Comparisons between characteristic lengths and fibre equivalent diameters in glass fibre and melamine foam materials of similar flow resistivity

Naoki Kino *, Takayasu Ueno

Shizuoka Industrial Research Institute of Shizuoka Prefecture, 2078 Makigaya, Aoi-ku, Shizuoka, Shizuoka 421-1298, Japan

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Abstract

Flow resistivity, tortuosity, viscous characteristic length and thermal characteristic length of three melamine foam and two glass wool samples have been measured. It has been found that a melamine foam sample with a bulk density 10.3 kg m\(^{-3}\) and a glass wool sample with a bulk density 28.0 kg m\(^{-3}\) have almost the same flow resistivity, however, the bulk density is as much as three times different. The cross-sectional pore shape factors, which are deduced with the non-acoustical parameters in the Johnson–Allard model, of the melamine foam are smaller than those of the glass wool. This paper also discusses a new relationship between the flow resistivity, the fibre equivalent diameter and the bulk density in melamine foam.

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1. Introduction

The fibrous structure of a glass wool is shown in Fig. 1a and the open cell structure of a melamine foam is shown in Fig. 1b. The most obvious difference between melamine foam and glass wool is that the former is an isotropic foam and the latter is an anisotropic layered fibrous material. Melamine foam has a cellular structure with open cells. Fig. 1b shows the absence of cell walls and short hexagonal cellular struts in the melamine foam. The short hexagonal struts of the cells in the foam are regarded as ‘fibre equivalents’ for the purposes of this paper. The diameter of glass fibre is almost equal to 7 \(\mu\)m. The melamine fibre equivalent diameter is slightly smaller than the diameter of the glass fibre. The cell size of melamine foam is about 100–150\(\mu\)m (diameter).

Melamine foam is a light sound-absorbing material. The flow resistivity for the melamine foam with a bulk density 10.3 kg m\(^{-3}\) is practically 13,100 Pa s m\(^{-2}\). According to the Bies model (Eq. (1) [1]) the flow resistivity of glass fibre with the same density and with mean fibre diameter 7 \(\mu\)m is 2300 Pa s m\(^{-2}\).

\[
\sigma_{\text{Bies}}d_{\text{Bies}}^{2} \rho_{1}^{-1.53} = 3.18 \times 10^{-9},
\]

where \(\sigma_{\text{Bies}}\) is the flow resistivity and \(d_{\text{Bies}}\) is the diameter of a circular cross-sectional glass fibre.

To demonstrate the acoustical effects of the different internal structure of the melamine foam, the two characteristic lengths of the two materials (glass wool and melamine foam) with the same flow resistivity are investigated. Additionally, the bulk density and the flow resistivity of the two materials with a same viscous characteristic length are also investigated. The non-acoustical parameters of the melamine foam are found to be different from usual rigid-framed fibrous material. In Section 2, the non-acoustical parameters in the Johnson–Allard model are described. Subsequently, in Section 3, careful measurements of the non-acoustical parameters of glass wool and melamine foam materials are described. For the melamine foam the
The Johnson–Allard model

According to the Johnson–Allard model, the flow resistivity [2–6], porosity [7], tortuosity [8], viscous characteristic length [8] and thermal characteristic length [9] are indispensable non-acoustical parameters, used to determine the effective density and the bulk modulus of rigid-framed fibrous materials. Eq. (2) is the equation of the effective density of rigid-framed materials, at the audible frequency, as proposed by Johnson et al. [8]. Eq. (5) is the equation of the bulk modulus of rigid-framed materials, at the audible frequency, as proposed by Allard et al. [10]. The viscous characteristic length $\lambda$ depends only on the geometry of the frame. Johnson et al. [8] pointed out Eq. (4) as the relation between $\lambda$ and $(8x_{\infty}H/\phi)^{1/2}$. The thermal characteristic length $\lambda'$ is the surface to volume ratio of pores with no weighting. Champoux and Allard [9] proposed Eq. (7) as the relation between $\lambda'$ and $(8x_{\infty}H/\phi)^{1/2}$.

$$\rho_0(\omega) = \rho_0 x_{\infty} \left(1 + \frac{\sigma \phi}{2x_{\infty} \rho_0 \omega} G_1(\omega)\right),$$

(2)

with

$$G_1(\omega) = \left(1 + \frac{4i \omega x_{\infty} \rho_0 \omega}{\sigma^2 A^2 \phi} \right)^{1/2},$$

(3)

$$\lambda = \frac{1}{c} \left(\frac{8x_{\infty}H}{\sigma \phi}\right)^{1/2},$$

(4)

$$K(\omega) = \gamma P_0 \left[\gamma - (\gamma - 1) \left(1 + \frac{8H}{2x_{\infty} \rho_0 P_0 \omega} G_1(\Pr \omega)\right)^{-1}\right],$$

(5)

with

$$G_1(\Pr \omega) = \left(1 + \frac{i \rho_0 \lambda^2 \Pr \omega}{16 \eta}\right)^{1/2},$$

(6)

$$\lambda' = \frac{1}{c'} \left(\frac{8x_{\infty}H}{\sigma \phi}\right)^{1/2},$$

(7)

where $\rho_0$ is the density of the air, $x_{\infty}$ is the tortuosity, $\sigma$ is the flow resistivity, $\phi$ is the porosity, $\omega$ is the angular frequency, $i = \sqrt{-1}$, $\eta$ is the viscosity of the gas, $P_0$ is the atmospheric pressure, $\gamma$ is the specific heat ratio of the gas, $Pr$ is the Prandtl number of the gas, and $c$ and $c'$ are the cross-sectional shape factors of the pore.

Eqs. (4) and (7) in the Johnson–Allard model show that the two characteristic lengths are related to the pore shape factors, tortuosity, porosity and flow resistivity. So materials with the same flow resistivity but different pore structures should have different characteristic lengths.

<table>
<thead>
<tr>
<th>Sample number</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measurements</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$ (kg m$^{-3}$)</td>
<td>28.0</td>
<td>31.8</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>25.0</td>
<td>25.0</td>
</tr>
<tr>
<td>$\sigma$ (Pa s m$^{-1}$)</td>
<td>11,900</td>
<td>16,800</td>
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<tr>
<td>$x_{\infty}$</td>
<td>1.0108</td>
<td>1.0093</td>
</tr>
<tr>
<td>$\lambda$ (µm)</td>
<td>143</td>
<td>132</td>
</tr>
<tr>
<td>$\lambda'$ (µm)</td>
<td>302</td>
<td>237</td>
</tr>
<tr>
<td><strong>Estimations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.989</td>
<td>0.987</td>
</tr>
<tr>
<td>$c$</td>
<td>0.78</td>
<td>0.71</td>
</tr>
<tr>
<td>$c'$</td>
<td>0.37</td>
<td>0.40</td>
</tr>
<tr>
<td><strong>Predictions (Johnson–Allard model)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_0$ (µm)</td>
<td>6.6</td>
<td>6.1</td>
</tr>
<tr>
<td>$L \times 10^3$ (m$^{-3}$)</td>
<td>3.273</td>
<td>4.301</td>
</tr>
<tr>
<td>$\lambda$ (µm)</td>
<td>147</td>
<td>121</td>
</tr>
<tr>
<td>$\lambda'$ (µm)</td>
<td>294</td>
<td>242</td>
</tr>
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</table>
Table 2
Measured, estimated and predicted parameters of three melamine foam samples

<table>
<thead>
<tr>
<th>Sample number</th>
<th>31</th>
<th>32</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurements</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_1 ) (kg m(^{-3} ))</td>
<td>8.6</td>
<td>10.3</td>
<td>13.27</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>32.0</td>
<td>25.5</td>
<td>10.5</td>
</tr>
<tr>
<td>( \sigma ) (Pa s m(^{-2} ))</td>
<td>10,500</td>
<td>13,100</td>
<td>17,500</td>
</tr>
<tr>
<td>( \varepsilon_{\infty} )</td>
<td>1.0059</td>
<td>1.0053</td>
<td>1.0055</td>
</tr>
<tr>
<td>( \lambda ) (( \mu \text{m} ))</td>
<td>240</td>
<td>199</td>
<td>161</td>
</tr>
<tr>
<td>( \lambda' ) (( \mu \text{m} ))</td>
<td>470</td>
<td>445</td>
<td>375</td>
</tr>
<tr>
<td>Estimations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.995</td>
<td>0.993</td>
<td>0.992</td>
</tr>
<tr>
<td>Predictions (Johnson–Allard model)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>0.49</td>
<td>0.53</td>
<td>0.57</td>
</tr>
<tr>
<td>( c' )</td>
<td>0.25</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>Predictions (Kino and Allard models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{Kino}} ) (( \mu \text{m} ))</td>
<td>5.43</td>
<td>5.58</td>
<td>5.86</td>
</tr>
<tr>
<td>( L \times 10^3 ) (m(^{-2} ))</td>
<td>2.367</td>
<td>2.684</td>
<td>3.135</td>
</tr>
<tr>
<td>( \lambda_{\text{Kino}} ) (( \mu \text{m} ))</td>
<td>248</td>
<td>213</td>
<td>173</td>
</tr>
<tr>
<td>( \lambda'_{\text{Kino}} ) (( \mu \text{m} ))</td>
<td>495</td>
<td>425</td>
<td>347</td>
</tr>
</tbody>
</table>

By transforming Eq. (4), Eq. (8) was obtained. Similarly, by transforming Eq. (7), Eq. (9) was obtained. For the highly porous fibrous materials (\( \varepsilon_{\infty} \equiv 1 \) and \( \phi \equiv 1 \)), the pore shape factors and the two characteristic lengths are important for the flow resistivity.

\[
\sigma = \frac{8\varepsilon_{\infty}\eta}{\phi\lambda^2c^2}, \quad (8)
\]

\[
\sigma = \frac{8\varepsilon_{\infty}\eta}{\phi\lambda'^2c^2}. \quad (9)
\]

3. Experiment method and results

3.1. Material

The important non-acoustical parameters of two glass wool and three melamine foam samples used in the experiment are listed in Tables 1 and 2, respectively. The bulk densities of glass wool samples were 28 kg m\(^{-3} \) and 31.8 kg m\(^{-3} \). The bulk densities of melamine foam samples were all about 10 kg m\(^{-3} \). Measurements of the flow resistivity were made with a device in accordance with the ISO standard 9053 [11]. Measurements of the tortuosity and the two characteristic lengths were made by a method similar to that proposed by Leclaire et al. [12] involving saturation by two different gases, in this case air and argon [13].

Porosity was estimated from Eq. (10).

\[
\phi = 1 - \rho_1/\rho_m, \quad (10)
\]

where \( \rho_1 \) and \( \rho_m \) are the densities of the porous medium and the raw material, respectively.

The assumed densities of glass and melamine were 2500 kg m\(^{-3} \) and 1570 kg m\(^{-3} \), respectively.

The cross-sectional shape factors were predicted from the measurements of flow resistivity, tortuosity and two characteristic lengths according to the Johnson–Allard model. Measured and estimated values of \( \varepsilon_{\infty} \), \( \sigma \), \( \lambda \), \( \lambda' \) and \( \phi \) were substituted in Eqs. (4) and (7) and the cross-sectional shape factors were predicted.

Allard and Champoux [14] showed that sound propagation in rigid-framed fibrous materials depends on the total length of fibres per unit volume of a material. Eqs. (11)–(13) show the relationship of the two characteristic lengths and the total length of fibres per unit volume of a material. They are applicable to a case where the velocity of the air is perpendicular to the direction of the fibres.

\[
L = 4\rho_1/\pi d^2 \rho_m, \quad (11)
\]

\[
\lambda_{\lambda} = 1/2\pi rL, \quad \text{where} \quad r = d/2, \quad (12)
\]

\[
\lambda'_{\lambda} = 2\lambda_{\lambda}, \quad (13)
\]

where \( d \) is the diameter of a fibre, \( L \) is the total length of fibres per unit volume of a material, \( \lambda_{\lambda} \) is the viscous characteristic length, and \( \lambda'_{\lambda} \) is the thermal characteristic length.

The flow resistivities \( \sigma \) of the glass wool samples in Table 1 were measured. By using Eq. (1) with \( \sigma_{\text{Bies}} = \sigma \), the glass fibre diameter \( d_{\text{Bies}} \) in Table 1 was predicted. The glass fibre equivalent diameter \( d_{\text{Bies}} \) predicted from Eq. (1), \( d = d_{\text{Bies}} \) was substituted for Eq. (11) so that the total length of fibre per unit volume \( L \) was obtained. The total length of fibres per unit volume was substituted in Eq. (12) so that the two characteristic lengths \( \lambda_{\lambda} \) and \( \lambda'_{\lambda} \) were predicted.

The following similar relationship (Eq. (14)) for melamine fibre equivalents was obtained from measurements of flow resistivity, bulk density and two characteristic lengths.

\[
\sigma_{\text{Kino}} d_{\text{Kino}}^2 \rho_1^{1.53} = 11.5 \times 10^{-9}, \quad (14)
\]

where \( \sigma_{\text{Kino}} \) is the flow resistivity, \( d_{\text{Kino}} \) is the diameter of a hexagonal cellular strut in the melamine foam and the constant “11.5 \times 10^{-9}” was obtained from a manual fit as described in the following paragraph.

The flow resistivities \( \sigma \) of the melamine foam samples in Table 2 were measured. The melamine fibre equivalent diameter \( d_{\text{Kino}} \) predicted from Eq. (14), \( d = d_{\text{Kino}} \) was substituted for Eq. (11) so that the total length of fibre equivalents per unit volume \( L \) was obtained. The total length of fibre equivalents per unit volume was substituted in Eq. (12) so that the two characteristic lengths \( \lambda_{\lambda} \) and \( \lambda'_{\lambda} \) were predicted. Then, the right side value of Eq. (14) was adjusted, so that the value of \( \lambda_{\lambda} \) and \( \lambda'_{\lambda} \) were suitable for the measured characteristic lengths of \( \lambda \) and \( \lambda' \) shown in Table 2.

By using Eq. (14) with \( \sigma_{\text{Kino}} = \sigma \), the fibre equivalent diameter \( d_{\text{Kino}} \) in Table 2 was predicted. The predicted diameter of melamine fibre equivalents in Table 2 is about 5.5 \( \mu \text{m} \). It is in good agreement with visual inspection [see Fig. 1b]. The measurements of \( \lambda \) and \( \lambda' \) shown in Table 2 are compared with the predictions of \( \lambda_{\lambda} \) and \( \lambda'_{\lambda} \) shown in Table 2. The predictions are close to the measurements.
The discrepancies between the measurements of $\lambda$ and $\lambda'$ and the predictions of $\lambda_A$ and $\lambda'_A$ are examined in detail. For the viscous characteristic lengths of the three melamine foam samples shown in Table 2 the discrepancy is represented as $100 \times |\lambda_A - \lambda|/\lambda$. The mean value for the melamine foam data is 5.9%. For the thermal characteristic lengths of the three melamine foam samples shown in Table 2 the discrepancy is represented as $100 \times |\lambda'_A - \lambda'|/\lambda'$. The mean value for the melamine foam data is also 5.9%. The predictions are very close to the measurements, so that the predictions of the two characteristic lengths are judged to be accurate. For the melamine foam samples, it is found that the two characteristic lengths are derivable from the fibre equivalent diameter as shown in Eq. (14).

### 3.2. Verification of the measurements of non-acoustical parameters

In this section, the measurements of the flow resistivity data, the tortuosity data and the two characteristic lengths data in Tables 1 and 2 are verified. The measured flow resistivities were used to evaluate the characteristic impedance ($Z_c$) and propagation constant ($\Gamma$) of the porous medium using the Delany and Bazley expressions [5].

$$Z_c(\omega) = \rho_0 c_0 \left[ 1 + 0.0571 (\rho_0 f/\sigma)^{-0.754} - i0.087 (\rho_0 f/\sigma)^{-0.732} \right],$$  
$$\Gamma(\omega) = \frac{\sigma}{c_0} \left[ 0.189 (\rho_0 f/\sigma)^{-0.595} + i \left[ 1 + 0.0978 (\rho_0 f/\sigma)^{-0.700} \right] \right],$$  

where $c_0$ is the sound velocity in the air, and $f$ is the frequency.

The normal incidence plane wave absorption coefficient ($\alpha$) for a hard-backed porous layer was then calculated from

$$\alpha = 1 - |r|^2,$$

where $\alpha$ is the normal incidence plane wave absorption coefficient.

The absorption coefficient was also measured using an impedance tube in accordance with the ISO standard transfer-function method [15]. The predicted normal incidence absorption coefficients are compared with the measured data in Fig. 2. The predictions and measurements are close. Thus the flow resistivity data in Tables 1 and 2 were judged to be sufficiently accurate.

The measurement results for the ultrasonic propagation in air and argon have been used to determine the two characteristic lengths and the tortuosity. The results for the squared refraction index as a function of the square root of the inverse frequency are shown in Fig. 3. The accuracy of the measurements of tortuosity and two characteristic lengths was tested by using the measured values to predict the sound velocity through Eq. (19). By transforming the wave number equation at high frequencies [16], Eq. (19) is obtained. Subsequently the predicted and measured sound velocities were compared as shown in Fig. 4. The temperature was 22°C during the experiment.
where \( c_{\text{high}} \) is the sound velocity in the materials at high frequencies, and \( \delta \) is the viscous skin depth.

The discrepancy between measured and predicted sound velocities is calculated as follows:

\[
100 \times \frac{|c_m - c_{\text{high}}|}{c_m},
\]

where \( c_m \) is the measured frequency-dependent sound velocity and \( c_{\text{high}} \) is the predicted frequency-dependent sound velocity.

For the glass wool sample 2, the mean value of the sound velocity discrepancy between measurement and prediction in the frequency range between 100 kHz and 800 kHz was 0.06%. For the sample 2, the maximum value of the sound velocity prediction difference was 0.88 m s\(^{-1}\). For the melamine foam sample 32, the mean value of the sound velocity prediction discrepancy in the frequency range between 100 kHz and 800 kHz was 0.06%. For the sample 32, the maximum value of the sound velocity prediction difference was 0.54 m s\(^{-1}\). The predicted sound velocities are very close to the measured ones, so that the measurements of the tortuosity and the two characteristic lengths in Tables 1 and 2 are judged to be highly accurate.

4. Discussion

The predicted absorption coefficient for hard-backed material samples by the Johnson–Allard model and the measured one are shown in Fig. 5. The predictions are close to measurements. The different pore structures of the two materials (glass fibre and melamine foam) are discussed using the Johnson–Allard model. For the two glass wool samples in Table 1, the values of \( c \) are predicted to be 0.71 and 0.78. For the melamine foam samples in Table 2, the values of \( c \) are predicted to be between 0.49 and 0.57. According to the Johnson–Allard model, it is found that the melamine foam is an efficient material that achieves large flow resistivity by lowering the cross-sectional shape factors, though the two characteristic lengths are much larger than those for glass wool.

Next, two glass wool examples in Table 3 are prepared to investigate the melamine foam sample 32 more in detail. Using 7.0 \( \mu \text{m} \) as the diameter of a glass fibre, the flow resistivity of the glass wool examples is predicted by the Bies model [Eq. (1)]. The two characteristic lengths are predicted by using the Allard model [Eqs. (11)–(13)]. Using 1.0 as the tortuosity, the cross-sectional shape factors are predicted by using the Johnson–Allard model [Eqs. (4) and (7)].
By comparing melamine foam sample 32 and glass wool example 1 with a same flow resistivity 13,100 Pa s m$^{-2}$, it is found that the bulk density for melamine foam is 0.32 times smaller than that for glass wool. It is also found that the viscous characteristic length for melamine foam is 1.46 times larger than that for glass wool and the thermal characteristic length for melamine foam is 1.63 times larger than that for glass wool. Additionally, it is found that the cross-sectional shape factor $c$ for melamine foam is 0.68 times smaller than that for glass wool and $c'$ for melamine foam is 0.62 times smaller than that for glass wool.

By comparing melamine foam sample 32 and glass wool example 2 with a same viscous characteristic length 199 $\mu$m, it is found that the bulk density of melamine foam is 0.47 times smaller than that for glass wool. It has been also found that the flow resistivity for melamine foam is 1.79 times larger than that for glass wool. Additionally, it is found that the cross-sectional shape factor $c$ for melamine foam is 0.75 times smaller than that for glass wool and $c'$ for melamine foam is 0.69 times smaller than that for glass wool. As a result, it is found that the structure of melamine foam is clearly different from that of glass wool.

The predicted absorption coefficient for hard-backed material samples by the Johnson–Allard model for the melamine foam sample 32 shown in Fig. 5b is less accurate than the glass wool sample 2 shown in Fig. 5a. The predictions by our new model [17] as shown in Eqs. (22)–(25) have been executed and are shown in Fig. 6.

\[
\rho(\omega) = \rho_0 x_a \left( 1 + \frac{\sigma \phi}{\omega} \right) G_N(\omega), \quad (22)
\]

with

\[
G_N(\omega) = \left( \sqrt{2\eta\rho_0 \omega} \frac{x_a (1+i)\sqrt{N_1}}{\sigma \phi \omega} \right)^{1/2}, \quad (23)
\]

\[
K(\omega) = \gamma P_0 \left[ \gamma - (\gamma - 1) \left[ 1 + \frac{8\eta}{\ln^2 \rho_0 \omega} \right] \right], \quad (24)
\]

\[
G_N'(Pr\omega) = \left( \sqrt{2\eta\rho_0 \omega} \frac{\lambda'(1+i)\sqrt{N_2}}{8\eta} \right)^{1/2}, \quad (25)
\]

where $N_1$ and $N_2$ are the correction factors.

The predicted and measured absorption coefficients for hard-backed material are shown in Fig. 6a. Those with rear air layer of 20 mm are shown in Fig. 6b. The two correction factors were 8.5($N_1$) and 250($N_2$). The discrepancy between measurement and prediction in Fig. 6 was calculated by

\[
100 \times \frac{|x_m - x_p|}{x_m}, \quad (26)
\]

where $x_m$ is the measured frequency-dependent narrow band absorption coefficient and $x_p$ is the predicted frequency-dependent narrow band absorption coefficient.

The discrepancy using our new model is 5% or less in the frequency range between 400 and 5 kHz, so it has been found that our prediction model with the two correction factors is effective for the melamine foam.

5. Concluding remarks

According to the Johnson–Allard model, it has been found that the melamine foam achieves large flow resistivity by lowering the cross-sectional pore shape factors, despite the fact that the two characteristic lengths are much larger than those of the glass wool with a diameter of a
glass fibre of 7 μm. We also showed the possibility that the two characteristic lengths for the melamine media are derivable using the measured flow resistivity data and bulk density data.

Acknowledgement

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References

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