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An optimal mandatory lane change decision model for autonomous vehicles in urban arterials

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ABSTRACT

Autonomous driving has become a popular topic in both industry and academia. Lane-changing is a vital component of autonomous driving behavior in arterial road traffic. Much research has been carried out to investigate discretionary lane changes for autonomous vehicles. However, very little research has been conducted on assisting autonomous vehicles in making mandatory lane changes (MLCs), which is the core of optimal lane-specific route planning for autonomous vehicles. This research aims to determine the best position for providing MLC instruction to autonomous vehicles. In this article, an optimization model is formulated to determine the optimal position at which an instruction to change lanes should be given through automotive navigation systems. First, the distribution of time spent waiting for safe headway to make a lane change is modeled as an exponential distribution. Lane-specific travel times are then calculated for vehicles in various situations by applying traffic shockwave theory and horizontal queuing theory. Finally, the expected travel time is derived for a vehicle receiving a lane change instruction to change lanes at an arbitrary position along the road. The proposed model is validated by a comparison with a simulation model in VISSIM. Additional experiments show that the instruction should be given earlier in the case of denser traffic or higher travel speed in the target lane and that vehicles can save considerable time, if they follow the guidance provided by the proposed model. The proposed model can be applied to guide autonomous vehicles to travel an optimal route.

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Introduction and motivation

In recent years, autonomous driving has become a popular topic in both industry and academia (Urmson et al., 2008; Ross, 2014; Reuters, 2014; Le Vine, Zolfaghari, & Polak, 2015; Ulbrich & Maurer, 2015; Xie, Zhang, Gartner, & Arsava, 2017). The application of autonomous driving promises to lead to great benefits, such as a decline in automobile accidents and death tolls, improvements in fuel economy, reductions in stress, and optimizations in the use of highways (Le Vine et al., 2015; Luettel, Himmelsbach, & Wuensche, 2012; Payre, Cestac, & Delhomme, 2014). Taking control from human drivers, autonomous vehicles conduct self-driving with real-time perception of the surrounding environment through various sensing devices. It is well acknowledged that the driving decision-making process of an autonomous vehicle and a human-driven vehicle are different to some extent because of different “brains.” In addition, an autonomous vehicle communicates with the intelligent transportation systems (ITS) (such as receiving navigation information) directly and seamlessly, instead of going through the middleman–human driver, whose driving decisions are uncontrollable and unpredictable, for a human-driven vehicle. In real time, an autonomous vehicle shares locally perceived information (lane changes, overtaking, etc.) with ITS and receives global information (traffic states, incidents, navigation, etc.) from ITS. This information transfer makes it possible for ITS to monitor the microscopic driving behaviors of autonomous vehicles and to assist vehicle self-driving to achieve certain global optimization objectives (such as minimal travel time on a route).

Similar to human-driven vehicles (Donges, 1999), the driving tasks of autonomous vehicles are classified into navigation tasks (strategic level), guidance tasks (tactical level), and stabilization tasks (operational level) (Ulbrich & Maurer, 2013). A route navigation task at the strategic level cannot be adequately completed without
considering lane-changing behaviors at the tactical level. It is well known that lane-changing is a vital microscopic driving behavior and significantly affects traffic operations (Sun & Elefteriadou, 2012; Qi, Wang, Chen, & Bie, 2014). Most existing lane-changing strategies in automated driving try to imitate human-driving behaviors and apply lane-changing models originally proposed for human-driving vehicles (Naranjo et al., 2008; Al-Shihabi & Maurant, 2001; Wan et al., 2011; Ulbrich & Maurer, 2013; Ulbrich & Maurer, 2015). In general, lane-changing behavior is categorized as (Gipps, 1986; Hidas, 2005; Sun & Elefteriadou, 2010): (1) a discretionary lane change (DLC) to gain better traffic conditions (avoid following trucks, avoid merging traffic, etc.) or (2) a mandatory lane change (MLC) to follow the specific route. DLC involves dealing with information about surrounding vehicles, while MLC requires knowledge of traffic states of the desired path. For autonomous driving, many researchers have investigated strategies specifically for DLC maneuvers (Naranjo, Gonzalez, Garcia, & De Pedro, 2008) or generally without specifically considering the differences of DLC and MLC (Al-Shihabi & Maurant, 2001; Wan et al., 2011; Ulbrich & Maurer, 2013; Ulbrich & Maurer, 2015; Leonard et al., 2008; Kuwata et al., 2009); however, to the best of our knowledge, there has been little research specifically on MLC decisions for obtaining an optimal global route.

This paper aims to investigate MLC decisions for autonomous vehicles. Specifically, we investigate the impacts of the MLC decision on travel time through a signalized road if an autonomous vehicle makes a lane change decision (or receives a lane change instruction) at a different position on this road and provide the autonomous vehicle with optimal instruction for making lane changes through automotive navigation systems (ANS). This problem involves at least two prerequisites: the subject vehicle is positioned with high precision in real time, and traffic states on planned routes are acquired (Chen, Qin, & Shaon, 2017). The former is supported by lane identification technique in autonomous driving itself (Lai & Yung, 2000; Cheng, Jeng, Tseng, & Fan, 2006; Du & Barth, 2008; Alam, Balaei, & Dempster, 2012; Rompis, Cetin, & Habtemichael, 2017) and positioning technology from more advanced satellite systems (Suzuki, Kubo, & Takesu, 2014; Montenbruck et al., 2013), while the latter is related to many existing technologies (Ban, Hao, & Sun, 2011; Liu, Wu, Ma, & Hu, 2009; Cao, 2014) in ITS (traffic monitoring, traffic state estimation, prediction, etc.). Information from these two sources is applied to determine the optimal lane change decision, which is transferred to the autonomous vehicle from ANS.

Another motivation for this study originates from a single observation: that travel speed in the outer lane (slow or curbside lane) is slower than that in the inner lane (fast or overtaking lane) for urban arterial roads since there is often an adjacent bike lane or sidewalk, and drivers must always be aware of bicycles, pedestrians, and/or vehicles entering from side roads. An autonomous vehicle entering a section of road on the inner lane that wishes to exit the road via the outer lane must decide at which point to change lanes (mandatory). Intuitively, if the decision to change lanes is made too early, the vehicle will spend more time in the slower speed lane (or the outer lane); conversely, if the vehicle delays the change until it gets too close to the intersection, time might be lost waiting for safe headway to change lanes because of denser traffic.

The literature predicts that fully automated vehicles may begin in the 2020s and will likely become common and affordable in the 2040s to 2060s, so there will be at least 10 years during which autonomous vehicles run in a low penetration rate together with human-driven vehicles and without cooperation from other vehicles (Litman, 2014). Based on this scenario, this paper investigates lane-level routes on a signalized road for autonomous vehicles without vehicle-to-vehicle and vehicle-to-infrastructure communications. Applying Lighthill–Whitham–Richards (LWR) theory (Lighthill and Whitham, 1955; Richards, 1956) and horizontal queueing theory, lane-specific travel times are calculated for autonomous vehicles entering the road at various times in one light cycle. Finally, expected travel time is modeled as a function with respect to lane change decision. In the experiment, it is proven that there exist optimal lane change decision points associated with minimal lane-level route travel time and that the optimal point can be determined by the proposed model with high accuracy.

The remainder of this article proceeds as follows: The following section describes the problem to be addressed. The third section describes several basic assumptions for modeling. This section is followed by the development of the optimization model for giving lane change instructions. The proposed model is then validated, and its performance is analyzed. Finally, conclusions are given, and directions for future research are discussed.

**Problem statement**

Consider an urban arterial road of length of $L$ with left-hand traffic from upstream to downstream approaching an intersection under the control of a traffic signal. There are two lanes of traffic in this direction: the outer lane, which is adjacent to the bike lane or sidewalk, is denoted Lane 1, and the inner lane is denoted Lane 2. Lane 1 is used for both left-turning traffic and traffic moving straight ahead, while Lane 2 is only for traffic proceeding...
Figure 1. The schematic of an autonomous vehicle traveling on an arterial road.

The road is divided into two sections along its length, a restricted section with length of $L_r$, distant from the stop line and an upstream section with length of $L - L_r$, bounded by the entrance of the restricted section. The restricted section in this paper refers to the area in which vehicles are prohibited from changing lanes. The vehicles are prohibited from making lane changes in the restricted section.

Assume that an autonomous vehicle equipped with a navigation system enters the road in Lane 2 and expects to turn left at the downstream intersection. This vehicle must move into Lane 1 before reaching the restricted section. As shown in Figure 1, the autonomous vehicle experiences five critical points. The vehicle enters the road at point A and travels forward along Lane 2; it is assumed to receive a lane change instruction from the navigation system at point B (the distance from point B to the stop line is denoted $X$). The vehicle then begins to seek a safe gap in the target lane (Lane 1); after time $t_c$, a safe gap occurs at point C and the vehicle begins to change lanes; by point D, the vehicle has fully entered Lane 1 from Lane 2 and the lane change is complete; thereafter, the vehicle travels forward on Lane 1 until it leaves this road and turns left at point E. The point of this research is to determine the optimum location for the navigation system to give the lane change instruction to the autonomous vehicle such that a minimum average travel time on the arterial road is achieved.

To explicitly explain the proposed problem, we suppose the free flow speeds of Lane 1 and Lane 2 are $v_1$ and $v_2$, respectively. If $v_1 > v_2$, the autonomous vehicle should change lanes as early as possible since it can travel faster in Lane 1 and spend less time on the road. This means the optimal $X$ is equal to $L$ in this case. When $v_1 < v_2$, the situation becomes much more complicated. If lane-changing occurs too early, the vehicle would spend too much time in the slow lane (Lane 1); if the lane-changing decision is too late, the vehicle may need to stop at the boundary of the restricted section and wait until a safe gap appears, which also increases travel time. Therefore, there is an optimal $X^*$ that needs to be determined to ensure the minimum travel time for the autonomous vehicle. The objective of this paper is to derive the function $\bar{T}(X)$ of the average travel time of the autonomous vehicle with respect to $X$.

Model assumptions

In reality, urban traffic is affected by many factors that are certain (road attributes and traffic lights) and many that are uncertain (driving behaviors, types of vehicles, weather, etc.). Traffic flow is driven by the formation and
dissipation of queues at intersections. The dynamics of queues are characterized by shockwaves, which form at the interfaces of traffic flows with different densities (Ban et al., 2011; Liu et al., 2009). To model urban traffic, we make some standard assumptions (as commonly made in the transportation engineering literature) as well as some specific assumptions related to the problems discussed in this paper.

1. The macroscopic traffic flow is hydrodynamically fluid. The LWR traffic flow theory (Lighthill and Whitham, 1955; Richards, 1956) models vehicular flow as a continuum, and vehicular flow is represented by macroscopic variables of flow $q$ (veh/s), density $\rho$ (veh/m) and velocity $v$ (m/s). Their relationship is $q = \rho v$. We make the assumption of a triangular FD (fundamental diagram) (Figure 2a), which is also used for arterial traffic estimation and control in the literature (Ban et al., 2011; Liu et al., 2009; Geroliminis & Skabardonis, 2010; Hofleitner et al., 2012). The triangular fundamental diagram is determined by several parameters: $q_{max}$, the capacity (veh/s), $v_f$, the free flow speed (m/s), and $\rho_{max}$, the jam density (veh/m). Then, we can derive the critical density $\rho_c = \frac{q_{max}}{v_f}$. For an arterial road, if the arrival traffic flow rate is $q_a$, we can derive the speed of queue formation $u = \frac{q_a v_f}{(\rho_{max} - \rho_c)}$, and the speed of queue dissipation $w = \frac{\rho_c v_f}{(\rho_{max} - \rho_c)}$.

2. The free flow speed $v_1$ of Lane 1 is less than the free flow speed $v_2$. This assumption is reasonable in reality as explained in the problem statement section. Additionally, we assume that both Lane 1 and Lane 2 are subject to the same traffic signal with a red light time $R$ and cycle time $C$, and that Lanes 1 and 2 have the same arrival flow rate $q_a$. Then, the triangular fundamental diagrams for the two lanes are as depicted in Figure 2b. There are also some derived relations for Lane 1 and Lane 2: $q_{max1} < q_{max2}$, $u_1 > u_2$ and $w_1 < w_2$.

3. The traffic is stable. During each time interval of interest, the arrival traffic flow rate $q_a$ and signal timings (red light time $R$ and cycle time $C$) are constant. See Ban et al. (2011), Liu et al. (2009), Hofleitner et al. (2012) and Sun and Ban (2013).

4. The duration of lane change is neglected for simplicity. In other words, the vehicle is assumed to change its position from the current lane to the target lane in an instant (from point C to point D in Figure 1). See Gipps (1986), Ahmed, Ben-Akiva, Koutsopoulos, and Mishalani (1996).

5. The MLC of autonomous vehicles must be completed before the restricted section. That is, if an autonomous vehicle does not make the MLC in the upstream section, it will stop at the entrance of the restricted section to wait for the opportunity to make a lane change. We make this assumption for two reasons: on the one hand, at present, there is no explicit report on an MLC maneuver for both test autonomous vehicles and academic research; on the other hand, by transferring the failure rate of lane change into time loss, we can intently model the time cost under various MLC decision positions.

Mathematical model

Preliminary analysis

For an arterial road that is controlled by traffic signals, a queue forms in each lane (Liu et al., 2009). In addition, the queue length $L_{q1}$ in Lane 1 and the queue length $L_{q2}$ in Lane 2 can be calculated as $L_{q1} = Rw_1 u_1 / (w_1 - u_1)$ and $L_{q2} = Rw_2 u_2 / (w_2 - u_2)$, respectively. Applying the derived relationships $u_1 > u_2$ and $w_1 < w_2$ from assumption (3), we can know that $L_{q1} > L_{q2}$. Then, we can depict the space-time diagrams for Lane 1 and Lane 2 (Figure 3).

In Figure 3, we set the time origin as the beginning of a red light and space (distance) origin at the stop line. For Lane 1, $t_{1}^{ups}$ is a queuing shockwave with a slope of $u_1$, $t_{1}^{dis}$
is a discharge shockwave with a slope of \( w_1 \), and \( l_{1}^{\text{dep}} \) is a departure shockwave with a slope of \( v_1 \). For Lane 2, similar denotations are defined. Additionally, \( l_{1}^{\text{last}} \) is the trajectory of the last vehicle leaving Lane 1 just before the light turns red, and \( l_{2}^{\text{first}} \) is the trajectory of the first vehicle in Lane 2 that stops at the stop line just after the light turns red.

An autonomous vehicle with starting time \( t_s \) receives a lane change instruction at location \( X \). If the vehicle makes a lane change before stopping in one of the queues or at location \( L_r \), the duration of the wait for safe headway is denoted by \( t_e \). If the vehicle stops to wait for safe headway, the duration of the wait is denoted by \( t_{tc} \). These factors will directly affect the calculation of travel time. Among them, \( X \) is our decision variable, \( L_r \), \( L_q2 \), and \( L_q1 \) relate to location and in some sense are determined, whereas \( t_e \) and \( t_{tc} \) relate to the wait for safe headway and are random, depending on the distribution of arriving traffic in Lane 1.

Obviously, we have \( X > L_r \) since there is no reason to instruct a vehicle to change lanes along the restricted section. Then, the location-based parameters are regarded as the first-level factors, and the time-based variables are the second-level ones in our discussion. If \( L_r \leq L_q1 \), we may have \( L_q2 \leq L_r \leq L_q1 \), or \( L_r \leq L_q2 < L_q1 \). Fortunately, the vehicle trajectory is not affected by the relationship between \( L_q2 \) and \( L_r \). As long as the target lane (Lane 1) is occupied by the queue, the vehicle cannot change lanes from Lane 2 to Lane 1. Therefore, the feasible area for changing lanes is determined by the queue in Lane 1, not in Lane 2.

To clarify the trajectory of a specific vehicle, we define several critical points as shown in Figure 3. \( o_1 \), \( e_1 \), \( e_2 \), \( e_2 \) are the intersection points of \( L_r \) and \( l_{2}^{\text{first}}, l_{1}^{\text{que}}, l_{1}^{\text{dis}}, l_{1}^{\text{dep}}, \) and \( l_{1}^{\text{last}} \), respectively. Point \( e_5 \) is the intersection of \( L_r \) and the queuing shockwave on Lane 1 in the next cycle time. \( e_3 \) and \( e_4 \) are the intersections of line \( X \) and \( l_{1}^{\text{que}} \) and \( l_{1}^{\text{dis}}, \) respectively. \( e_5 \) is the intersection of \( l_{1}^{\text{que}}, l_{1}^{\text{dep}} \), and \( l_{1}^{\text{last}} \).

Note that \( e_1 \) and \( e_2 \) are two important points to indicate whether it is possible for the vehicle to change lanes at location \( L_r \). Specifically, lane-changing is possible only before \( e_1 \) or after \( e_2 \). The time coordinates of these points are derived as

\[
\begin{align*}
    t_{e_1} &= \frac{L_r}{u_1} \quad (1) \\
    t_{e_2} &= -\frac{L_r}{v_1} + L_{q1} \ast \left( \frac{1}{u_1} + \frac{1}{v_1} \right) \quad (2)
\end{align*}
\]

Without loss of generality, we assume a one-cycle time range \([-L/v_2, -L/v_2 + C]\) for \( t_s \). Because the travel time calculations are different for vehicles entering Lane 2 at different times \( t_s \), we divide the starting time range into four parts, as in Figure 3: \([-L/v_2, 0], [0, s_1], [s_1, s_2], [s_2, s_3] \) and \([s_3, -L/v_2 + C]\). Of these, \( s_1 \) is calculated by deducting point \( P_1 \) backward to the entry into Lane 2. We have

\[
s_1 = L_r \left( \frac{1}{u_1} + \frac{1}{v_2} \right) - \frac{L}{v_2} \quad (3)
\]

Similarly, \( s_2 \) and \( s_3 \) can be deduced from point \( e_3 \) and \( e_4 \), respectively. However, \( e_3 \) and \( e_4 \) merge to one point \( e_5 \) at location \( L_{q1} \) when \( X \geq L_{q1} \). At the same time, the range
Lane-specific travel time of an autonomous vehicle

As discussed above, we have two scenarios that depend on the relationship between $L_r$ and $L_{q1}$. Which scenario an autonomous vehicle belongs to depends on the traffic state ($q_a$) when it travels on the lane and lane parameters ($C$, $R$, $w$, $u$, etc.), and not on traveling parameters ($t_s$, $t_c$, $t_{fs}$, etc.). Therefore, only one of them corresponds to the calculation of travel time for a specific autonomous vehicle.

The calculation of travel time is discussed case by case for vehicles starting in different time ranges. Each case is then divided into subcases according to the different lane-changing scenarios for the vehicle.

Scenario 1: $L_r > L_{q1}$

If $L_r > L_{q1}$, we have $L_r > L_{q1} > L_{q2}$. An autonomous vehicle would change lanes before arriving at location $L_r$ or first stop at location $L_r$ and then change lanes.

If $t_c \in \left(0, \frac{X - L_r}{v_2}\right)$

In this case, the vehicle makes the lane change before the restricted section. The autonomous vehicle follows Trace 1 in Figure 4. The travel time is then the sum of the free flow travel time in Lane 2 before receiving the instruction, the time spent waiting for safe headway $t_c$, the free flow travel time in Lane 1, and the queuing time in Lane 1. It is calculated as

$$T_1 = \frac{L - X}{v_2} + t_c + \frac{X - t_c v_2}{v_1}$$

$$+ \max \left(0, \frac{u_1 - \omega_1}{\omega_1 (v_1 + u_1)} \left(t_c v_1 + \frac{(L - X) v_1}{v_2}\right) + t_c (v_1 - v_2) + X + R\right), \ for \ t_c \in \left(0, \frac{X - L_r}{v_2}\right).$$

If $t_c \in \left(\frac{X - L_r}{v_2}, +\infty\right)$

In this case, the autonomous vehicle stops at location $L_r$ and waits time $t_{fs}$ to change lanes (Trace 2 in Figure 4). The travel time is the sum of the free flow travel time in Lane 2, the time spent waiting for safe headway $t_{fs}$, the free flow travel time in Lane 1, and the queuing time in Lane 1. It is

$$T_2 = \frac{L - L_r}{v_2} + \frac{L_r}{v_1} + \max \left(0, \frac{u_1 - \omega_1}{\omega_1 (v_1 + u_1)} \left(t_c v_1 + \frac{(L - L_r) v_1}{v_2}\right) + t_{fs}, \right. \left. \frac{(L - L_r) v_1}{v_2} + L_r + R + t_c, \right)$$

for $t_c \in \left(\frac{X - L_r}{v_2}, +\infty\right), \ t_{fs} \in (0, +\infty)$.

Scenario 2: $L_r \leq L_{q1}$

Case 1: $t_c \in \left[-\frac{L}{v_2}, s_1\right)$

In this case, an autonomous vehicle would change lanes either before or after stopping at location $L_r$. Since the different lane-changing points lead to different calculations of travel time, we discuss them separately.

Case 1.1: $t_c \in \left(0, \frac{L - L_r}{v_2}\right)$, the lane change is made before the vehicle stops at location $L_r$. In other words, the vehicle encounters safe headway while moving. A representative vehicle trajectory in this case is depicted as Trace 1.1 in Figure 5a. The travel time is the sum of four components: the free flow travel time in Lane 2 before receiving the lane change instruction, the time spent waiting for safe headway, the free flow travel time in Lane 1, and the queuing time in Lane 1. It is calculated as

$$T_{11} = \frac{L - X}{v_2} + t_c + \frac{X - t_c v_2}{v_1} + \max \left(0, \frac{u_1 - \omega_1}{\omega_1 (v_1 + u_1)} \left(t_c v_1 + \frac{(L - X) v_1}{v_2}\right) + t_c (v_1 - v_2) + X + R\right)$$

for $t_c \in \left(0, \frac{L - L_r}{v_2}\right)$.

Case 1.2: $t_c \in \left(\frac{L - L_r}{v_2}, +\infty\right)$

If $t_c \in \left(\frac{L - L_r}{v_2}, +\infty\right)$, safe headway for lane-changing is not available. The vehicle has to stop at location $L_r$ and wait an additional time $t_{fs}$ for safe headway. However, the time of the lane change will determine which traffic signal cycle the vehicle passes in Lane 1. Therefore, different situations must be analyzed based on the instant of the lane change.

As shown in Figure 5a, the lane-changing point is denoted as $k_c = (t_k, d_k)$. If $k_c$ is between $o_1$ and $e_1$, the vehicle will experience one queue in Lane 1 (Trace 1.2.1); if $k_c$ is between $e_2$ and $o_2$, the vehicle will not encounter a queue and will travel in Lane 1 at free flow speed (Trace 1.2.2); if $k_c$ is between $o_2$ and $e_1$, the vehicle will experience a queue in Lane 1 during the next cycle (Trace 1.2.3).

As previously analyzed, it is impossible for $k_c$ to be located between $e_1$ and $e_2$, since lane-changing cannot occur as long as the target location is occupied by a queue.
The time interval $\Delta_1$ between $e_1$ and $e_2$ can be given as

$$\Delta_1 = (L_{q1} - L_1) \times \left( \frac{1}{u_1} + \frac{1}{v_1} \right)$$

(9)

The number of $\Delta_1$ that a vehicle has encountered can be derived as

$$M = \text{ceil} \left( \frac{1}{C - \Delta_1} \left( t_{c3} - \left( \frac{L_1}{v_1} - \frac{L - L_1}{v_2} - t_s \right) \right) \right)$$

(10)

Consequently, the time coordinate of $k_c$ can be calculated as

$$t_k = t_s + L - L_1 \frac{v_1}{v_2} + t_{c3} + M^* \Delta_1$$

(11)

Additionally, we use $N$ to represent the number of cycles that the vehicle has experienced at $k_c$, and $N$ is given as

$$N = \text{floor} \left( \frac{1}{C} \left( t_k + L_1 \frac{v_2}{v_1} \right) \right).$$

(12)

Based on this analysis and using the formulas above, we can easily calculate the travel time for different situations

$$T_{12} = \begin{cases} 
  t_k - t_s + \frac{L_1}{v_1} + \frac{u_1 - \omega_1}{\omega_1(v_1 + u_1)} (t_k v_1 + L_1) + R, \\
  \text{If } t_k \in \left[ \frac{L_1}{v_1} + \frac{L - L_1}{v_2}, \frac{L_1}{v_1} + L_1 \right], \\
  \text{If } t_k \in \left[ \frac{L_1}{v_1} + L_{q1} \left( \frac{1}{u_1} + \frac{1}{v_1} \right) + N + 1 \right] \times C, \\
  \frac{L_1}{v_1} + (N + 1) \times C, \\
  \frac{L_1}{v_1} + (N + 1) \times C 
\end{cases}$$

(13)

We have now derived expressions for travel time for all possible subcases of Case 1. Based on the state of motion of the autonomous vehicle and on when it changes lanes, we first divided this case into two subcases: Case 1.1, where the lane change is made when the vehicle is moving at the free flow speed (i.e., before the vehicle stops at location $L_1$) and Case 1.2, where the lane change is made when the vehicle is stationary (i.e., after the vehicle stops at location $L_1$). For Case 1.2, the travel time is calculated by one of the three formulas for three different situations, depending on whether the vehicle experiences a queue in one cycle time or the next cycle time or not at all. This idea will be applied to the other cases (Cases 2, 3, and 4) with only minor modifications needed. Meanwhile, variables defined in Case 1 are also suitable for these latter cases and will be used if no special comments are made.

Case 2: $t_2 \in [s_1, s_2)$

As with Case 1, this case is also divided into two subcases based on whether the vehicle is moving forward when the lane change is made: if so, it is Case 2.1; if not, it is Case 2.2.

Case 2.1: If $t_c \in (0, \frac{L_1 + L_{q1}}{v_1} - t_s - \frac{L - L_1}{v_2})$

In Case 2.1, the vehicle cannot change lanes as long as the place alongside it in Lane 1 is occupied by a queue (see Figure 5b). Thus, the domain of $t_c$ for Case 2.1 is determined by the queue formulation shockwave instead of line $L_1$, as in Case 1.1.

A typical trajectory of the autonomous vehicle in this case is depicted as Trace 2.1 in Figure 5b. Calculation of the travel time is the same as in Case 1.1 and thus, we have

$$T_{21} = \frac{L - X}{v_2} + t_c + \frac{X - t_c v_2}{v_1} + \max \left( 0, \frac{u_1 - \omega_1}{\omega_1} (v_1 + u_1) \right) \times \left( t_c v_1 + \frac{(L - X) v_1}{v_2} + t_c (v_1 - v_2) + X \right) + R$$

(14)

Case 2.2 If $t_c \in (\frac{L_1 + L_{q1}}{v_1 + v_2} - t_s - \frac{L - L_1}{v_2}, \infty)$. 

Figure 4. Space-time trajectory of an autonomous vehicle in Scenario 1.

Figure 5. Space-time trajectory of an autonomous vehicle in Scenario 1.
This case is very similar to Case 1.2. In both cases, various trajectories are possible for the vehicle depending on the lane change point $k_c$. One difference is that $k_c$ cannot be between $o_1$ and $e_1$ in Case 2.2 since the autonomous vehicle, which has not changed lanes before the place alongside it in Lane 1 is occupied by the queue, will move forward until it stops at location $L_r$ and wait for the queue in Lane 1 to disperse, at which point the vehicle will wait for safe headway before changing lanes from point $e_2$. Therefore, in this case, $k_c$ can be between $o_1$ and $e_1$ (Trace 2.2.1) or between $o_2$ and $e_2^*$ (Trace 2.2.2), which is similar to the corresponding situations in Case 1.2.

Unlike Case 1.2, in Case 2.2, the target lane is already occupied by the queue (after point $e_1$) when the autonomous vehicle arrives at location $L_r$. Thus, we modify $M$ as

$$M = \frac{t_{cs}}{C - (L_{q1} - L_r) \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right)}$$

(15)

And consequently,

$$t_k = -\frac{L_r}{v_1} + L_{q1} \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right) + t_{cs} - t_s$$

$$+ M \ast \left( L_{q1} - L_r \right) \ast \left( \frac{1}{u_1} + \frac{1}{v_1} \right)$$

(16)

$$N = \frac{t_k + L_s}{C}$$

(17)

Finally, the travel times are drawn from corresponding situations in Case 1.2

$$T_{22} = \begin{cases} 
  t_k - t_s + \frac{L_s}{v_1}, & \text{If } t_k \in \left[ -\frac{L_r}{v_1} + L_{q1} \ast \left( \frac{1}{u_1} + \frac{1}{v_1} \right) + N \ast C, \right] \\
  -\frac{L_r}{v_1} + (N + 1) \ast C \\
  t_k - t_s + \frac{L_s}{v_1} + \frac{L_{q1} - L_r}{v_1 + u_1} \ast (t_k - C) \ast v_1 + L_r + R, & \text{If } t_k \in \left[ -\frac{L_r}{v_1} + (N + 1) \ast C, \frac{L_s}{v_1} + (N + 1) \ast C \right]
\end{cases}$$

(18)

Case 3: $t_s \in [s_2, s_3]$

Compared with the other cases, Case 3 is a special one. It is only valid for $X_s$ smaller than $L_{q1}$, and the lane-changing point must be at location $L_r$, since the range $[s_2, s_3]$ yields an empty set when $X > L_{q1}$ (Figure 5c). Fortunately, this case is almost the same as Case 2.2, and we can directly use the same formulas:

$$M = \frac{t_{cs}}{C - (L_{q1} - L_r) \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right)}$$

(19)

$$t_k = -\frac{L_r}{v_1} + L_{q1} \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right) + t_{cs} - t_s$$

$$+ M \ast \left( L_{q1} - L_r \right) \ast \left( \frac{1}{u_1} + \frac{1}{v_1} \right)$$

(20)
\[
N = \frac{t_k + \frac{t_c}{v_1}}{C} \tag{21}
\]

\[
T_3 = \begin{cases}
  t_k - t_s + \frac{t_c}{v_1}, & \text{If } t_k \in \left[-\frac{t_c}{v_1} + L_{q1} \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right) + N \ast C, -\frac{t_c}{v_1} + (N + 1) \ast C\right] \\
  -\frac{t_c}{v_1} + (N + 1) \ast C, & \text{If } t_k \in \left[-\frac{t_c}{v_1} + L_{q1} \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right) + N \ast C, -\frac{t_c}{v_1} + (N + 1) \ast C\right]
\end{cases} \tag{22}
\]

Case 4: \( t_s \in [s_3, -L/v_2 + C] \)

Case 4 is first divided into three subcases, for which

\[
T_{41} = L - X \frac{v_2}{v_1} + t_c + \frac{X - t_c v_2}{v_1} + \frac{u_1 - \omega_1}{\omega_1 (v_1 + u_1)} \left( t_c v_1 + \frac{L - X v_1}{v_2} + t_c (v_1 - v_2) + X - C v_1 \right) \tag{23}
\]

Case 4.2: If

\[
t_c \in \left( \max \left(0, \min \left( \frac{X - L \frac{v_2}{v_1} + L}{v_2 - v_1}, t_s - \frac{L - \frac{X}{v_2}}{v_2 - v_1} \right) \right), \min \left( \frac{X - L \frac{v_2}{v_1} + L}{v_2 - v_1}, t_s - \frac{L - \frac{X}{v_2}}{v_2 - v_1} \right) \right)
\]

\[
T_{42} = L - X \frac{v_2}{v_1} + t_c + \frac{X - t_c v_2}{v_1} \tag{24}
\]

Case 4.3: If \( t_c \in (\min(\frac{L - \frac{X}{v_2}}{v_2 - v_1}, t_s - \frac{L - \frac{X}{v_2}}{v_2 - v_1}), \infty) \)

\[
M = \max \left(0, t_s + \frac{L - L \frac{v_2}{v_1} + \frac{t_c}{v_1} - L_{q1} \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right)}{C - (L_{q1} - L_r) \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right)} \right)
\]

\[
t_k = t_c + \max \left( -\frac{L}{v_1} + L_{q1} \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right), \frac{L}{v_1} - L_r \right) + M \ast (L_{q1} - L_r) \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right) \tag{25}
\]

\[
t_s + \frac{L - L_{r}}{v_2} \tag{26}
\]

\[
N = \frac{t_k - \frac{t_c}{v_1}}{C} \tag{27}
\]

4.3.1 If \( t_k \in [-\frac{L}{v_1} + L_{q1} \ast \left(\frac{1}{u_1} + \frac{1}{v_1}\right) + N \ast C, -\frac{L}{v_1} + (N + 1) \ast C] \)

\[
T_{431} = t_k - t_s + \frac{L}{v_1} \tag{28}
\]

4.3.2 If \( t_k \in [-\frac{L}{v_1} + (N + 1) \ast C, -\frac{L}{v_1} + (N + 1) \ast C] \)

\[
T_{432} = t_k - t_s + \frac{L}{v_1} + \frac{u_1 - \omega_1}{\omega_1 (v_1 + u_1)} \left( (t_k - C) v_1 + L_r \right) + R \tag{29}
\]

**Distribution of waiting time for safe headway**

The type of lane-changing considered in this study is the MLC (or objective-driven) to make a turn at an intersection (Sun & Elefteriadou, 2012; Qi et al., 2014). Waiting time for safe headway refers to the period of time that an autonomous vehicle spends locating a safe headway for a lane change. We break this waiting time into two types in this study: \( t_s \), the waiting time for a vehicle in motion; and \( t_{cs} \), the waiting time for a vehicle stationary at the boundary of the restricted section. Qi et al. (2014) formulated the probability of objective-driven lane-changing as an exponential function. Using a similar idea, we assume that waiting time follows an exponential distribution. We denote \( f_c(t_c) \) as the probability density function of \( t_c \), and \( f_{cs}(t_{cs}) \) as the probability density function of \( t_{cs} \). Thus, we have,

\[
f_c(t_c) = \lambda_c e^{-\lambda_c t_c} \tag{30}
\]

\[
f_{cs}(t_{cs}) = \lambda_{cs} e^{-\lambda_{cs} t_{cs}} \tag{31}
\]

where \( \lambda_c \) and \( \lambda_{cs} \) are parameters.

Actually, \( \lambda_c \) and \( \lambda_{cs} \) can be regarded as arrival rates of safe headway. Since the only difference between them is whether the autonomous vehicle is moving or not, they are related as

\[
\lambda_c = \lambda_{cs} \ast \frac{v_2 - v_1}{v_1} \tag{32}
\]

Assuming traffic arrival is a Poisson process with a flow rate \( q_a \), the time headway between adjacent vehicles follows an exponential distribution with parameter \( q_a \). According to rule-based models (Hidas, 2005; Sun & Elefteriadou, 2014), if the minimum safe headway for lane-changing is \( h_m \), the probability of a safe headway arising is

\[
p = P(h > h_m) = e^{-q_a h_m} \tag{33}
\]

From the definition of \( f_{cs}(t_{cs}) \) and \( p \), we have

\[
f_{cs}(t_{cs})(1 - p) = f_{cs}(t_{cs} + h_m) \tag{34}
\]

Then, we have

\[
\lambda_{cs} = -\frac{\ln(1 - p)}{h_m} \tag{35}
\]
\[ \lambda_c = -\frac{\ln (1 - p)}{h_m} \cdot \frac{v_2 - v_1}{v_1} \]  

That is, as long as the minimum safe headway \( h_m \) is defined, \( \lambda_c \) and \( \lambda_e \) can be calculated.

**Expected travel time**

We have derived calculations of travel time for one single autonomous vehicle in various possible cases. As we can see, the derived formulas are inherently functions of \( t_s, t_c, \) and \( t_e \). We denote the travel time of an autonomous vehicle as \( T(t_s, t_c, t_e) \), and thus, the expected travel time of the autonomous vehicle can be calculated as

\[ \tilde{T}(X) = \int_{-\lambda}^{C-\frac{1}{2}} \int_{0}^{+\infty} \int_{-\lambda}^{+\infty} T(t_s, t_c, t_e) dt_e P_c(t_e) dt_c P_c(t_c) dt_s \]

Assuming that \( t_s \) follows a uniform distribution (i.e., \( P(t_s) = 1/C \)), the above formula can be simplified as

\[ \tilde{T}(X) = \frac{1}{C} \lambda_c \rho \int_{-\lambda}^{C-\frac{1}{2}} \int_{0}^{+\infty} \int_{-\lambda}^{+\infty} e^{-\lambda_c \rho} T(t_s, t_c, t_e) dt_e dt_c dt_s \]

**Validation and performance analysis**

This section is divided into two subsections: model validation and performance analysis. In the former subsection, we present a simulation network in VISSIM (PTV, 2012) and compare results from both simulation and the proposed model for various scenarios. In the latter subsection, we evaluate the model performance under various traffic conditions (traffic flow rate and free flow speed). Specifically, we demonstrate the travel time distribution with respect to various MLC decision points, the variability of optimal lane change instruction position, and the travel time savings for autonomous vehicles (AVs) using MLC strategies provided by the proposed model under various traffic conditions.

**Model validation**

To validate the proposed model, we consider a specific road with some basic parameters as follows: The length of the road \( L = 1000 \) m, the length of restricted section \( L_r = 30 \) m, and the free flow speed of Lane 2 (the fast lane) is set as \( v_2 = 15 \) m/s as default. For the traffic light, the cycle time \( C = 120 \) s, and red time \( R = 60 \) s. In VISSIM, we built a network as in Figure 1 using the above parameters. A preliminary simulation shows that the maximum flow rate (i.e., capacity) is approximately 880 veh/h for Lane 2 and 810 veh/h for Lane 1 when \( v_1 = 10 \) m/s. To avoid interactions among autonomous vehicles, the flow rate of autonomous vehicles is set to be 20 veh/h. The total simulation time is 5 h, so we simulated approximately 100 autonomous vehicles on average. The lane change instruction is tested at position \( X \), where \( X \) ranges from 80 to 1000 m in 50 m steps. We did not test \( X = 30 \) m, since at the boundary of the restricted section, there is no suitable way to simulate the behavior of the autonomous vehicle in VISSIM.

For the proposed model, we need to determine other parameters in addition to the basic parameters above. The minimum spacing is estimated as 7.0 m, and the jam density \( \rho_{max} = 0.143 \) veh/m. The critical density \( \rho_c \) also needs to be calibrated. However, many studies (Li & Prevedouros, 2002; Hung, Tian, & Tong, 2003) have revealed that the critical density is not constant due to various factors (e.g., dissipating speed, start-up lost times), and consequently, estimating the critical density is difficult. Therefore, we assume the relationship \( \rho_c = 0.5 \rho_{max} \) for simplicity. Note that although this assumption may have a negative impact on the accuracy of the proposed model, it improves the practicality when applying this model for guiding autonomous vehicles. Furthermore, the minimum safe headway is set as a constant \( h_m = 5.3 \) s, which is the median value of an empirical distribution in Wakasugi’s work (2005) (200\( q_a = 5005 \)).

We test the proposed model in four typical situations. They are: \( q_a = 200 \) veh/h and \( v_1 = 6 \) m/s, \( q_a = 200 \) veh/h, and \( v_1 = 10 \) m/s, veh/h and \( v_1 = 6 \) m/s, and \( q_a = 500 \) veh/h and \( v_1 = 10 \) m/s. For simplicity, we use \( \langle q_a, v_1 \rangle \) to refer to a situation with \( q_a \) and \( v_1 \). The maximum queue lengths for these four situations are calculated as 26.8, 25.3, 86.3, and 72.4 m, respectively. Since the lane change section is 30 m long, the former two ((200, 6) and (200, 10)) belong to Scenario 1, and the latter two ((500, 6) and (500, 10)) belong to Scenario 2. Additionally, we consider \( v_1 = 6 \) m/s as an extreme case for which the speed of the slow lane is much slower than the speed of fast lane, and \( v_1 = 10 \) m/s is regarded as a modest case.

The validation results are shown in Figure 6. We can observe that the expected travel times calculated using the proposed model are similar in distribution to the average travel times obtained with the simulation model for all test situations. Both have a unique extreme point at which the expected travel time is minimal. This is consistent with the hypothesis described in the introduction. Furthermore, the difference between the proposed model and simulation data in the downstream section is larger than that in the upstream section. The reason lies in that many AVs would make lane change at the queuing area.
Figure 6. Expected travel times for the proposed and simulation models.

<table>
<thead>
<tr>
<th>RMSE of travel time (s)</th>
<th>$X^*$ From simulation (m)</th>
<th>$X^*$ determined by model (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(200, 6)</td>
<td>2.34</td>
<td>80</td>
</tr>
<tr>
<td>(200, 10)</td>
<td>2.07</td>
<td>180</td>
</tr>
<tr>
<td>(500, 6)</td>
<td>1.85</td>
<td>130</td>
</tr>
<tr>
<td>(500, 10)</td>
<td>2.87</td>
<td>280</td>
</tr>
</tbody>
</table>

The proposed model performs well in most test situations. For the (500, 10) situation, there is a 50 m difference between proposed and simulation models, which is reasonable because lane change instruction position is set at every 50 m in the simulation. In addition, since there are less than 3 s of difference for travel times among near points of 280 m in Figure 6, there is not much difference for an autonomous vehicle to make a lane change decision at these points.

Performance analysis

Since the arrival flow rate and free flow speed are the two most important parameters for measuring traffic conditions in our model, we test the model performance by changing these two parameters.

First, we change the arrival flow rate $q_a$ while keeping $v_1 = 10$ and $v_2 = 15$ m/s. The results are shown in Figure 7a. Expected travel time trends to increase with larger arrival flow rates, which is consistent with common sense. However, this trend does not hold for cases where the lane change instruction is given close to the start of the modeled section of road, which occurs because expected travel time is determined by two groups of vehicles: those vehicles that do not change lanes until they arrive at the queue area and those that find safe headway...
and complete the lane change before the queue area. The ultimate expected travel time is determined by the trade-off between the two groups of vehicles. If the lane change instruction is given to the vehicle too early, the expected travel time is dominated by the latter with a low flow rate because safe headway is more likely to be encountered. Conversely, for a high flow rate, the former dominates. A notable observation from Figure 7a is the trend in the optimal instruction point. Both horizontal and vertical coordinates increase as the flow rate rises. This means that lane change instruction should be given earlier when traffic is denser, and travel time increases for an autonomous vehicle in denser traffic.

Next, in order to demonstrate the model performance for various traffic speed differences between Lane 1 and Lane 2, we change the free flow speed \( v_1 \) of Lane 1 while keeping \( v_2 = 15 \text{ m/s} \) and \( q_a = 500 \text{ veh/h} \). The results are shown in Figure 7b. The overall expected travel time is greater with lower \( v_1 \), especially at the optimal point for lane change instruction. The optimal position for the instruction to be given gets close to the start of the section of road when \( v_1 \) becomes larger, which also tells us it is advantageous for an autonomous vehicle to change lanes later if the travel speed in the target lane is slower. Additionally, the curve becomes flatter for higher travel speed \( v_1 \), which means that, if vehicles follow the instruction given by the proposed model, they can save more time in the case of lower travel speeds in the target lane. In addition, if the travel speed in Lane 1 is close to that in Lane 2, there is no major difference as to when an autonomous vehicle changes lanes.

Then, we specifically investigate the variability of optimal lane change instruction position \( X^* \) under situations with various flow rates and free flow speeds of Lane 1. \( q_a \) ranges from 100 to 700 veh/h in 100 veh/s steps, while \( v_1 \) ranges from 6 to 12 m/s in 2 m/s steps and \( v_2 \) remains 15 m/s. Distribution of \( X^* \) is presented in Figure 8. With more significance than that in Figure 7, the optimal lane change position trends to approach the upstream of the road (i.e., the lane change instruction comes earlier) along with the increase of arrival flow rate or and free flow speed in the slow lane (Lane 1). This is reasonable because larger arrival flow leads to a longer queue in the downstream and a longer waiting time for safe headway, and consequently, it is better for a vehicle to make lane changes earlier. Additionally, the smaller speed difference between Lane 1 and Lane 2 indicates that it is less necessary for a vehicle to risk spending more time for safe headway. Figure 8 can also be referred to as a guide for determining the optimal lane change instruction position as long as

![Figure 8. Distribution of \( X^* \) for various flow rates and free flow speeds of Lane 1.](image)

![Figure 9. Travel time savings of using the instruction by the proposed model.](image)
the arrival flow rates and free flow speeds in both lanes are given.

Finally, we compare the efficiency of two mandatory lane change decision strategies for autonomous vehicles: one uses the optimal instruction and the other occurs when a vehicle makes a lane change decision once it enters this road. We calculate the expected travel times for both strategies using the proposed model and subtract them to obtain the travel time saved. Figure 9 shows the travel time savings for various arrival flow rates and free flow speeds of Lane 1. As we can see, free flow speed is the principal determinant of time savings of the proposed model. Actually, free flow speed difference between lanes is common for urban arterials. The arrival flow rate is also an important factor, and travel time savings are reduced with denser traffic.

Conclusions and future directions

In this paper, we formulate an optimization model for determining the best position at which an automotive navigation system should give a lane change instruction to an autonomous vehicle. First, an exponential distribution is used to model the distribution of waiting times for safe headway to make a lane change. Then, applying hydrodynamic theory and horizontal queuing theory, we calculate the lane-based travel times for autonomous vehicles in various scenarios. The travel time for each vehicle is actually a function of random variables. Based on this, we derive the expected travel times for an autonomous vehicle receiving an instruction to change lanes at arbitrary positions along a section of road.

A validation is conducted by comparing the results of the proposed model with those obtained using a simulation model in VISSIM. These results show that the model not only gives a reasonable determination of the optimal position for the lane change instruction but also provides a good estimate of the average travel time for various instruction positions. According to experiments with various arrival flow rates and travel speeds, we conclude that the instruction should be given earlier in the case of denser traffic or a higher travel speed in the target lane and that vehicles can gain considerable time, especially in the case of lower travel speed in the target lane, by following instructions provided by the proposed model.

This study is a pilot research project against the background of autonomous driving. Future research will likely include analyzing the effect of giving a lane change instruction on total traffic or extending the proposed model for multiple lanes. Additionally, calculation of lane-specific travel time will be an interesting topic in answering the question: would it be worthwhile to change lanes from the viewpoint of a single vehicle or total traffic?

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