Method of Optimal Deployment for Radar Netting
Based on Detection Probability

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Abstract—The optimal deployment for radar netting must treat of many factors that restrict the capability of radar network. Traditional decision-making model only considered the maximum detecting range of radar, the conditions used are simple. So, the optimal deployment schemes got by these models are not applicable to modern battle environment. Now, Electronic jamming, Anti-radiation missiles, Stealth targets and the Low-level attack of High Speed and High Maneuvering Target have become the “The four major threats” which Radar need be faced with. According to the four countering abilities, this paper establishes the decision-making model based on the detection probability. And it gives the modling method using the genetic algorithm. The result indicates that the model is efficient.

Detection probability ; Grid ; Radar netting ; Optimal deployment

I. INTRODUCTION

Radar is the main sensor that is contested by the two warring sides for information superiority in the fight of the modern warfare. Electronic jamming, Anti-radiation missiles, Stealth targets and the Low-level attack of High Speed and High Maneuvering Target have become the “The four major threats” which Radar need be faced with. For improving the confronting ability to “The four major threats” for Radar, there are usually two methods, The first is advancing the detecting probability of Radar; The second is increasing the overall detection performance through radar netting that include different systems, different frequency bands, different working patterns radar equipments. The practice has proved that Radar netting is the important mean to improve the Radar detection probability. The research for radar netting mainly involves three aspects[1][2] ; 1) Configuration and optimal deployment; 2) Working cooperation; 3) Data fusion. The optimal deployment among them is the first problem to consider, which is relating directly with the distribution for the detecting ability (such as detecting and fusion) in the space and deciding the choices of the data fusion airspace, the communicate net forming and the data fusion algorithms[3]. Presently, the researches of Optimal Deployment for Radar Netting mainly focus on the three aspects: the principle of the Optimal Deployment[4][5] the decision model and the optimal algorithms of solving decision model[6][7]. Literature[8] expatiates how to building the grid model of the problem of sensor optimal deployment. Though the results of these studies to the optimal deployment for radar netting has a strong in-depth study of the guiding role, in order to construct the practical radar network, it need consider a variety of factors that constraints the performance of radar network. The level structure model in [9] is the integrated model which reflect a more comprehensive radar netting four countering abilities. Since the tactical significance of the result of the model induction calculation is not clear and which contains a lot of value to determine for man-made (such as the weight coefficient), this fact results in the difficulty to use for the commanders. Accordingly, this article is going to abandon the level hierarchy of four countering comprehensive abilities and directly selects the basic goals whose tactical significance is the more obvious as the decision goals. On the basis of the detection probability in radar netting, this paper has established the multi-objective programming model of the optimal deployment and made the simulation. As a discussion of the methods, among the modeling, this paper selected the representative largest radar detecting range, the polarization type, frequency overlap coefficients and spatial overlap coefficients, and the remaining guidelines can refer to the above-mentioned indicators Modeling Method.

II. NETTING DETECTION PROBABILITY MATRIX

The building for the decision-making model of the radar netting optimal deployment mostly adopts the coverage range by intersecting the detecting ranges of the single radar. In fact, the coverage of radar network is not exactly the coverage by intersecting the detecting ranges of the single radar, but the coverage in the sense of the detection probability according to the different fusion rules. As the fusion rule with rank 1, the radar detection probability after netting [9][10]

\[ P_{net} = 1 - \prod_{i=1}^{N} (1 - P_i) \]  \hspace{1cm} (1)

where \( P_{net} \) is the netting detection probability, \( P_i \) is the detection probability of the radar by order \( i \), \( N \) is the number of the radar in network.

According to the idea of grid, this paper divides the responsible region of radar network into grid with equal borders, and encodes the grid (as shown in Figure 1); the border length of grid is able to elected in accordance with the minimum distance resolution \( R_0 \) of the radar that can be selected. Each grid in the responsible region can be confirmed uniquely in accordance with horizontal encoding \( m \) ( \( m=1,2,…,N_x \) ) and the vertical encoding \( n \) ( \( n=1,2,…,N_y \) ), that is, the grid coordinate is \( (m,n) \). Then, the \( j \)th station’s
coordinate is \((m_j, n_j)\), while the arbitrary distance \(R_j\) between the grid \((m, n)\) and the station \((m_j, n_j)\) in the region is
\[
R_j = \sqrt{(m-m_j)^2 + (n-n_j)^2}
\]
(2)

It assumes that the \(i\)th radar’s detection probability is \(P_0\), the maximum detecting range is \(R_{\text{max}}\), converting \(R_{\text{max}}\) to grid coordinates, that is
\[
R_{\text{max}}' = \frac{R_{\text{max}}}{R_0}\]
(3)

Known single-radar detection probability and detecting range is

\[
P = (P_0)^{(R_j/R_{\text{max}})\text{*}}
\]
(4)

where \(D_0\) is the radar detection range when the detection probability is \(P_0\), \(D\) is the distance between the target relative radar.

After deploying the radar of type \(i\) on the \(j\)th station, the single radar detection probability of any grid in the responsible region is
\[
P_{\text{ijmn}} = (P_0)^{(R_j/R_{\text{max}})\text{*}}
\]
(5)

According to the formula (1), for a specific deployment scheme, the netting detection probability of any grid in region is
\[
P_{\text{mn}} = 1 - \prod_{j=1}^{N_r} (1 - \sum_{i=1}^{N_r} u_{ij} P_{\text{ijmn}})
\]
(6)

where \(N_p\) is the number of stations; \(N_r\) is the number of radars, \(u_{ij}\) is for 0-1 variables, indicating that the \(i\)th type of radar is whether deployed in the \(j\)th station. All the \(P_{\text{mn}}\) form a matrix of \(N_r \times N_p\), denoted as \(P_{\text{Matrix}}\), that is
\[
P_{\text{Matrix}} = [P_{\text{mn}}]_{N_r \times N_p}
\]
(7)

\(P_{\text{Matrix}}\) is named netting detection probability matrix; it has described the coverage condition of the responsible region after radar netting; it is more accurate than the coverage range for simply merging the single radar detection range, so that the commander could be in control of the conditions of the target detecting. Therefore, on the basis of the netting detection probability matrix, the optimal deployment of the radar netting is very significant.

III. DECISION-MAKING MODEL

A. Deployment Matrix

Assuming that the numbers of the selectable radars and radar stations are \(N_r\) and \(N_p\), and serial number of the radar types is 1, 2, ..., \(N_r\), and the serial number of the radar station is 1, 2, ..., \(N_p\), and uses 0-1 variables \(u_{ij}\) indicating that the \(i\)th type of radar is deployed in the \(j\)th station, that is
\[
u_{ij} = \begin{cases} 1 & \text{the } i\text{th type of radar in the } j\text{th station} \\ 0 & \text{otherwise} \end{cases}
\]
(8)

\(u_{ij}\) has formed a matrix of \(N_r \times N_p\), with the \(U\) indicating, that is
\[
U = [u_{ij}]_{N_r \times N_p}
\]
(9)

\(U\) is named the deployment matrix. Assuming the \(i\)th type of radar’s number is \(N_r\), then there is
\[
\sum_{j=1}^{N_p} u_{ij} \leq 1, \quad (j = 1, 2, ..., N_p)
\]
(10)

\[
\sum_{j=1}^{N_p} u_{ij} \leq N_r
\]
(11)

Formula (10) indicates that a station can only deploy one radar. Formula (11) indicates that the number of the deployed radar of the type \(i\) is constrained by the total number. So, optimal deployment will be converted to the problem of solving the deployment matrix.

B. Detecting Blind Area

Aiming at different battle requirement, the targets in the radar network have some detection probability requirements, assuming that the radar network needs the detection probability not smaller than \(P_T\), defining 0-1 variables \(A_{\text{mn}}\) indicating that grid \((m, n)\) whether satisfy the requirement, that is
\[
A_{\text{mn}} = \begin{cases} 1 & P_{\text{mn}} \geq P_T \\ 0 & P_{\text{mn}} < P_T \end{cases}
\]
(12)

Therefore, the detecting blind area \(B\) is
\[
B = A - \sum_{m=1}^{N_r} \sum_{n=1}^{N_r} A_{\text{mn}}
\]
(13)
where \( A \) is the total number of grid in the responsible region, and \( A = N_x \times N_y \). For a optimal deployment scheme, there is \( B \to 0 \).

C. Overlap Coefficient

The overlap coefficient describes the tight grade of radar network to responsible region, especially for focus region to be protected, which usually needs multi-overlap. Assuming that the single radar detection probability is not smaller than \( P_{0F} \), defining 0-1 variables \( A_{ij,mn} \) indicating the coverage condition of the \( j \)th type of radar deployed in the \( j \)th station to grid \((m,n)\), that is

\[
A_{ij,mn} = \begin{cases} 1 & P_{ij,mn} \geq P_{0F} \\ 0 & P_{ij,mn} < P_{0F} \end{cases}
\]

So the overlap coefficient of grid \((m,n)\) is

\[
O_{mn} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} u_{ij} A_{ij,mn}
\]

Setting the overlap coefficient of the focused protected region not smaller than \( O_t \), defining 0-1 variables \( C_{mn} \) indicates that grid \((m,n)\) is weather satisfying the requirement of the overlap coefficient, that is

\[
C_{mn} = \begin{cases} 1 & O_{mn} \geq O_t \text{ and } (m,n) \text{ is in the region} \\ 0 & \text{otherwise} \end{cases}
\]

And

\[
N_o = A_e - \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} C_{mn}
\]

where \( A_e \) is the total number of the grid in the protected region. For one optimal deployment scheme, there is \( N_e \to 0 \)

D. Frequency-dependent Factor

In order to improve the anti-jamming capability of the radar network, the total working bandwidth constructed by netted radar should be large as far as possible, more importantly, the operating frequency of the adjacent radar should be no overlap, the first can improve the anti-jamming ability in the detecting region, the second can avoid the same frequency interference between the radars. Therefore, it need define the frequency-dependent factor \( F_{kj} \) of station \( k \) and station \( q \) to indicate that the radar working frequency between two radar stations is weather related. Assuming the operating frequency range and maximum detecting range are \([F_{imin},F_{imax}]\) and \(R_{imax} \) respectively, Combined with the deployment matrix, the radar operating frequency range and maximum detecting range deployed in the station \( k \) are

\[
\left[ \sum_{i=1}^{N_x} u_{ik} F_{imin}, \sum_{i=1}^{N_x} u_{ik} F_{imax} \right] \text{ and } \sum_{i=1}^{N_x} u_{ik} R_{imax}\text{ respectively, and}
\]

\[
R_{k_{max}} = \frac{\sum_{i=1}^{N_x} u_{ik} R_{imax}}{R_0}
\]

\[F_i = \left[ \sum_{i=1}^{N_x} u_{ik} F_{imin}, \sum_{i=1}^{N_x} u_{ik} F_{imax} \right] \text{ and } \sum_{i=1}^{N_x} u_{iq} F_{imin}, \sum_{i=1}^{N_x} u_{iq} F_{imax}\]

So the frequency-dependent factor \( F_{kj} \) is

\[
F_{kj} = \begin{cases} 1 & k \neq q \text{ and } R_{ij} \leq R_{k_{max}} + R_{q_{max}} \text{ and } F_i = 0 \\ 0 & \text{otherwise} \end{cases}
\]

and

\[
N_F = \sum_{k=1}^{N} \sum_{q=1}^{N} F_{kj}
\]

For a optimal deployment scheme, there is \( N_F \to 0 \).

E. Polarization Type Factor

There are four kinds of horizontal polarization, vertical polarization, circular polarization and elliptical polarization for the Signal level of the polarization type, namely, that by the number of 1, 2, 3, 4, while the universal set of the signal polarization type is \( G_{pl} = \{1,2,3,4\} \). Assuming the \( k \)th type of radar whose polarization type is \( PL_k \), combined with the deployment matrix, the polarization type \( PL_i \) of the radar deployed in the \( j \)th station is

\[
PL_j = \sum_{k=1}^{N} u_{ik} PL_k \quad (j = 1,2,...,N_p)
\]

For a assured scheme of deployment, the radar polarization types constitute a set, that is

\[
G_{pl-net} = \{PL_j \mid PL_j \in G_{pl}, j = 1,2,...,N_p\}
\]

Defining the polarization factor \( N_{pl} \) is

\[
N_{pl} = 4 - \text{Num}(G_{pl-net} \cap G_{pl})
\]

where \( \text{Num()} \) is the quantity operator for the elements in the set. For a optimal deployment scheme, there is \( N_p \to 0 \).

F. Netting Cost Indicator

Defining the netting cost indicator is the ratio of the costs of the deployment radar and the costs of all the selectable radars, that is

\[
C = \frac{\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} u_{ij} C_i}{\sum_{i=1}^{N_x} C_i}
\]

where \( C \) is the cost indicator of radar network, \( C_i \) is the cost for the \( i \)th type of radar. For a optimal deployment scheme, \( C \) should be small as far as possible.

G. Decision Model

To sum up, the decision model of problem for the radar netting optimal deployment is

\[
\begin{align*}
\min & B \\
\min & N_0 \\
\min & N_F \\
\min & N_{pl} \\
\min & C
\end{align*}
\]
This is a multi-goal programming. It have a number of ways to solve it, the most commonly used is the goal of solving the linear weighted sum as an alternative objective function, which will convert the multi-goal programming problem into single-goal programming problem to solve. However, this method need to determine the relative weight goals on the one hand, on the other hand, the weighted linear goal function would loss the clear tactical meaning of each single-goal, which make the commander be difficult to understand. Therefore, this paper retains the multi-goal decision-making characteristics, and makes the use of the genetic algorithm for the multi-goal programming to solve the problem.

IV. GENETIC ALGORITHM TO SOLVE THE DECISION-MAKING MODEL

A. Genetic Algorithm based on Compromise

The optimal solution of Multi-goal programming problem is named Pareto optimal solution, pareto optimal solution is a collection of solutions, but evaluating all to solutions and is named Pareto optimal solution, pareto optimal solution is frequency-dependent factor.

\[
\left\{ \begin{array}{l}
\sum_{j=1}^{m} u_{ij} \leq 1, (j = 1, 2, ..., N_p) \\
\sum_{i=1}^{n} u_{ij} \leq N_p, (i = 1, 2, ..., N_r)
\end{array} \right.
\]

(26)

Selection operation uses the best individual preserving method.

Cross operation uses one line cross method, namely choosing one line randomly form the deployment matrix of the two pairs, all the lines under that line carry out exchanging, to generate two new individuals (as shown in Figure 2). Mutation operation randomly select one column from the deployment matrix, and put the position of the 1 in that column changed.

B. Genetic Algorithm of Optimal Deployment

(1) Chromosome coding

Using the deployment matrix in formula (9) as the chromosome coding.

(2) Fitness function

Using formula (27) as a fitness function. According to the analysis of the paper, we can see that the ideal value of the detecting blind area B, the overlap coefficient factor N, the frequency-dependent factor N_P and the polarization type factor N_p is 0 respectively, the ideal netting cost value can be chosen the estimated cost of building radar network, to simplify the calculation, using numerical 0 as the ideal value of the cost of the radar netting, the ideal value of the goal is \(z^* = (0, 0, 0, 0, 0)\), and the value of \(p\) is 2.

\[
r(z; \rho, w) = \left\| z - z^* \right\|_{\rho,w} = \left[ \sum_{j=1}^{n} w_j^p (z_j - z^*_j)^p \right]^{1/p}
\]

(27)

where \(z_j\) is the \(j\)th target value of current individual, \(z_j^*\) is the ideal value of the \(j\)th goal, \(w_j\) is the weight factor of the \(j\)th goal, \(q\) is the target number.

C. Simulation Results

The new individuals generated during the genetic process may be not satisfied with constrain conditions of the decision-making model, so the new individual needs filtered according to constrain conditions (26) with discarded directly that are not satisfied with constrain condition.
V. CONCLUSION

Radar network optimal deployment problem is a complex systematic project, many factors need to be considered, especially in today’s complex battlefield environment and the application of information fusion technology, which make optimal deployment problem more complex. In this paper, on basis of the comprehensive assessment model of the four countering abilities of radar netting, by calculating the detection probability, establishing the multi-goal programming mode, simulating with genetic algorithm based on a compromise method. The simulation results show that the decision model can effectively solve the radar netting optimal deployment.

REFERENCES
