a^\gamma\) and \(a^\gamma^\prime\) are measured for single- or for two-phase materials, one obtains the unconstrained misfit \(\delta\) or the constrained one \(\delta^\prime\), respectively. In the case of two-phase materials, both phases elastically adjust their lattice constants somewhat to each other. Therefore \(|\delta^\prime|\) is smaller than \(|\delta|\) [39,41–50].

Since the thermal expansion coefficient of the \(\gamma\)-phase exceeds that of the \(\gamma^\prime\)-phase, the derivatives \(\partial\delta^\prime/\partial T\) and \(\partial\delta/\partial T\) are negative. If at high temperatures \(\delta^\prime\) is derived from a two-phase material, \(\delta^\prime\) and consequently \(\delta^\prime\) are affected by the increase of the solubility of the \(\gamma^\prime\)-forming elements in the \(\gamma\)-matrix. The thermodynamic equilibrium composition of the \(\gamma^\prime\)-precipitates varies much less with temperature [5,44,51]. The change in \(\gamma\)-composition may render \(\partial\delta/\partial T\) still more negative [50].

The lattice misfit is of exceeding technical importance because it governs the rafting process of cuboidal \(\gamma^\prime\)-precipitates during creep exposure at high temperatures. The application of an external uniaxial stress breaks the cubic symmetry of the misfit stress field. During the application of an external stress along a \(\langle 011\rangle\)-direction, the \(\gamma^\prime\)-forming elements diffuse such that the \(\gamma^\prime\)-cuboids are transformed to plates parallel to \(\langle 001\rangle\)-planes [39,47–49,52,53].

In the following it will be demonstrated that the temperature variation of the lattice misfit may give rise to peaks of the functions \(\sigma_y(T)\) and \(\tau_y(T)\) of \(\gamma^\prime\)-strengthened superalloys. In the analyses presented below it will be assumed: (a) that \(\delta^\prime\) is negative in the temperature range concerned and (b) that no changes of the compositions of the two phases occurred. Such changes brought about by the dissolution of \(\gamma^\prime\)-precipitates would lower \(\sigma_y(T)\) and \(\tau_y(T)\) and produce no peak.

Several groups [e.g. 42–46] calculated the stress fields surrounding cubic \(\gamma^\prime\)-precipitates in commercial \(\gamma^\prime\)-strengthened superalloys. Finite element methods were applied. Here the results published by Glazier and Feiler-Kniepmeier [42] and by Müller et al. [44,45] for the commercial \(\gamma^\prime\)-strengthened superalloy SRR 99 (composition in at.\%: 5 Co, 10 Cr, 3 W, 1 Ta, 12.0 Al, 2.8 Ti, 0.7 Ni, \(\gamma\)-volume fraction: 0.70) are referenced. These authors performed two-dimensional (2D) plane stress as well as three-dimensional (3D) calculations for \(T = 1253 K\). The results are expressed best by quoting the von Mises stress \(\sigma_{VM}\), which relates a three-dimensional state of stress to an equivalent stress in a uniaxial tension or compression test [42,43]. \(\sigma_{VM}\) turned out to reach higher values in the \(\gamma\)-matrix-channels than in the \(\gamma^\prime\)-cubes. Since the high stresses in the \(\gamma\)-matrix-channels govern the macroscopic plastic deformation, throughout the remainder of this Section 2.5 only these lattice stresses will be considered. Here three relevant results are listed:

(i) \(\sigma_{VM}\) assumes its maximum value \(\sigma_{VM_{\text{max}}}\) in the middle of the \(\gamma\)-\(\gamma^\prime\)-interfaces.

(ii) In 2D-calculations, allowing for the elastic anisotropy or disregarding it, leads to nearly the same stresses in the \(\gamma\)-matrix-channels.

(iii) \(\sigma_{VM_{\text{max}}}\) is proportional to Young’s modulus \(E_{111}\) along \(\langle 001\rangle\)-directions and to the absolute value of the unconstrained lattice misfit \(\delta^\prime\).
The following numerical values are inserted; they are typical of $\sigma_{\text{M max}}$-strengthened superalloys: $E_{011}$ (900 K) = 106 GPa, $\delta_{011} = 0.0001$, $\Delta T = 1.5$. $\Delta T$ is raised from 800 to 1000 K. In $\sigma_{\text{M max}}$-strengthened superalloys, the leading1 term of the function $\tau_{\text{f}}(\delta)$ and thus rendered $\sigma_{\text{M max}}$ was 336 MPa. $\delta_{011}$ (900 K) = 0.002, $R = 0.008$, $r = 10$ nm of NIMONIC PE16 increases from 133.4 to 141.1 MPa, i.e. by 5.8%. The yield strength $\tau_{\text{f}}(\delta)$ of NIMONIC PE16 increases from 133.4 to 141.1 MPa, i.e. by 5.8%. $\tau_{\text{f}}(\delta)$ of NIMONIC PE16 is based on the curvature of the bow of the leading $\delta_{011}$-dislocations between neighboring $\gamma$-precipitates. 

Inserting for $A_1$, $A_2$, and $S$ from Ref. [30] and allowing for $\tau_{\text{f}}$ as described in Refs. [30,55] leads to the result that $\tau_{\text{f}}(\delta) = 0.089$, $r = 10$ nm of NIMONIC PE16 decreases by 11.7%. Around 620 K, the calculated CRSSs $\tau_{\text{f}}(T)$ reach their minimum value. Below about 620 K, the negative derivative $\sqrt{\tau_{\text{f}}(T)}$ overweights the positive one $\sqrt{\tau_{\text{f}}(T)}$ and thus renders $\tau_{\text{f}}(T)$ negative. The main cause for the precipitate decrease of $\tau_{\text{f}}(T)$ above about 1000 K is the change of the $\gamma$-precipitate dispersion during heating the specimen to the deformation temperature and deforming it [30]. Evidently the decrease of $\delta$ with increasing $T$ gives rise to a quite pronounced peak of the function $\tau_{\text{f}}(\delta = \text{const.}, r = \text{const.}, T)$ of underaged $\gamma$-strengthened superalloys. It is reiterated that this peak producing mechanism is only possible in underaged specimens, neither in peak nor in overaged ones [55].

3. Assessment of the six peak producing mechanisms and conclusions

The most important interpretation of the peaks of the yield strength $\sigma_{\text{f}}$-versus-$T$ curves, respectively of the CRSS $\tau_{\text{f}}$-versus-$T$ curves, is probably the one detailed in Section 2.1: $\sigma_{\text{f}}(T)$ and $\tau_{\text{f}}(T)$ reflect the anomalies of the CRSS $\tau_{\text{f}}(T)$ of the strengthening $\gamma$-phase. But the models based on this idea, do not describe all aspects of the experimentally observed functions $\sigma_{\text{f}}(T)$ and $\tau_{\text{f}}(T)$ consistently. In Sections 2.2–2.6 five alternative peak producing mechanisms have been presented. Of course there may be even more such mechanisms. The relative importance of the six presently discussed mechanisms depends on many parameters of the superalloy under consideration: the temperature at which $\tau_{\text{f}}(T)$ of the strengthening $\gamma$-phase reaches its maximum, the size, shape, volume fraction, and lattice mismatch of the $\gamma$-precipitates, etc. In addition, there may be synergisms between the different peak producing mechanisms: e.g. the lattice misfit affects the probability of cross-slip. Hence without detailed analyses of the temperature dependences of all parameters governing $\sigma_{\text{f}}(T)$ and $\tau_{\text{f}}(T)$ it is impossible to single out one peak producing mechanism as the
dominant one. The operation of more than one peak producing mechanism can explain the appearance of two peaks of $\tau_c(T)$ of MAR-M200 reported by Copley et al. [8] (see Section 1). At first glance the mechanisms described in Sections 2.2–2.6 produce neither an orientation dependence (anisotropy, see Section 1.1) nor a tension/compression asymmetry (see Section 1.2) of $\tau_c(T)$. There have, however, been reports of anisotropies of the CRSS of single-phase fcc materials: pure Al [56] and Ni-40at.%Co [57]. Hence the anisotropy of the CRSS. Naturally this applies also to the yield strength -matrix of superalloys.

Similarly on the basis of the mechanism (e) described in Section 2.1, it appears as possible that the probability of cross-slip from the primary $\{111\}$-glide plane onto a secondary $\{111\}$-glide plane is different in tension and compression tests of any fcc material. This gives rise to an asymmetry of their yield strength. At high strains, cross-slip is also manifest at relatively low strains. Hence it is concluded that it is quite possible that more than one peak producing mechanism contributes substantially to the peaks of the yield strength and of the CRSS and that it depends on the superalloy under investigation whether the contribution of one mechanism outweighs those of all others. Some potential peak producing mechanisms may not become manifest, because they operate only in temperature ranges where the yield strength drop rapidly as the temperature is raised, i.e. below about 500 K and above about 1000 K.

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References
