A distributed plasticity model for cyclic analysis of concrete-filled steel tube beam-columns and composite frames

Jerome F. Hajjar and Aleksandr Molodan
Department of Civil Engineering, University of Minnesota, 500 Pillsbury Drive SE, Minneapolis, MN 55455-0220, USA

Paul H. Schiller
Barr Engineering Company, 8300 Norman Center Drive, Minneapolis, MN 55437-1026, USA

This paper presents the constitutive formulation and cyclic analysis capability of a three-dimensional fiber-based distributed plasticity finite element for square or rectangular concrete-filled steel tube (CFT) beam-columns. A related paper outlines the geometrically nonlinear formulation and the element's formulation for interlayer slip between the steel tube and concrete core. The present paper discusses the steel and concrete cyclic material models, which are each based on stress–space bounding–surface plasticity formulations. The paper details the calibration of these models to steel coupon tests and to experiments of short CFTs in flexure which yield moment–curvature–thrust results. Verification of the model is provided by comparing to several experiments of CFT beam-columns loaded monotonically and cyclically, proportionally and non-proportionally, and in single and reverse curvature. A final example presents a comparison of the analysis to the experimental results of a composite subassemblage consisting of three steel I-girders framing rigidly into a CFT beam-column, which is thus subjected to axial force plus cyclic biaxial bending. © 1997 Elsevier Science Ltd.

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1. Introduction

In the last several years, efforts have been increasing worldwide to determine the effectiveness of using concrete-filled steel tube (CFT) beam-columns as part of unbraced composite framing systems in areas subjected to seismic activity. These structural systems typically consist of steel I-girders framing rigidly into CFT beam-columns to form either one-way or two-way unbraced composite frames. Such composite systems may be most amenable for low- to moderate-rise structures, and they take advantage of the many beneficial features exhibited by CFTs. This paper presents a three-dimensional, cyclic, geometrically and materially nonlinear distributed plasticity finite element model for square or rectangular CFT beam-columns used in these types of composite frames. This formulation is most effective for studying the entire load–deformation behavior of CFTs as part of complete composite frames subjected to either monotonic or cyclic loading. It is also suitable for conducting comprehensive parametric studies of individual CFT beam-columns or composite subassemblages, which, in conjunction with experiments, may provide the data required to improve the accuracy and scope of current non-seismic and seismic design specification provisions for CFTs.

The distributed plasticity model, developed within the context of a stiffness-based beam-column finite element formulation, discretizes the CFT cross-sections at the beam ends into a grid of fibers, and the stress–strain behavior of each steel and concrete fiber is monitored explicitly during
the loading history. Frame members are modeled as line beam-column elements with their degrees-of-freedom located at the centroidal axis at each element end. The cross-sectional stiffness of each element end is obtained using numerical integration over all the fibers, and interpolation functions are used to integrate along the element length. This type of finite element has been successfully used by many researchers to model structural frames. For analyzing individual CFT beam-columns, Kawaguchi et al. has presented limited details of a similar two-dimensional cyclic formulation.

A comparable fiber-based finite element formulation for steel wide-flange members developed by White and Tomii is adopted in this work for use in composite frame analyses, and it forms much of the basis for the development of the geometrically nonlinear formulation used for the CFT elements. The modeling of geometric nonlinearity is based upon a co-rotational formulation, and the terms retained in the stiffness matrix and force recovery account sufficiently for all significant $P-\Delta$ and $P-\Delta$ effects within the CFT beam-column (Schiller and Hajjar and Hajjar et al. provide details of the geometric nonlinear stiffness formulation, the force recovery procedure, and the Newton-Raphson incremental/iterative nonlinear solution strategy).

The steel and concrete constitutive formulations account for the significant inelastic phenomena which are exhibited in cyclic CFT experiments and which greatly affect the behavior of CFTs. The steel formulation, adapted from Shen et al., models the rounded shape of the stress-strain curve found in cold-formed tube steel, a decreasing elastic zone with increased cyclic excitation, cyclic hardening, ratchetting, and the different stress-strain behavior exhibited in the corners and flanges of cold-worked steel tubes. The concrete formulation, adapted from Chen and Buyukozturk, models strength and stiffness degradation by means of a cumulative damage parameter, and the post-peak behavior of the concrete is calibrated to account implicitly for the effects of confinement of the concrete core by the steel tube. The concrete formulation also models fibers which cycle into tension and then back into compression. In addition, slip is accounted for between the steel and concrete components of the CFT by incorporation of a nonlinear slip interface. This interface allows axial movement of the concrete core with respect to the steel tube. It is able to capture behavior ranging from perfect bond to perfect slip, and it permits modeling of the gradual transfer of stress between the steel tube and concrete core in the connection region of CFT beam-columns.

A related paper outlines the geometrically nonlinear formulation for this beam-column element, including the capability to model interlayer slip. That paper also presents the calibration and verification of the slip model, and verification of the beam-column finite element formulation for monotonic loading, including comparisons to a number of tests of CFT beam-columns. Over 30 verification problems were chosen to represent a wide range of material strengths and cross-section dimensions so as to verify the accuracy and robustness of the formulation. The current paper presents the constitutive models, briefly summarizes the verification of the beam-column model vs monotonically loaded experiments, and presents verification of the model results of cyclic CFT experiments. A final experimental comparison consists of a cyclically loaded three-dimensional subassembly of three steel I-beams framing into a CFT beam-column, which is thus subjected to biaxial bending plus axial compression.

The current work is part of an ongoing research program to develop analysis algorithms capable of modeling the geometrically and materially nonlinear behavior of three-dimensional CFT beam-columns and their connections as part of composite frame systems. Previous work has included the development and calibration of a macro nonlinear CFT beam-column finite element, and the corresponding general purpose software is suitable for conducting static, transient dynamic, and dynamic or buckling eigenvalue analysis. The current research has to date included the development of the fiber element beam-column model. The corresponding software may currently be used for performing both monotonic and cyclic static analysis.

The scope of this research is limited to square or rectangular CFTs which are completely filled with concrete. The CFTs are presumed to have no shear connectors or internal reinforcing bars so as to facilitate construction; composite action is presumed to be provided solely by the steel-concrete bond at the material interface. Time and rate dependent effects of the materials are not modeled. In addition, flexural-torsional and lateral-torsional buckling are not modeled, since these failures modes are rarely dominant in CFTs due to the significant torsional stiffness of the composite section. Also, since shear and torsional forces are expected to be small in frame members, nonlinearity due to these forces is neglected, and shear deformations due to flexure are neglected (Euler-Bernoulli beam theory is assumed). All connections are assumed to be fully-restrained. Residual stresses in the steel tubes are not explicitly modeled, since the through-thickness residual stresses, which tend to be the largest for cold-formed welded steel tubes, are accounted for indirectly in the steel coupon tests used to calibrate the steel model. In conjunction with this, residual straining is modeled by accounting for initial plastic strains, as will be discussed in the section describing the steel model calibration. Local buckling is currently not modeled in this formulation.

Monotonic experimental load-deformation curves have been used for verification of CFTs having width-to-thickness ratios varying from approximately 15 to 50, concrete strengths ranging from 20 to 50 MPa, and typical values of steel yield strengths. For cyclic analysis, sections with a similar range of $D/t$ values and concrete strengths of 20–35 MPa were analyzed and verified.

2. Constitutive modeling

2.1. Cyclic behavior of CFT beam-columns

The requirements for the steel and concrete constitutive models on the stress-strain level must be selected by identifying the key behavioral characteristics of the structural response of CFTs on the force-deformation level. Figure 1 illustrates typical behavior of a CFT beam-column subjected to cyclic loading, showing the load-deformation response of a square CFT beam-column tested by Sakino and Tomii. A schematic illustration of the test setup is shown in the inset. The specimen was subjected to a constant axial load, $P = 0.3P_t$ (where $P_t = 528$ kN is the CFT peak axial strength), and a cyclic shear loading, $Q$, applied nonproportionally for three full cycles at each increasing increment of chord rotation, $R$, which was varied from 0.5
Elastic unloading: CFTs unload elastically, such that the stiffness at the load reversal is nearly equal to the stiffness of a virgin specimen. However, the elastic stiffness degrades somewhat due to the concrete crushing after the first half-cycle of loading before reaching a nearly constant value (e.g. compare the stiffness of segment A'B' to that of segment AB in Figure 1).

Decreasing elastic zone: the size of the 'elastic zone' of a CFT reduces with cyclic loading, mainly due to steel local buckling and concrete crushing, as well as to the reduction in size of the elastic zone of the steel material. Eventually, the elastic zone stabilizes at a non-zero value (this is seen in comparing segments AB and A'B' in Figure 1).

Strength degradation: the maximum strength achieved within each hysteretic cycle degrades as the analysis proceeds, due primarily to local buckling of the steel tube and to damage of the concrete (compare peaks Q_{max} to Q_{max} in Figure 1). However, in the earliest cycles of loading, the experiments of Sakino and Tomii show that an increase of CFT capacity beyond the nominal monotonic strength is sometimes observed. This increase may be attributed to steel cyclic hardening, or to the structure being stiffer than expected due to support fixity in the experiments.

Bauschinger effect: the Bauschinger effect typically exhibited at the stress level by the steel tube is seen in Figure 1 to propagate to the stress-resultant level of a CFT.

Gradual stiffness reduction: during a loading excursion, the stiffness of a CFT reduces gradually from its initial elastic value, due to both geometric and material nonlinearity, as evidenced in each cycle in Figure 1.

Bounding stiffness: CFTs exhibit a limiting bounding stiffness, which may be seen most clearly in the last two hysteretic cycles in Figure 1. The bounding stiffness evolves due to the stabilization of the steel tube, even after local buckling.

This stress-resultant behavior exhibits the cumulative effects of material nonlinearity at the stress-point level in both the steel and concrete, coupled with the effects of geometric nonlinearity of the member. Thus, the above stress-resultant behavior must be reflected in the cyclic stress-strain models of the steel and concrete.

Bridge, Tomii and Sakino, Kawaguchi et al., Tsuji et al. and other researchers have presented related two-dimensional fiber model formulations for monotonic loading, primarily to assess cross-section strength of CFTs. They invoked relatively simple constitutive relations for their models, which yielded good results for monotonic loading, especially for evaluation of cross-section strength. However, for cyclic excitation, a more thorough assessment of the requirements of the constitutive relations is needed. The following sections summarize the primary features of the constitutive models used in this research.

2.2. Steel constitutive model

The steel constitutive model developed by Shen et al. is adapted to this research for modeling each fiber in the steel tube. The model utilizes a bounding surface formulation in stress-space. This plasticity formulation uses two nested surfaces, an inner loading surface and an outer bounding surface. The inner loading surface represents the locus of stresses which cause the initiation of yielding at the stress point (i.e. at the fiber). The outer surface represents the stress state at which a limiting stiffness of the steel fiber is achieved. When the stress point lies within the loading surface, the material undergoes elastic behavior. Once the stress point has contacted the loading surface, the response is governed by a number of hardening rules which determine subsequent inelastic behavior. As the steel fiber is loaded in elastically, both the loading and the bounding surfaces may translate (kinematic hardening), or contract or expand (isotropic hardening), to model key material phenomena. The plastic modulus of the fiber changes non-linearly from infinity to a limiting bounding value as the stress point moves from initial contact with the loading surface to the bounding surface. The von Mises surface establishes the shape for both the loading and bounding surfaces. The value of the plastic modulus, E, is a function of the distance, δ, between the two surfaces and is taken as:
where $E^p_0$ is the bounding plastic modulus after contact with the bounding surface, and $E^p$ is the virtual value of $E$ at first contact with the loading surface. In this work, the distance $\delta$ is measured from the beginning-of-step stress point to conjugate point on the bounding surface, as defined by Mroz and Dafalias and Popov.

This steel model has several key features geared specifically to capturing appropriate stress–strain behavior necessary for obtaining the CFT stress-resultant behavior discussed in the previous section. A summary of the features most relevant to CFT behavior follows. For further details of this formulation.

1. **Lack of yield plateau**: coupons taken from cold-formed steel tubes usually do not exhibit a yield plateau, especially for the material in the corners of the tube, due to the cold-working process. Consequently, no yield plateau is modeled in this work.

2. **Elastic unloading**: the steel formulation unloads elastically until the loading surface is recontacted. As concrete does not retain its initial stiffness during unloading, the immediate elastic unloading behavior seen in CFTs, even after large plastic excursions, is due to the behavior of the steel.

3. **Cyclic hardening**: to capture the effects of cyclic hardening in the early stages of loading (as exhibited for the CFT of Figure 1), the bounding surface hardens isotropically. To stabilize this hardening behavior during cycling within a constant plastic strain range, the rate of change of the bounding surface size is a function of the size of a plastic strain–space surface, often referred to in the literature as a nonhardening surface. The nonhardening surface hardens both isotropically and kinematically to envelop the loading path in the plastic strain space. During cycling, in which the plastic strain point is within the nonhardening surface, cyclic hardening of the bounding surface stabilizes. The hardening rules for the plastic strain–space surface are formulated so as to incorporate a nonhardening memory of past straining, such that a repetitive excursion in a particular plastic strain range produces the same effect on the cyclic hardening stabilization as a single excursion in the same range. Thus, if a specimen is cycled with a constant plastic strain range which is symmetric about the origin, the nonhardening region expands during the first cycle, and does not change thereafter.

4. **Decreasing elastic zone**: the loading surface in this model softens isotropically to provide a decreasing zone of elastic behavior of the steel. Several researchers have reported this behavior in structural steel, and its effects are seen at the stress-resultant level in CFTs (Figure 1). The decrease is a function of the size of the nonhardening surface in plastic strain–space. The change in size is rapid at low levels of plastic strain, but it stabilizes after achieving a plastic strain of approximately 1%. In addition, for cold formed steel tubes, the cold-working process introduces plastic prestraining, so that the size of the initial elastic zone is noticeably smaller than the yield stress of the virgin material. As will be discussed in the section on calibration, this effect is accounted for in this research by specifying a non-zero initial size of the nonhardening surface in plastic strain space.

5. **Stiffness degradation**: the decreasing elastic zone indirectly contributes to stiffness degradation with increased cyclic excitation, an effect seen in Figure 1.

6. **Bounding stiffness**: as is seen in Figure 1, CFTs exhibit a limiting bounding stiffness after sustaining a substantial plastic excursion. Correspondingly, the steel bounding surface model achieves a limiting bounding modulus once the stress point contacts the bounding surface, and this value is retained until a load reversal occurs. The value of the bounding modulus gradually decreases with accumulation of plastic work, so as to match experiments of structural steel which exhibit this phenomenon.

7. **Overshooting**: a common shortcoming of bounding surface models is that they exhibit overshooting. Overshooting is most pronounced for uniaxial loading, and it manifests itself in predicting unrealistically stiff response for plastic loading which follows a small plastic excursion in the reverse direction. To address the overshooting phenomenon, the shape parameter $h$, is specified to be a linear function of $\delta$ in equation (1).

8. **Bauschinger effect and ratchetting**: the Bauschinger effect in CFTs occurs predominantly due to the behavior of steel. Kinematic hardening of the loading and bounding surfaces capture this behavior. In addition, for the strain levels typically achieved in steel tubes of CFTs, it is desirable to capture a small amount of ratchetting, exemplified by an accumulation of strain when cycling within a constant stress range about a non-zero mean stress. A common shortcoming of traditional bounding surface models is that they often exhibit excessive ratchetting relative to the behavior exhibited in structural steel experiments. To correct this phenomenon, Shen et al. proposed a simple addition of using a memory surface and a virtual bounding surface in stress–space. The centroids of the memory and virtual bounding surfaces coincide with that of the bounding surface, and they take the same shape as the bounding surface. The size of the memory surface is always less than or equal to that of the bounding surface, while the virtual bounding surface is always larger than the bounding surface by an amount equal to the distance from the stress point at reversal to the memory surface. The size of the memory surface is related to the largest stress achieved in the loading history at the steel fiber. Through the use of the memory surface, it is possible to identify smaller plastic excursions relative to previous larger plastic excursions. For smaller plastic excursions (i.e. if the memory surface has not been contacted by the stress point at reversal), the distance $\delta$ is measured to the larger virtual bounding surface, rather than to the actual bounding surface, when assessing the plastic modulus, $E^p$, in equation (1). The virtual bounding surface is used until the memory surface is recontacted, at which point the actual bounding surface is retained for calculating the distance $\delta$. This use of the memory and virtual bounding surfaces directly decreases the amount of ratchetting, and it affects the magnitude of the Bauschinger effect. Shen et al. show excellent comparisons to cyclic experiments of structural steel using this feature of the model.
In addition to the above features, the different stress-strain characteristics in the corners and flanges of the steel tube are accounted for in this research. This will be described in the section on calibration.

2.3. Concrete constitutive model

The concrete model developed by Chen and Buyukozturk\textsuperscript{13} is adapted to this research for modeling each fiber in the concrete core. This model is also based on a bounding surface formulation, and it contains several features critical to capturing the complex behavior of concrete in CFT cores subjected to cyclic excitation. For details of this formulation, see References 13, 16, 32–34. Key features of the model which have been adapted and extended in this work include the following.

2.3.1. Vanished elastic zone and achieving peak strength

Since the stress–strain curve of concrete in compression is observed to be nonlinear even at low stress values, the concrete model does not assume the existence of an elastic zone. In the context of a more traditional bounding surface formulation, this model may be thought of as having a point loading surface\textsuperscript{35}. Consequently, prior to contacting the bounding surface, the plastic modulus, \( H \), varies continuously as a function of the normalized distance between the current stress point and an image point on the bounding surface measured in the deviatoric plane. The size of the bounding surface defines the peak concrete strength that may be achieved prior to initiating softening behavior. Thus, after the stress point at a concrete fiber contacts the bounding surface, the peak concrete strength is achieved (initially equal to \( f' \) for uniaxial loading), and the concrete softens, asymptotically approaching a zero stress level as it crushes fully in this post-failure region.

2.3.2. Damage accumulation

A scalar damage parameter, \( K \), is used to account approximately for the microcrack formation in the concrete at every stage of compressive loading and unloading, including the post-failure region. In this model, loading and unloading in stress space are treated separately in the deviatoric plane and along the hydrostatic axis. Deviatoric loading (unloading) is defined as loading which increases (decreases) the normalized distance in the deviatoric plane between the current stress point and an image point on the bounding surface, while hydrostatic loading and unloading are defined by the sign of the hydrostatic component of the stress increment. Correspondingly, the increment in the damage parameter within a load step is defined differently for the cases of deviatoric loading, deviatoric unloading, and post-failure behavior.

The damage parameter is assumed to increase during deviatoric loading and post-failure behavior, but it decreases during deviatoric unloading. Since the damage parameter accounts only for the microscopic damage due to the compressive stresses, it is assumed to remain constant during tensile loading in this work.

For the cases of deviatoric loading and unloading, the damage parameter increment is proportional to the increment of the normalized distance between the current stress point and the image point on the bounding surface. For post-failure behavior, the current stress point remains on the bounding surface, so that this increment is equal to zero. Ameur-Moussa and Buyukozturk\textsuperscript{33} thus defined the increment in the damage parameter in the post-failure region to be proportional to the octahedral plastic shear strain increment, since this value was thought to be an accurate indicator of the overall plastic deviatoric straining in the post-failure range. The octahedral plastic shear strain increment is a ratio of the octahedral shear stress increment and a plastic shear modulus, \( H \), defined below.

2.3.3. Shear compaction and dilatancy

The separation of deviatoric and hydrostatic loading states in the concrete formulation permits modeling of important phenomena observed experimentally, including the shear compaction–dilatancy phenomenon characteristic of concrete\textsuperscript{32}.

2.3.4. Strength degradation

Permanent strength degradation occurs in this formulation during each excursion in the post-failure region and during deviatoric loading (when the maximum damage parameter, \( K_{\text{max}} \), is increasing) by decreasing the size of the bounding surface. Such behavior is critical for capturing the load–deformation behavior seen in Figure 1. As will be discussed in the next section, the softening branch of the stress–strain curve has been calibrated to account implicitly for the effects of confinement on the steel tube.

The equation of the bounding surface is given as:

\[
F = \frac{0.25 \cdot \sqrt{J_2} + 3.10 \cdot J_2}{4 \cdot f_t + 3.48} \cdot [\cos(3\theta) + 5] - \frac{40}{39 + K_{\text{max}}^2} = 0.
\]

In this equation, \( J_2 \) is the normalized first stress invariant, \( J_2 \) is the normalized second deviatoric stress invariant, \( \theta \) is the Lode angle (defined as an angle between the projection on the deviatoric plane of the current stress vector and a tensile principal stress axis), and \( K_{\text{max}} \) is the maximum value of the damage parameter, \( K \), ever experienced by the fiber. This equation was obtained by fitting to the data of several multiaxial experiments\textsuperscript{35}. The term \( H = 40/(39 + K_{\text{max}}^2) \) reflects the size of the bounding surface. It is intended to give values close to 1.0 for the pre-failure region during monotonic uniaxial compression. For the peak stress value in monotonic uniaxial compression, the maximum damage parameter, \( K \), is equal to 1.0 and, thus, \( H \) is also equal to 1.0.

The bounding surface is dependent upon the mean pressure and Lode angle, and it forms a non-circular cone in the Haigh–Westergaard\textsuperscript{36} space that is open in the compression direction of the hydrostatic axis and centered about that axis. The surface shrinks isotropically with increased damage accumulation to model strength degradation. It is not allowed to expand during the load history, even when the value of the damage parameter is decreased during deviatoric unloading. No kinematic hardening of the bounding surface is used.

2.3.5. Post-failure behavior for substantial strain levels

The concrete model of Chen and Buyukozturk\textsuperscript{13} was selected for use primarily because it provides comprehensive and accurate modeling of the cyclic behavior of concrete within the context of a consistent plasticity theory. However, typical monotonic and cyclic CFT beam-column experiments\textsuperscript{7,19,37,38,39} yield strains substantially beyond the values to which Ameur-Moussa and Buyukozturk\textsuperscript{33} were able to calibrate their concrete model, particularly for uni-
axial loading. Buyukozturk and Ameur-Moussa\textsuperscript{32} correctly indicate that only limited cyclic stress–space experiments of concrete have been carried out well into the post-failure range. Consequently, several modifications to the concrete model regarding post-failure behavior were required in the current research to accommodate these typical experimental straining levels in CFTs.

In the post-failure range, Ameur-Moussa and Buyukozturk\textsuperscript{33} chose to calculate the damage parameter increment using an empirical expression for the plastic shear modulus, $H_P$, as mentioned above. In this range, this expression must be independent of the distance between the stress point and the bounding surface, which becomes zero. Using an empirical expression that is a function of the strain level is in effect using a deformation theory of plasticity to supplement the incremental plasticity theory of this model. The equation of the plastic shear modulus in the post-failure region was thus chosen to provide a fit to the post-failure branch of the uniaxial concrete stress–strain curve. This approach leads to a satisfactory prediction of the material response, provided that the strain range covered is relatively small. The equation is given by\textsuperscript{33}:

$$H_P = -0.15 \cdot \exp[-0.025 \cdot (K_{\text{max}} - 1)^2] \cdot F_2$$

where $F_2$ is a multiaxial factor described in the next section. This equation causes the plastic shear modulus to decrease to zero very rapidly with the increase of the maximum damage value.

When recovering forces from incremental deformations, the plastic modulus, $H_P$, affects both the change in size of the bounding surface through the damage parameter calculations, and the value of the stress increment obtained using a tangent predictor. Since Ameur-Moussa and Buyukozturk\textsuperscript{33} were able to verify this equation only for relatively small excursions into the post-failure region, particularly for uniaxial and biaxial loading, when equation (3) is used the resulting stress point may lie far off the new bounding surface. However, in the post-failure region at the end of a load step, the resulting stress point should lie on the new bounding surface. Thus, a consistency condition specifying the relation between the stress increment and the maximum damage parameter increment should be satisfied for each loading step. Nevertheless, Buyukozturk and Ameur-Moussa\textsuperscript{32} and Ameur-Moussa and Buyukozturk\textsuperscript{33} do not discuss using either a consistency condition, or a subincrementation or return mapping algorithm, in their plasticity formulation to insure the end-of-step stress point lies on the bounding surface.

The consistency condition of the bounding surface is thus implemented in this work to permit, in certain circumstances, the accurate calculation of a new bounding surface size in the post-failure region. Specifically, using the consistency condition, the increment of $K_{\text{max}}$ can be obtained as a function of stress, the stress increment, and the current value of $K_{\text{max}}$ [recall equation (2)]. However, $K_{\text{max}}$ is defined as the largest value of $K$ during the entire load history, and it typically should not be obtained directly from the consistency equation, but rather from the equation $K_{\text{max}} = \max(K_{\text{max},\text{old}}, K)$, except under special circumstances. In particular, when at the beginning of a load step the current value of damage parameter, $K$, is equal to its maximum value, $K_{\text{max}}$, and the loading condition is post-failure, it is guaranteed that the damage parameter will breach its previous maximum value as loading continues. For this case, the value of the damage parameter is equal to its maximum value for the entire load step, and it is appropriate to calculate the damage parameter increment directly from the consistency equation. The consistency condition is thus used in this work to obtain the size of the bounding surface under these circumstances.

The first-order consistency condition for the bounding surface has a general form

$$dF = \frac{\partial F}{\partial \sigma_j} \cdot d\sigma_j + \frac{\partial F}{\partial K_{\text{max}}} \cdot dK_{\text{max}} = 0$$

where $\sigma_j$ represents the state of stress in the concrete fiber. For the case of modeling only uniaxial stress and strain at each concrete fiber, the only non-zero component of stress is the longitudinal stress, $\sigma$, and the consistency equation can be simplified to yield:

$$dK = \frac{40 - (39 + K^2) \cdot (0.167 \cdot \sigma + 1.79)}{(0.167 \cdot \sigma^2 + 3.58 \cdot \sigma) \cdot K} \cdot d\sigma.$$  \hspace{1cm} (4)

Thus, if the consistency condition is used, the increment of the damage parameter is directly proportional to the stress increment, and is a nonlinear function of stress and the current value of damage parameter. On the other hand, when the value of damage parameter, $K$, is not equal to $K_{\text{max}}$ at the beginning of the load step, the damage parameter increment is not calculated in this work based upon the use of equation (4), but rather it is computed as a linear function of the octahedral shear strain using the empirical expression for $H_P$ in equation (3)\textsuperscript{33}.

The plastic shear modulus, $H_P$, approaches zero for large values of the damage parameter, $K$, and thus its inverse approaches infinity, which causes the damage parameter to grow unbounded. In order to address this problem, the value of $H_P$ was capped from below in this work, so that it is never less than a small positive constant. Similarly, concrete fibers with the values of the damage parameter, $K$, greater than 63 (corresponding to the bounding surface achieving 1.0% of its initial size) are neglected in all subsequent strength and stiffness calculations; effectively, the fiber is considered pulverized. Finally, a return mapping algorithm is invoked at each load step if the concrete fiber is in the post-failure range, so as to adjust the end-of-step stress point location and insure that it lies on the bounding surface\textsuperscript{16}. These extensions of the algorithms in the post-failure regime yield a more robust and accurate concrete constitutive model that permits modeling of concrete strains well in excess of the strain at achievement of peak concrete stress, reflecting strains typically seen in cyclic CFT experiments.

2.3.6. Stiffness degradation The stiffness of the concrete, through the behavior of the plastic modulus $H_P$, decreases continuously as a function of the cumulative damage parameter. This effect propagates to the stress-resultant level as is seen in Figure 1.

2.3.7. Fully reversed loading The tensile behavior of the concrete is based on a formulation presented in ASCE\textsuperscript{30} and it is modeled as being linear elastic (with a stiffness equal to the tangent stiffness at the point at which the stress changes from compressive to tensile) up to the rupture stress. After rupture, the fiber loses all capacity to retain
stress in tension, thus modeling the opening of the crack. Upon reversed loading, compressive behavior is reinitiated at the point at which tensile behavior first initiated, thus simulating crack closure.

2.4. Calibration of the concrete-filled steel tube fiber model

While the present research models only uniaxial stress and strain at each fiber, the constitutive models used in this research are multiaxial. These constitutive models provide powerful cyclic analysis capabilities within the framework of compact, versatile, and consistent plasticity formulations. In addition, these multiaxial stress–space models were adopted to permit possible future extension for direct inclusion of the effects of confinement of the concrete core by the steel tube. In the present work, confinement is accounted for implicitly within the uniaxial stress–strain curves, and only the uniaxial behavior of these materials is presented in this paper.

The calibration of the CFT fiber element involves explicit calibration of both the steel and the concrete constitutive models. Shen et al. calibrated their cyclic steel model to tests of structural steel box columns. This steel showed a definitive yield plateau when stressing a virgin specimen. Material taken from either the corners or flanges of a cold-formed tube steel often does not display this characteristic, unless the tube is first annealed. Sully and Hancock presented a comprehensive set of coupon tests for cold-formed tube steel, including coupons extracted both from the flanges and the corners of a steel tube that was used for a stub–column test. The stress–strain curves exhibited a distinctive rounded shape with a proportional limit that was substantially below the 0.2% offset stress of the coupons (Figure 2 — the experimental data are represented by the solid lines). This is predominantly due to residual plastic strain caused by cold-working of the steel. In this research, the size of the strain–space nonhardening surface is initialized to a non-zero value (rather than having a typical initial size of zero) to account for this straining. The loading surface size, which is a function of the size of the nonhardening surface, is then modified according to the hardening rules of the model to properly reflect the initial plastic strain value. This calibration produces a value of the initial proportional limit that is approximately 60% of the 0.2% offset yield stress of the steel tube. The initial plastic strain, \( \epsilon_p^{\text{initial}} \), differs for the fibers in the corners and the flanges of the steel tube, with a value of initial strain of 0.0006 taken in the corners, and 0.0004 taken in the flanges (for the modeling of annealed tubes, these values are taken as zero). The value adopted for the corners is greater to account for the higher degree of cold-working experienced by the corners, relative to the flat portions of the tube. This causes the ratio of the initial elastic zone to the yield stress to be smaller for the corners, producing a more rounded stress–strain curve, which corresponds to the experimental observations. Figure 2 indicates the strong correlation achieved between the experimental data and the constitutive model using these values of initial plastic strain. While these initial strains seem small, they substantially affect the reduction of the elastic zone upon reversed loading. If these initial strains are not accounted for, the decrease in the elastic zone, as per Shen’s calibration parameters, is significant in the early cycles of straining. Such a decrease is inappropriate for structural steel which has an initial yield plateau, but is excessive for the behavior found in cold-formed tube steel. It should be noted that since the multiple coupon tests of Sully and Hancock showed some scatter, two coupon tests in the mid-range of their results were used for these calibrations.

In addition to the ratio of the proportional limit to the yield stress being different in the corners and flanges, Bridge and Sully and Hancock showed that the initial yield stress of the steel tube material differs in the corners and flanges, with the yield stress in the corners as much as 25% higher. Thus, at the next stage of the calibration process, the average stress–average strain curve was obtained from the results of the stub-column test for which the actual yield stresses of the flange and corner portions of the tube were reported in Sully and Hancock. From this curve, a nominal yield stress of the steel tube was calculated, where the nominal yield stress was assumed to be measured as a 0.2% offset on the average stress–average stress curve obtained from the stub–column experiment. Ratios of the measured yield stresses at the flats and corners (attained from coupon tests) to this nominal average yield stress were then calculated. These ratios were assumed to be representative of the cold-formed tubes. As a result, in this work, the yield stress of the material is varied from a minimum value in the flanges (0.96 times the reported nominal yield stress) to a peak in the corners (1.2 times the reported nominal yield stress), with an eighth-order polynomial used to vary the yield stress continuously along the depth of each flange of the tube. The eighth-order polynomial was chosen to approximate the variation of the yield stresses along the tube side reported by Bridge. A constant yield stress is used for all fibers if the steel tube is annealed. The ratio of the measured yield stress in the flange portion of the tube to the nominal yield stress obtained from a stub column curve was also recorded by Shakir-Khalil and Moll. This ratio was reported to vary between 0.95 and 0.99, with an average value of 0.96.

After the steel parameters were calibrated, an analysis of the steel stub column using the measured yield stresses was performed, and the calculated load–deformation curve matched well to the experimental results of Sully and Hancock. Additionally, an axially loaded steel stub–column reported by Lu and Kennedy was analyzed (only the nominal yield stress was reported for this experiment, labeled Section 2), and the results also compared well to the experimental data (Figure 3).

Once the monotonic steel behavior was calibrated, the monotonic stress–strain curve of the concrete was calibrated to match the results from several CFT flexural experiments conducted by Tomii and Sakino which yielded moment–curvature–thrust data. In particular, the post-failure plastic modulus of the uniaxial concrete stress–strain response was increased substantially from the modulus established for unconfined concrete by Ameur-Moussa and Buyukozturk in their original calibration [i.e., equation (3)]. This more gradual loss of strength represents the added ductility commonly exhibited in square and rectangular CFTs as a result of the moderate confinement exhibited in these members.

The post-failure behavior of the concrete cannot easily be modeled using the stress level experiments only, as it is recognized that post-peak concrete response largely represents structural, rather than material, behavior. The situation is further complicated by the fact that a triaxial stress state usually exists in the concrete core of CFTs due to the confinement by the steel tube. In equation (3), \( F_2 \) is...
a factor which accounts for the multiaxial stress state of the concrete. It is calculated by Buyukozturk and Ameur-Moussa\textsuperscript{32} as:

$$F_2 = \frac{0.14}{I_{1,\text{max}} - 0.86}, \quad I_{1,\text{max}} \leq 2.54$$

$$F_2 = 0.025, \quad I_{1,\text{max}} > 2.54$$

where $I_{1,\text{max}}$ is the maximum value of the normalized first stress invariant experienced in the load history by the material fiber (for post-failure behavior, its value is greater than or equal to 1.0). In the implementation of Buyukozturk and Ameur-Moussa\textsuperscript{32}, the value of $F_2$ varies between 1.0 for unconfined concrete and 0.025 for highly confined concrete.

The value of the $F_2$ parameter of equation (3) was altered in the current research so as to provide a strong correlation with experimental results, and in particular a sufficient level of curvature ductility. It was determined that the factor which most affected the post-peak ductility of the concrete is the $D/t$ ratio of the steel tube\textsuperscript{14}. The CFT monotonic moment–curvature–thrust results of Tomii and Sakino\textsuperscript{37} were chosen in order to investigate the effect of the $D/t$ ratio on the post-peak behavior of the concrete, and on the curvature ductility of individual CFT members. The experimental results used for this calibration included two groups of CFTs, each loaded first with a constant axial force, and then in flexure with a uniform bending moment. For the first group, $D/t$ was equal to 24; for the second group, $D/t$ was equal to 44. Each group consisted of five square CFT beam-columns, with similar geometric and material properties. The ratio of applied force to the axial capacity of the column varied between the tests. The steel tubes were
annealed and had yield stresses ranging from 280 to 340 MPa. The concrete strength was approximately 25 MPa (see Refs.\(^{16,37}\) for details of these experiments).

The moment–curvature–thrust diagrams for several of these specimens are shown in Figures 4 and 5. Those with a \(D/t\) of 24 exhibited positive stiffness for the duration of the experiment. For all specimens but the one with the highest axial force, no specific points of failure were reported. The post-failure modulus of the concrete for this group of columns was thus chosen to provide a curvature ductility at least equal to the values reported.

The specimens having a \(D/t\) equal to 44 all had moment–curvature–thrust diagrams with an ascending portion, a peak of the moment, and a descending branch. For these CFTs, the downward slope of the concrete was selected to provide enough curvature ductility to reach the peak moment for these experiments.

While it is possible to vary \(F_2\) as a function of \(D/t\), a constant value of 0.05 was found to be sufficient to obtain strong correlation with the experimental results (Figures 4 and 5). This value was used in equation (3) for all subsequent analyses. For cyclic analysis, the parameters calibrated by Shen et al.\(^{12}\) and Ameur-Moussa and Buyukozturk\(^{33}\) for steel and concrete, respectively, were retained in this work.

3. Verification of the concrete-filled steel tube fiber model

The verification of the CFT fiber element is performed by comparing the finite element results with several different experimental studies of individual CFT beam-columns. A representative set of CFT experiments reported in the literature was selected for verification of the formulation\(^ {43}\). The selection was made to include experiments having a wide variety of parameters, the most important being length-to-depth ratio, width-to-thickness ratio of the steel tube, material properties, and methods of load application. The details of comparisons to more than 30 experiments having monotonic loading, holding all calibration parameters fixed, can be found in Schiller and Hajjar\(^{10}\) and Hajjar et al.\(^{11}\). A sample of this comparison consisting of four monotonic load–deflection curves vs the computed response is presented in Figure 6a–d. The nomenclature of the original experiments is used for all comparisons. Figures 6a, b present eccentric load tests, with the beam-columns loaded proportionally by a combination of axial force and bending moment. Figure 6a shows a comparison of a short, square CFT loaded eccentrically by Bridge\(^ {20}\). Since the CFT was turned about its longitudinal axis by 30°, it is subjected to axial force plus biaxial bending. The results for this biaxial
bending experiment are as accurate as those for tests in uniaxial flexure. Alternately, Figure 6b represents a long, rectangular CFT loaded in uniaxial flexure combined with axial force by Shakir-Khalil (out-of-plane section values are given in parentheses). Figure 6c, d are annealed specimens, loaded nonproportionally, with varying levels of constant axial force being applied in the different tests, followed by application of uniform bending moment. This last set of tests differs from those by the same authors that were performed by Morino and his colleagues, loaded nonproportionally, with varying levels of constant axial force by Shakir-Khalil (out-of-plane section values are given in parentheses).

A statistical comparison for the entire monotonic verification set shows that the current formulation provides an excellent prediction of the nonlinear behavior of these members. The computational model overpredicts the strength of CFT beams and beam-columns under proportional or nonproportional monotonic loading by an average of 1.8%, with a standard deviation of 7.0%, when compared to the experimental results; this comparison is made at the deformation level corresponding to the experimental maximum load, or at the level corresponding to the end of the analysis, if this occurs first. When the computational and experimental loads are compared at one half of this deformation level, the average error is only 0.15%, with a standard deviation of 6.7%, suggesting that the gradual plastification of the CFT elements is predicted well by the current formulation.

4. Verification of a three-dimensional cyclic CFT subassemblage

To verify the capability of analyzing composite subassemblages, the final verification problem consists of a three-dimensional cyclic CFT subassemblage of steel I-girders framing rigidly into a CFT beam-column. The setup of the experiment, performed by Morino and his colleagues, is shown in Figure 8. The specimen, labeled SCC2038, was designed...
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Figure 7 Comparison of cyclic CFT experimental and computational results

(a) Rotation (%)

Tube: 100x100x4.21
\( f_y = 295.0 \text{ MPa} \)
\( f_{e} = 24.0 \text{ MPa} \)
\( L/D = 6.0 \)
\( D/t = 24 \)

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(b) Rotation (%)

Tube: 100x100x2.96
\( f_y = 298.0 \text{ MPa} \)
\( f_{e} = 27.0 \text{ MPa} \)
\( L/D = 6.0 \)
\( D/t = 34 \)

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such that the CFT beam-column, rather than the fully-restrained connections, would reach its limiting capacity first, while the steel beams were designed to remain elastic for the duration of the experiment. The CFT was loaded first with a constant value of axial load, \( P = 0.15P_0 \) (\( P_0 = 1300 \text{ kN} \) for this specimen), while simultaneously a constant load of \( W = 21.0 \text{ kN} \) was applied to one end of the steel beam in the \( y-z \) plane (see Figure 8a). The specimen was then loaded cyclically in the \( x-z \) plane. The anti-symmetric beam shear forces, \( Q \), were varied to obtain increasing increments of rotation, \( R \), equal to 0.005, 0.01, and 0.02, and 0.03 radians. For each rotation increment, two full cycles of loading were performed. This loading pattern created a combination of cyclic biaxial bending and axial force in the CFT beam-column.

The CFT beam-column consists of a square tube with cross-section dimensions \( 125 \times 125 \times 5.74 \) and material strengths of \( f_{e} = 20.0 \text{ MPa} \) and \( f_y = 395 \text{ MPa} \). The steel I-beams are built-up sections consisting of flanges measuring \( 125 \times 9 \text{ mm} \) and a web measuring \( 231 \times 6 \text{ mm} \), and having a yield strength of \( f_y = 400 \text{ MPa} \). Figure 8b illustrates a schematic representation of the computational model of the subassemblage. The drift rotation is calculated by summing the displacements, \( D_1 \) and \( D_2 \), at locations \( A \) and \( B \), respectively, and dividing by the length \( L \) between these points, thus simulating the approach used by the experimentalists. Both the connection rotation and the beam flexure between the connection and the measuring points, \( A \) and \( B \), contribute to the calculated values of the drift rotation. The beam shear shown in the results is calculated by averaging the

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Figure 8 Three-dimensional composite CFT subassembly (after Morino et al.)
values of shear forces in the x-z planes at both ends of the beam. These forces, although close in value, are not equal, due to the boundary conditions and axial displacement of the CFT beam-column. Ten fibers were used in each flange of the steel tube (with one fiber through the tube thickness), and the concrete core was meshed with a 10 by 10 grid of fibers. Twelve elements were used along the length of the member, and two steel wide-flange fiber elements were used for each steel beam. Since the steel elastic modulus was not reported for the experiment, a value of 210 GPa was selected to provide a match to the initial elastic slope in the first half-cycle of loading. In addition to the boundary conditions shown at the ends of the CFT beam-column in Figure 8b, each beam had its out-of-plane translation restrained.

Figure 9 illustrates the CFT fiber model results compared to the experimental results provided by Morino et al. The entire hysteresis curve is predicted well by the fiber model, with the stiffness prediction being particularly accurate throughout the loading history. While the maximum shear capacity is somewhat underpredicted by the analysis, with the largest error being approximately 10% at a rotation level of 0.03 radians, the overall shear capacity is predicted well, with the error significantly below its maximum level for most of the analysis.

Figures 10 and 11, respectively, show the stresses in two of the steel fibers and two of the concrete fibers in the model. The fibers are taken from the cross-section at mid-height of the specimen. Steel fiber ‘a’, as shown in the inset of Figure 10, is located in a corner of the steel tube that is initially subjected to approximately zero mean stress and strain due to the offsetting effects of the column axial force and the tip load on the out-of-plane girder. Its cyclic stress-strain behavior is approximately symmetric for much of the load history, and thus little ratchetting is evident. Alternatively, fiber ‘b’ is located in a steel tube corner that is initially subjected to compressive stress and strain due to the static loads. As it oscillates about non-zero mean stress, substantial ratchetting behavior is seen, with each successive cycle showing a significant increase in accumulated plastic strain. The benefits of using the memory and virtual bounding surfaces are realized in several of the cycles, where a large compressive inelastic excursion is followed by a smaller plastic excursion in tension. Upon reloading in compression, ratchetting is apparent, but its magnitude is smaller than would occur without the use of these additional surfaces. In addition, the subtle decrease in the elastic zone is exhibited in this stress-strain curve.

Figure 11 similarly shows two concrete stress-strain curves, both located in corners identified as ‘a’ and ‘b’. Concrete fiber ‘a’, with approximately zero mean stress at the initiation of cyclic activity, shows several tensile and compressive cycles until, in a late cycle, substantial post-peak softening is exhibited in two successive cycles, after which the concrete fiber permanently loses nearly all of its strength. Concrete fiber ‘b’ shows similar behavior, but at a more extreme level due to the initial compressive straining seen in this fiber due to the static loading. The fiber crushes completely much earlier in the loading history, losing most of its strength. Substantial stiffness and strength degradation are seen in these curves. Also note two small elastic excursions into the tensile stress region in fiber ‘b’. The third tensile excursion breaches the rupture strength of the concrete, after which further tensile stress is prohibited to simulate cracking. Together these stress-strain curves exhibit the key behavior required to capture the CFT load-deformation behavior outlined earlier.

It should be noted that slip was prevented at each end of each CFT beam-column in both the monotonic and cyclic experiments presented here (although slip was permitted along the length of the CFT as per the formulation and calibration presented in Ref[10]). These boundary conditions are reflective of the experimental setups. Thus, the slip was minimal within these specimens. However, for comparison purposes, the 3D subassemblage was rerun permitting slip of the concrete core at the top of the specimen. The results showed no perceptible change, indicating that this type of anti-symmetric loading did not induce high bond stresses at the steel-concrete interface. Preliminary parametric studies of the effects of slip in CFT beam-columns and composite frames are reported in Schiller and Hajjar and Hajjar et al.

5. Conclusions

This paper outlines a geometrically and materially nonlinear fiber-based distributed plasticity finite element formu-

![Figure 9](image-url) Comparison of experimental and computational shear-rotation hysteresis curve for three-dimensional subassembly
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Figure 10 Cyclic stress-strain curves from steel tube for three-dimensional subassembly

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Figure 11 Cyclic stress-strain curves from concrete core for three-dimensional subassembly

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References
5 American Institute of Steel Construction, Inc. (AISC) Load and resistance factor design specification for structural steel buildings (2nd edn), AISC, Chicago, IL, 1993
8 White, D. W. 'Material and geometric nonlinear analysis of local planar behavior in steel frames using interactive computer graphics', M.S. thesis, School of Civil and Environmental Engineering, Cornell University, Ithaca, NY, 1985
9 Morales, L. E. 'Object-oriented software for advanced analysis of steel frames', M.S. thesis, School of Civil Engineering, Purdue University, West Lafayette, IN, 1994
10 Schiller, P. H. and Hajjar, J. F. 'A distributed plasticity formulation for three-dimensional rectangular concrete-filled steel tube beam-columns and composite frames', Report no. ST-96-5, Department of Civil Engineering, University of Minnesota, Minneapolis, MN, 1996
412 A distributed plasticity model for cyclic analysis of CFT beam-columns: J. F. Hajjar et al.

16 Moladan, A. and Hajjar, J. F. 'A cyclic distributed plasticity model for analysis of three-dimensional concrete-filled steel tube beam-columns and composite frames', Report No. ST-96-6, Department of Civil Engineering, University of Minneapolis, Minneapolis, MN, 1997
18 Sully, R. M. and Hancock, G. J. 'Behaviour of cold-formed SHS beam-columns', Research Report no. R696, School of Civil and Mining Engineering, University of Sydney, Sydney, Australia, 1994
20 Bridge, R. Q. 'Concrete-filled steel tubular columns', Res. Rep. no R283, School of Civil and Mining Engineering, University of Sydney, Sydney, Australia, 1976
25 Shen, C. 'Development of a cyclic two-surface model for structural steels with yield plateau', Ph.D. dissertation, Department of Civil Engineering, Nagoya University, Nagoya, Japan, 1993
31 Hashiguchi, K. 'Mechanical requirements and structures of cyclic plasticity models', Int. J. Plastic., 1993, 9 (6), 721–748
36 Chen, W. F. and Han, D. J. Plasticity for structural engineers, Springer, New York, 1988
42 Shaker-Khalil, H. and Mouli, M. 'Further tests on concrete-filled rectangular hollow section columns', The Struct. Engineer, 1990, 68 (20), 405–413
43 Gourley, B. C., Hajjar, J. F. and Schiller, P. H. 'A synopsis of studies of the monotonic and cyclic behavior of concrete-filled steel tube columns', Report no. ST-93-5.2, Department of Civil Engineering, University of Minneapolis, Minneapolis, MN, 1995