Bayesian methodology for diagnosis uncertainty quantification and health monitoring

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SUMMARY

This paper develops a Bayesian approach for the continuous quantification and updating of uncertainty in structural health monitoring. The uncertainty in each of the three steps of damage diagnosis—detection, localization, and quantification—is considered. Bayesian hypothesis testing is used for damage detection, thus facilitating easy quantification and updating of the uncertainty in damage detection. Qualitative damage signatures derived from the model are used for rapid damage localization; when the damage signatures fail to localize the damage uniquely, the uncertainty in damage localization is quantified using the principle of likelihood. Damage quantification is done through the method of maximum likelihood, and the uncertainty in damage quantification is estimated through Bayesian inference. The uncertainty in each of the three steps is continuously updated with the acquisition of more measurements. The overall uncertainty in diagnosis is also calculated, using the concept of total probability. The proposed methods are illustrated using two types of example problems—structural frame and a hydraulic actuation system. Copyright © 2011 John Wiley & Sons, Ltd.

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1. MOTIVATION

An ideal structural health monitoring system is expected to facilitate automated continuous monitoring and diagnosis, thereby enabling prognosis and online health management. Damage diagnosis consists of three major steps—damage detection, damage localization (or isolation), and damage quantification. Practical applications often consist of multi-level, multi-domain systems, where only a few response quantities are monitored, but the number of potential damage candidates is very high. This leads to non-unique solutions in damage diagnosis, which is natural for inverse problems.

The issue is further complicated due to the presence of various other sources of uncertainty such as physical variability, data uncertainty, and model uncertainty. The model inputs and parameters are physically variable in nature. System responses are measured through sensors, and the data may be noisy. Further, the sensors themselves may be damaged and imply deviation of system response from nominal behavior; the health monitoring system must distinguish such scenario from the deviation caused due to actual damage in the system. These are the different aspects of data uncertainty. The models used for diagnosis are not precise and may be uncertain as well. These different sources of uncertainty lead to uncertainty in detection, localization, and quantification of damage. Therefore, the quantification of uncertainty in damage diagnosis is an essential step to guide decision-making with respect to operations, maintenance, and risk management.

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Researchers have pursued two different types of approaches for damage diagnosis, that is, data-driven methods and model-based methods [1]. Data-driven approaches include neural networks [2], autoregressive moving average models [3], fuzzy logic [4], and similarity-based methods [5]. Model-based techniques attempt to emulate nominal system behavior using physics-based system models, such as state-space models [6], finite element models [7], bond graph models [8], and so forth. Hybrid approaches that combine physics-based and data-driven methods have also been pursued [9]. The focus of these studies has been on developing the diagnosis procedure and not on quantifying the uncertainty in diagnosis.

The uncertainty in detection [10] has been previously addressed with respect to non-destructive evaluation techniques through the quantity probability of detection (POD). In such POD calculations, nominally identical damage is introduced in a number of nominally identical specimens, and the number of successful detections is used to calculate the probability of detection. However, such an approach is only applicable to offline testing and not directly applicable to real-time diagnosis [11,12].

System identification techniques have been pursued by several researchers for the purpose of damage isolation and quantification. Fundamentally, these methods could be viewed as statistical calibration problems where the model parameters are calibrated and the uncertainty in calibration can be expressed through confidence bounds in the parameter estimation. Several studies have investigated classical statistics-based methods [13,14] and Bayesian methods [15–21] for uncertainty quantification in system identification and structural health monitoring. These methods involve estimating all the system parameters simultaneously; this makes the computation time-consuming and thus not suitable for online diagnosis. System identification-based methods also combine damage isolation and damage quantification into one procedure; the individual contributions from different stages of diagnosis (detection, localization, and quantification) to the overall uncertainty in diagnosis are not separated.

In our previous work [22], the uncertainty in damage diagnosis was quantified based on principles of classical statistics. Damage detection was based on statistical hypothesis testing, and the uncertainty in detection was quantified based on a chosen significance level. Damage localization was qualitatively based on cause–effect relationships generated from the system model; a prospective set of damage parameters was isolated, and the uncertainty in localization was quantified using a heuristic error sum of squares-based metric. Finite-differencing procedures may be employed to generate the cause–effect relationships; however, this may be computationally expensive. This paper uses a bond graph to represent the system model; the advantage of using bond graphs is that they can be used to derive a set of qualitative cause–effect relationships (referred to as damage signatures) between the model parameters and model outputs. Because the system bond graph model can be generated offline (i.e., before health monitoring) and the damage signatures are only qualitative, there is no computational expense involved in generating them. These damage signatures are then used for rapid online damage localization where sensor data are used to quickly localize the damage to a part of the overall system; then the damage quantification procedure needs to estimate only a smaller subset of possible damaged parameters, thereby considerably reducing the computational effort. Whereas Section 3 briefly discusses some aspects of bond graph modeling and its use in damage localization, these topics have been documented in several textbooks [23–25] and research articles [8][26].

The use of classical statistics to quantify the uncertainty in diagnosis is found to have several challenges such as choice of significance levels, use of heuristic measures for localization uncertainty, use of subjective weights in combining multiple types of sensor data, and so forth. Therefore, this paper develops a unified Bayesian approach to quantify the uncertainty in each of the three steps of diagnosis, leading to a rational, consistent basis for implementation and interpretation. An important additional motive of the current paper is to not only calculate the uncertainty in diagnosis but also continuously update this estimate of uncertainty with acquisition of more measurements. This updating of the uncertainty is not straightforward using a classical statistics-based approach, whereas Bayesian updating provides an efficient framework for updating the statistics when more data are available. The current paper is still focused only on single faults, similar to our previous work. Hence, in the damage quantification stage, when there are multiple damage candidates, a multidimensional calibration problem reduces to multiple one-dimensional calibration problems. Multiple simultaneous faults will need to be considered in future work.
Damage detection is based on Bayesian hypothesis testing of the residuals, that is, the difference between the model predictions and system measurements. In classical hypothesis testing, the choice of the significance level critically influences the detection result and the calculation of uncertainty in detection. In the Bayesian hypothesis testing-based approach, this choice is not necessary. The proposed approach directly quantifies the likelihoods of two scenarios, that is, ‘no fault’ and ‘fault’, and yields results that are easy to interpret. An additional advantage of this approach is the ability to quickly detect false alarms based on the trend in damage probability with acquisition of more measurements.

Damage isolation uses qualitative damage signatures derived from a bond graph representation to isolate a prospective set of damage candidates. A robust, direct measure in terms of probability using the principle of likelihood is proposed in this paper for uncertainty quantification in damage isolation as opposed to the previously used heuristic measure of uncertainty [22]. Bayes theorem for discrete variables is used to update the uncertainty in localization with the acquisition of more measurements.

Damage quantification is based on the principle of maximum likelihood, and the uncertainty in damage quantification is continuously quantified using Bayesian updating. When multiple quantities with different magnitudes and units are measured, combining them for diagnosis requires a robust approach. One option is to scale them and combine them using a weighted sum approach, which may require a subjective choice of weights. In the proposed Bayesian approach, multiple measurements are aggregated using the principle of probability, thus avoiding the choice of weights that may not be intuitive.

The Bayesian updating in the damage quantification stage is based on Bayes theorem for continuous variables; conjugate distributions may not be available in this case, and it may be necessary to resort to Markov chain Monte Carlo (MCMC) sampling, which involves repeated evaluation (in the order of 10,000) of the system model and hence is not suitable for online diagnosis. This paper replaces the MCMC sampling approach with a faster integration technique. The single fault assumption reduces this integration to a one-dimensional integration, which can be solved numerically using advanced quadrature methods. This drastically reduces the computational effort and results in faster diagnosis results.

The methods for uncertainty quantification in damage detection, localization, and quantification are developed in Sections 2. The proposed methods are illustrated using two different types of systems—structural frames (Section 5) and a hydraulic actuator (Section 6). (Note: The terms ‘fault’ and ‘damage’ are used interchangeably in this paper. Whereas the former is commonly used in electrical and electronic systems, the latter is used for mechanical and structural components. Also, the terms ‘localization’ and ‘isolation’ are used interchangeably here. Whereas localization implies narrowing down the spatial location of damage, isolation is a more general term referring to a parameter space; some of the parameters need not be associated with a particular location in space).

2. UNCERTAINTY IN DAMAGE DETECTION

This section outlines the methodology for damage detection and develops a methodology for quantifying and continuously updating the uncertainty in damage detection.

2.1. Damage detection

Detection is based on Bayesian hypothesis testing of residuals \( r(t) \), that is, the difference between the model prediction \( \hat{y}(t) \) and system measurement \( y(t) \). In practice, the residuals \( r(t) \) cannot be obtained continuously with time; usually, they are available as \( r_1, r_2, r_3, ..., r_n \) at discrete time instants \( t_1, t_2, t_3, ..., t_n \). In the context of structural health monitoring, it is conventional [20] to assume that these residuals are normally distributed, that is, \( r \sim \mathcal{N}(\mu, \sigma^2) \). In this paper, the variance \( \sigma^2 \) is calculated based on observed residuals (estimated using all available residuals before damage detection), and hypothesis testing is done only on the mean of the residuals. There is no damage if the mean of the residuals is zero; there is damage if otherwise. Hence, the null hypothesis and the alternate hypothesis are as follows:
H₀ : No damage \( \mu = 0 \)  

H₁ : Damage \( \mu \neq 0 \)

The mean of the residuals \( \mu \) is assumed to follow a prior distribution, which is then updated using the residuals available for damage detection. Let the prior distribution be \( N(\rho, \tau^2) \). If no additional information is available, Migon and Gamerman [27] suggest \( \rho = 0 \) and \( \tau^2 = \sigma^2 \). Then the posterior distribution of \( \mu \) can be calculated using the concept of conjugate distributions and can be proved to be normal [28] with mean and variance as follows:

Posterior mean of \( \mu = \frac{\rho \tau^2 + r_1 + r_2 + \cdots + r_n}{\tau^2 + n\sigma^2} \)  

Posterior variance of \( \mu = \left( \frac{1}{\tau^2 + n\sigma^2} \right)^{-1} \)

Damage detection can be achieved through the use of Bayes factor, which is defined as the ratio of likelihood of the two scenarios: ‘damage’ and ‘no damage’ as follows:

\[ B = \frac{P(D|H_1)}{P(D|H_0)} \]

In Equation (5), \( D \) refers to the data on the residuals obtained during health monitoring. Jiang and Mahadevan [20] derived the expression for the (natural) logarithm of Bayes factor.

\[ \log B = -\frac{1}{2} \log (n + 1) + \frac{n^2R^2}{2(n + 1)\sigma^2} \]

In Equation (6), \( R \) stands for the mean of the observed residuals, that is, \( R = (r_1 + r_2 + \cdots + r_n)/n \). Note that Jiang and Mahadevan [20] defined the Bayes factor as the reciprocal of the expression in Equation (5) and hence calculate \( \log(B) \) as the negative of the expression in Equation (6).

If the Bayes factor \( B \) is greater than 1, it implies that the data favor the hypothesis \( H_1 \) and hence suggests that there is damage. If the Bayes factor is less than 1, then there is no damage. According to Jeffreys [29], a Bayes factor such that \( 1 < B < 3 \) is ‘barely worth mentioning’, \( 3 < B < 10 \) is substantial, \( 10 < B < 30 \) is strong, \( 30 < B < 100 \) is very strong, and \( B > 100 \) is decisive. The advantage in using this approach is that the Bayes factor easily facilitates the calculation of uncertainty in damage detection as explained in the following subsection.

2.2. Assessing the uncertainty in damage detection

Suppose that the residuals have been obtained and the Bayes factor has been calculated as shown in the previous subsection. Then the probability that there is damage is

\[ P(H_1|D) = \frac{P(D|H_1)P(H_1)}{P(D|H_0)P(H_0) + P(D|H_1)P(H_1)} \]  

and the probability that there is no damage is

\[ P(H_0|D) = \frac{P(D|H_0)P(H_0)}{P(D|H_0)P(H_0) + P(D|H_1)P(H_1)} \]

In Equations (7) and (8), \( P(H_0) \) and \( P(H_1) \) denote the prior probabilities of the hypothesis \( H_0 \) and \( H_1 \) being true. If each of these prior probabilities is assigned a value of 0.5, then Equations (7) and (8) reduce to the following:

\[ P(H_1|D) = \frac{B}{B + 1} \]
This metric gives a direct measure of the probability of damage and hence is easy to compute and interpret in comparison with the classical statistics-based metric we previously developed in [22].

2.3. Continuous updating of uncertainty in damage detection

The Bayesian approach provides a convenient means for updating the uncertainty in damage detection with acquisition of new measurements. Suppose that Sections 2.1 and 2.2 used a first set of residuals to calculate the Bayes factor (Equation (5)) and the probability measures (Equations (9) and (10)). A new set of measurements is now available, and a new Bayes factor \( B' \) can be calculated for this new set of residuals. Then Equations (7) and (8) can be directly used to update the uncertainty in damage detection, by using the estimates in Equations (9) and (10) as priors.

\[
P(H_1|D) = \frac{B B'}{1 + B B'} \quad \text{and} \quad P(H_0|D) = \frac{1}{1 + B B'}
\]

(11)

This updating procedure can be continued with more measurements. If there is damage, the value of \( P(H_1|D) \) increases continuously with time. On the other hand, a reduction in \( P(H_1|D) \) following a high value indicates a false alarm.

3. UNCERTAINTY IN DAMAGE LOCALIZATION

This section outlines the methodology for damage localization and develops a methodology for quantifying and continuously updating the uncertainty in damage localization.

3.1. Damage localization

Damage localization is based on the comparison between damage signatures calculated from the system model and the symbols calculated from the system measurements.

First, the system model is used to derive cause–effect relationships between the model parameters (that correspond to damage parameters) and model outputs (that correspond to system measurements). These cause–effect relationships, collectively called as the damage signatures, describe the qualitative changes in the 0th and 1st derivatives of a system output as a result of a change in a system parameter. There are several ways to obtain these signatures, for example, by using finite difference with the system model to calculate approximate derivatives. This paper uses a bond graph-based methodology to derive the damage signatures; the advantage of this method is that the damage signatures can be derived in a qualitative manner, and these signatures can be used in rapid online damage localization of damage parameters [25,26].

Bond graphs can be used to simulate the dynamics of a complex system by modeling the flow of energy (expressed as the product of flow and effort), thus providing a systematic framework for building models of systems spanning multiple domains (e.g., structural, electrical, mechanical, hydraulic, and thermal) using energy as the connecting measure across domains. The bond graph theory is well established and is described in detail in several textbooks [23,24]. The bond graph can be used to derive a temporal causal graph (TCG) that depicts the qualitative cause–effect relationships between the parameters and outputs of the system model. The TCG is used to derive the qualitative damage signatures.

Consider the two-story frame shown in Figure 1. This structure has six parameters: damping \((D_1, D_2)\), stiffness \((k_1, k_2)\), and inertia \((m_1, m_2)\). The inputs to this model are the excitations \((F_1(t) \text{ and } F_2(t))\) at the two stories, and the outputs are the displacements \((u_1(t) \text{ and } u_2(t))\) at the two stories. The state-space equations for this system are as follows [30]:
The bond graph model for this frame structure was constructed by Moustafa et al. [26] and is shown in Figure 2.

In this bond graph model, the efforts are equal at every 0-junction, and the flows are equal at every 1-junction. Hence, by writing the equations for conservation of energy at every junction, it is possible to derive the set of state-space equations that is contained in the bond graph model. Moustafa et al. [26] completed this exercise and proved that the bond graph model yields the same set of equations as in Equation (12).

Thus, the bond graph model is simply a graphical representation of the state-space system equations. Note that the sensors for measuring displacements are also included as separate blocks in the bond graph model, thereby facilitating diagnosis of sensor faults as well. Two types of sensor faults were considered by Moustafa et al. [26]—sensor bias faults and sensor drift faults. Another advantage of the bond graph model in Figure 2 is that this bond graph is modular with respect to the number of floors and can easily be extended to multi-story frames. Refer to Moustafa et al. [26] for complete details regarding the construction of the bond graph model for structural frames.

The combined bond graph model of the system and sensors was used to construct a temporal causal graph, from which the damage signatures were derived. These damage signatures are shown in Table I. These signatures correspond to changes in the 0th derivative and the 1st derivative of the displacements due to changes in stiffness and damping parameters.

Once the presence of damage is detected, the residuals \( r(t) \) are measured, and two sets of symbols are generated for each measurement. The first symbol can be (0) or (+) or (−) depending on three different
scenarios: no discontinuity, discontinuity with a positive residual, and discontinuity with a negative residual, respectively. The second symbol refers to the slope of the residual. It can be either (0), (+), or (−).

This entire procedure of generating damage signatures from bond graphs can be automated [31]. Now, these damage signatures can be used in qualitative localization of damage parameters.

During continuous monitoring, several measurements are collected (for example, pressure, accelerations, etc.), and symbols are generated for each of these measurements. Starting from a list of possible damage scenarios, a particular damage parameter is dropped when the observed symbols do not correspond to the damage signatures. By repeating this process for every damage parameter, the incorrect parameters are all dropped, and it is possible to arrive at the correct damage parameter. As this isolation procedure is qualitative in nature, its implementation is inexpensive and hence aids in rapid online damage localization.

3.2. Assessing the uncertainty in damage localization

Practical engineering systems usually have a large number of parameters that could become faulty but only a small set of measurements. It may be difficult to exclusively isolate one particular damage parameter if several candidates have the same set of damage signatures. Instead, a set of potential candidate damage parameters, \( \theta_i \) (\( i = 1 \) to \( m \)) may be suggested by the isolation procedure. Recalling the single fault assumption made in Section 1, only one of these candidates corresponds to the actual damage parameter. In addition, because of measurement noise and modeling errors, none of these prospective candidate damage parameters can be isolated with 100% probability.

(Note: The symbol \( \theta_i \) is used to denote a particular model parameter that may be damaged, and the symbol \( q_i \) is used to denote the new value of this parameter. For example, \( \theta_i \) denotes ‘stiffness’ and \( q_i \) denotes the ‘new stiffness value reflecting the amount of reduction in stiffness’).

Let \( A \) denote the event that \( \theta_i \) is the true damage parameter. The proposed metric calculates the probability of event \( A \) after observing the data (\( D \)). A prior probability \( P_i^\prime \) is first assumed (a suitable initial assumption would be \( 1/m \) as any of the candidate damage parameters is equally likely to be the damage parameter) and then updated to calculate a posterior probability \( P_i^\prime\prime \) based on the following likelihood function.

\[
P(D|A) = P(D|\theta_i \text{ is the true damage parameter}) = L(\theta_i)
\]

(13)

This probability is the likelihood of event \( A \), denoted by \( L(\theta_i) \). The calculation of this probability is explained in the following paragraphs.

Consider an engineering system which is modeled by Equation (14). In this equation, \( \hat{y} \) represents the model prediction. Let \( B \) denote the event that \( q_i \) is the value of the damage parameter \( \theta_i \). Thus, for a given \( x \) and \( t \), \( \hat{y} \) depends on the value of \( \theta_i \).

\[
\hat{y} = f(x, t; q_i)
\]

(14)

Assume that damage has been detected at time ‘\( t_f \)’, when the value \( q_i \) of the parameter \( \theta_i \) changes to an unknown value. Let measurements continue to be available after damage detection and \( y \) denote the measurements available (for example, pressure, velocity, etc.) at any time instant from \( t_f \) to \( t_f+N \). The standard deviation of these measurements can be monitored until damage is detected and the covariance matrix (\( \Sigma \)) of the residuals can be calculated. Then the likelihood function corresponding to event \( A \cap B \) (i.e., the probability of observing the data conditioned on (i) event \( A \), i.e., \( \theta_i \) is the true damage parameter, and (ii) event \( B \), i.e., the true damaged value is \( q_i \)) can be calculated by assuming an appropriate distribution (normal, below) for the residuals:

\[
P(D|A \cap B) = L(\theta_i, q_i) \prod_{t = t_f}^{t_f+N} \exp \left[ -\left( \hat{y}(q_i) - y \right)^T \Sigma (\hat{y}(q_i) - y) \right]
\]

(15)

Note that \( y \) and \( \hat{y} \) are column vectors of length \( n \), and the covariance matrix \( \Sigma \) is of size \( n \times n \), where \( n \) is the number of measured quantities. Hence, the right-hand side of Equation (15) is a scalar.

The joint likelihood \( L(\theta_i, q_i) \) in Equation (15) can be used in Bayesian inference to update \( P_i \). Let \( f(q_i; \theta_i) \) denote the probability density function (PDF) of the damage value \( q_i \) conditioned on event \( A \). The posterior probabilities \( P_i^\prime\prime \) and \( f(q_i; \theta_i) \) can be calculated based on Bayes theorem. Whereas
the former indicates the uncertainty in damage localization, the latter indicates the uncertainty in damage quantification. Section 4 focuses on the latter topic of uncertainty in damage quantification; the choice of prior \( f'(q_i \mid \theta_i) \) and the estimation of the posterior \( f''(q_i \mid \theta_i) \) are discussed therein. This section deals only with the estimation of \( f''. \)

Consider the simultaneous Bayesian updating of two generic quantities, \( x \) and \( y \). If the likelihood function is given by \( L(x, y) \) and the prior probability densities are \( f'(x) \) and \( f'(y) \), then joint posterior density function \( f''(x, y) \) can be calculated as follows:

\[
f''(x, y) \propto L(x, y)f'(x)f'(y)
\]  

(16)

The marginal density \( f''(x) \) can be calculated by integrating Equation (16) over the space of the variable \( y \).

\[
f'(x) \propto \int f'(x)\int f'(y)dy L(x, y)dy \]

(17)

Hence, by comparing the right-hand side and left-hand side of Equation (17),

\[
L(x) \propto \int L(x, y)f'(y)dy
\]

(18)

Thus, the likelihood of \( x \) can be calculated by computing the product of the joint likelihood and the prior of \( y \) and integrating this product over the space of \( y \). Using this observation, \( L(\theta_i) \) can be calculated from \( L(\theta_i, q_i) \) in Equation (15) as

\[
P(D \mid A) = L(A) = L(\theta_i) \propto \int L(\theta_i, q_i)f'(q_i \mid \theta_i) dq_i
\]  

(19)

Hence, it follows that

\[
P_i' \propto P(D \mid A)P_i' \propto L(\theta_i, q_i)f'(q_i \mid \theta_i) dq_i
\]  

(20)

Note that there is a proportionality constant in Equation (20). This constant can be evaluated by imposing the constraint that the sum of all \( P_i' \)'s should be equal to unity. These probabilities can be continuously updated with the acquisition of measurements.

3.3. Continuous updating of uncertainty in damage localization

If measurements are continuously collected, then the probability calculated using Equation (20) can be updated continuously using Bayes theorem. For the sake of illustration, assume that the current set of estimates (based on time \( t_f \) to \( t_f + N \)) of uncertainty in damage localization is given by \( P_i \) (\( i = 1 \) to \( m \)), where \( P_i \) corresponds to the probability that \( \theta_i \) is the true damage parameter. This quantity is in fact the posterior probability resulting from Equation (20). This posterior probability from the previous data set (based on time \( t_f \) to \( t_f + N \)) becomes the new prior \( P_i' \) for the subsequent data \( D \) (time \( t_f + N \) to \( t_f + N + M \)). Note that these two data sets are not independent. The new posterior probability \( P_i'' \) can be calculated as follows:

\[
P_i'' = \frac{P(D \mid \theta_i \text{ is the true damage parameter})P_i'}{\sum_{i=1}^{m} \text{P}(D \mid \theta_i \text{ is the true damage parameter})P_i'}
\]  

(21)

In Equation (21), the expression \( P(D \mid \theta_i \text{ is the true damage parameter}) \) is again calculated based on Equations (15) and (20), now the difference being that the time window considered in Equation (15) is only from time \( t_f + N \) to \( t_f + N + M \). In order to account for the dependence between the two data sets, the probability of observing data \( D \) (time \( t_f + N \) to \( t_f + N + M \)) in Equation (15) is evaluated conditioned on the previous data set (based on time \( t_f \) to \( t_f + N \)).

Thus, the uncertainty in isolation can be updated continuously with the acquisition of measurements easily using the Bayesian framework.

4. UNCERTAINTY IN DAMAGE QUANTIFICATION

This section outlines the methodology for damage quantification and develops a methodology for quantifying and continuously updating the uncertainty in damage quantification.
4.1. Damage quantification

Given that there is damage and the true damage parameter is \( \theta_i \), the next step is to calculate the actual damage value. The amount of damage is commonly quantified damage using a least squares-based optimization approach. However, when multiple quantities are measured (different magnitudes and different units), the residuals may need to be aggregated using a weighted sum approach. In such a weighted sum approach, the choice of weights is subjective and may not be straightforward. Hence, this paper proposes a different approach based on the concept of likelihood.

A maximum likelihood estimate (MLE) \( \hat{q}_i \) of the damage value can be obtained easily by maximizing the right hand side (RHS) of Equation (15). Note that this equation aggregates different measurements by simply multiplying the probabilities corresponding to each measurement; the standard deviation corresponding to each measurement is estimated from the signal directly. The following subsection estimates the uncertainty in the damage quantification.

4.2. Assessing the uncertainty in damage quantification

Statistical confidence intervals are a common measure of uncertainty; they are not equivalent to calculating the entire probability distribution of the damage value of damage parameter. The interpretation of a 90% statistical confidence interval is very different from that of 90% credibility bounds based on the cumulative distribution function (CDF) [32]. This paper directly calculates the entire probability distribution without considering statistical confidence intervals.

The likelihood function calculated in Equation (15) can be used in Bayes theorem, and the entire probability distribution of the value \( q_i \) of the damage parameter \( \theta_i \) can be calculated as follows:

\[
\hat{f}(q_i|\theta_i) = \frac{L(\theta_i, q_i)\hat{f}(q_i|\theta_i)}{\int L(\theta_i, q_i)\hat{f}(q_i|\theta_i) dq_i}
\]

(22)

In Equation (22), \( f(q_i|\theta_i) \) is the prior density function and represents the knowledge about \( q_i \). As there is no ‘prior’ knowledge about the damage value before the damage quantification, a uniform distribution may be assumed for the prior distribution.

Note that this approach is very similar to Bayesian model calibration [33–35]; the calculation of the denominator on the RHS of Equation (22) is computationally expensive, and several researchers have used MCMC techniques to sample from the numerator in Equation (22). However, this approach requires numerous model evaluations (of the order of \( 10^6 \) [33]) and may not be suitable for online health monitoring and continuous uncertainty quantification using Bayesian updating. Hence, this paper discards the sampling approach and directly evaluates Equation (22) through numerical integration using the adaptive recursive Simpson’s quadrature algorithm.

Consider any one-dimensional integration problem and its approximation using Simpson’s rule as follows:

\[
\int_a^b f(x) dx \approx \frac{b-a}{6} \left\{ 7f(a) + 3f\left(\frac{a+b}{2}\right) + f(b) \right\} = S(a, b)
\]

(23)

The adaptive recursive quadrature algorithm calls for subdividing the interval of integration \((a,b)\) into two sub-intervals \((a,c)\) and \((c,b)\), \(a < c < b\) and then, Simpson’s rule is applied to each sub-interval. The error in the estimate of the integral is calculated by comparing the integral values before and after splitting. The criterion for determining when to stop dividing a particular interval depends on the tolerance level \( \epsilon \). The tolerance level for stopping [36] may be chosen as follows:

\[
S(a, c) + S(c, b) - S(a, b) < 15\epsilon
\]

(24)

This numerical integration algorithm can be used directly in Equations (20) and (22), and the probability distribution of the value \( q_i \) of the damage parameter \( \theta_i \) can be calculated. The next subsection proposes a methodology to continuously update this probability distribution with acquisition of more measurements.
4.3. Continuous updating of uncertainty in damage quantification

Consider the case where a fault was detected at $t_f$ and measurements from time $t_f$ through $t_f+M$ were used to calculate the probability distribution of the damage parameter. Let $f(q_i|\theta_i)$ denote this distribution. Suppose more measurements were collected from time $t_f+M$ through $t_f+N+M$. How do we update the uncertainty in damage quantification using this new information?

The classical statistics-based procedure developed earlier by Sankararaman and Mahadevan [22] does not facilitate such an analysis. It may be possible to recalculate confidence intervals; however, this needs to be done either (i) for the new data only, in which case it is not straightforward to combine these new results with the previously calculated confidence intervals, or (ii) for the entire measurement data again, which renders the previous computation (from time $t_f$ through $t_f+N$) useless.

The Bayesian approach pursued in this paper easily facilitates combining new data with already existing uncertainty estimates. After collecting the second set of measurements, the posterior PDF from the first set can now be used as the prior in Equation (22). Now, the likelihood function $L(\theta_i,q_i)$ is calculated for the newly available data, that is, from time $t_f+M$ through $t_f+N+M$.

This procedure can be repeated again and again, and the probability distribution of the damage parameter can be updated continuously with the acquisition of more measurements.

5. OVERALL UNCERTAINTY IN DIAGNOSIS

Sections 2, 3, and 4 developed methods to quantify the uncertainty in the three steps of diagnosis, that is, detection, localization, and quantification. However, the distribution of $q_i$ calculated in Section 4 is only the conditional probability distribution conditioned on two events: (i) damage is true and (ii) $\theta_i$ is the true damage parameter. The overall uncertainty in diagnosis can be quantified by calculating the overall unconditional probability distribution of $q_i$.

Let $P_d$ denote the uncertainty in damage detection, that is, the probability that there is damage as calculated in Section 2. Let $P_l$ denote the uncertainty in isolation, that is, the probability that $\theta_i$ is the true damage parameter, as calculated in Section 3. This is the probability that the damage is in $\theta_i$, given that there is damage. To calculate the unconditional distribution of $q_i$, the various possible scenarios that lead to damage detection and isolation need to be identified.

1. The damage detection system might have triggered a false alarm. The probability for this event is $(1-P_d)$. In this case, the distribution of the value $q_i$ of the damage parameter $\theta_i$ remains the same as the nominal (healthy) distribution, $F_q(q_{\text{healthy}})$.
2. The damage detection might be correct, but the isolation might be wrong, and $\theta_i$ might not be a correct damage parameter. This event has a probability equal to $P_d(1-P_l)$. In this case also, the distribution of $q_i$ remains the same as the nominal distribution $F_q(q_{\text{healthy}})$ because $\theta_i$ is not the damage parameter.
3. Both damage detection and isolation might be correct. This event has a probability $P_d \cdot P_l$. In this case, the distribution of the value of the damage parameter $q_i$ is what was calculated in Section 4. Hence, $F_q(q_i \mid \text{Damage is true, Damage is in } \theta_i)$ denotes the corresponding CDF.

Using the theorem of total probability [37], the unconditional probability of $q_i$ can be evaluated as in Equation (25), where $F(.)$ represents the cumulative probability distribution and $\ast$ refers to the multiplication operation.

$$F_q(q_i) = (1-P_d)F_q(q_{\text{healthy}}) + P_d \cdot P_lF_q(q_{\text{healthy}}) + P_d \cdot (1-P_l)F_q(q_{\text{healthy}}) \ast \text{Damage is true and in } \theta_i$$

The expression in Equation (25) quantifies the overall uncertainty in the diagnosis procedure by calculating the unconditional CDF of the damage parameter. Further, the corresponding PDF can also be calculated by differentiating the expression in Equation (25) with respect to $q_i$.

The following sections illustrate the proposed methods through two numerical examples, a structural frame and a hydraulic actuation system.
6. ILLUSTRATION USING A STRUCTURAL FRAME

Consider the two-story frame discussed in Section 3. This system has six parameters, $m_1$, $m_2$, $k_1$, $k_2$, $D_1$, and $D_2$. The inputs to the system are forces at the two levels, and the measured outputs are the accelerations at the two levels. The masses of the first and second floors are assumed not to change. Hence, there are only four parameters that could be affected by damage. A typical degradation of the system would be a decrease in the stiffness and/or damping values. The bond graph model of this system and the temporal causal graph were derived by Moustafa et al. [26]. The damage signatures derived from this TCG were given earlier in Table I.

It is evident that some candidate damage parameters have unique signatures and some candidates do not. For example, a bias in the first floor sensor has unique damage signature and hence can be isolated with a 100% probability. However, the stiffness and damping at a particular floor share the same set of damage signatures, and hence, it is not possible to isolate between them qualitatively. However, it is possible to localize the damage to a particular floor, either the first floor ($k_1$, $D_1$) or the second floor ($k_2$, $D_2$). These observations are presented in Table II.

Thus, using the damage signatures, it is possible to identify the floor that is damaged. Hence, it is sufficient to consider the equations governing the motion of that particular floor alone. The damage quantification scheme and uncertainty quantification procedures are carried out as explained in Sections 2. These are illustrated in this section by simulating damage in the structural frame.

The various parameters used in the system are listed in Table III. The stiffness parameters of each floor are represented by $k_1$ and $k_2$, the damping parameters are represented by $D_1$ and $D_2$, the mass of each floor is represented by $m_1$ and $m_2$, and the loading at each level is represented by $F_1(t)$ and $F_2(t)$. The system is measured at every 0.01 s, hence making 100 observations per second. Damage is triggered at 5 s.

6.1. Bayesian damage diagnosis—decrease in stiffness $k_1$

The value of $k_1$ is reduced to 80% at 5 s. This means a change from 30,700 to 24,560 N/m. Sensor noise is simulated by adding Gaussian white noise to the measurements.

Damage detection is done through Bayesian hypothesis testing, as explained in Section 2. A moving window is used to calculate the mean of the residuals, and this mean is tested to detect damage. The Bayes factor is calculated, and the uncertainty in damage detection is calculated until an acceptable level of probability is achieved. The results are tabulated in Table IV.

<table>
<thead>
<tr>
<th>Actual damage</th>
<th>Qualitative isolation results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decrease in $k_1$</td>
<td>Decrease in $k_1$ or decrease in $D_1$</td>
</tr>
<tr>
<td>Decrease in $k_2$</td>
<td>Decrease in $k_2$ or decrease in $D_2$</td>
</tr>
<tr>
<td>Decrease in $D_1$</td>
<td>Decrease in $k_1$ or decrease in $D_1$</td>
</tr>
<tr>
<td>Decrease in $D_2$</td>
<td>Decrease in $k_2$ or decrease in $D_2$</td>
</tr>
<tr>
<td>First floor sensor (+ bias)</td>
<td>First floor sensor (+ bias)</td>
</tr>
<tr>
<td>Second floor sensor (+ bias)</td>
<td>Second floor sensor (+ bias)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$ (N/m)</td>
<td>30,700</td>
</tr>
<tr>
<td>$k_2$ (N/m)</td>
<td>44,300</td>
</tr>
<tr>
<td>$D_1$ (Ns/m)</td>
<td>307</td>
</tr>
<tr>
<td>$D_2$ (Ns/m)</td>
<td>443</td>
</tr>
<tr>
<td>$m_1$ (Ns²/m)</td>
<td>136</td>
</tr>
<tr>
<td>$m_2$ (Ns²/m)</td>
<td>66</td>
</tr>
<tr>
<td>$F_1(t)$ (N)</td>
<td>75 sin(9t)</td>
</tr>
<tr>
<td>$F_2(t)$ (N)</td>
<td>100 sin(9t)</td>
</tr>
</tbody>
</table>
From Table IV, the evidence at \( t = 5 \) s in favor of damage is barely worth mentioning, whereas the Bayes factor at \( t = 5.04 \) s decisively supports the conclusion that there is damage. This probability increases steadily and asymptotically reaches 100% (up to four decimal places) after 5.1 s.

Having detected the damage, the next steps are to perform damage localization and damage quantification and assess the uncertainty in each of them. For the purpose of illustration, three sets of measurements are considered—(i) from 5 to 5.05 s; (ii) from 5.05 to 5.1 s; and (iii) from 5.1 to 5.15 s.

The qualitative damage localization results, as given in Table III, suggest that either \( k_1 \) or \( D_1 \) could be faulty, thereby reducing the number of candidate damage parameters from four to two. Hence, \( \theta \) is a vector of two damage parameters and equal to \( \{ k_1, D_1 \} \). Initially, a prior probability equal to 0.5 is assigned to each of these parameters being faulty. The first set of measurements is used to update these probabilities based on the likelihood corresponding to this set of measurements. These updated probabilities become the priors for the second set of measurements. This procedure can be continued until a reasonable estimate of uncertainty in damage localization is obtained. If the posterior probability is monotonically increasing, then, it could be concluded that the corresponding candidate is in fact the ‘true’ fault candidate. It can be proved that the corresponding posterior probability will reach unity asymptotically. The results of damage localization are shown in Table V.

From Table V, there is approximately 100% (up to four decimal places) probability that \( k_1 \) is the damage parameter. The damage quantification procedure is then performed, and the probability distribution of the damage parameter \( k_1 \) is calculated using the first set of measurements. This distribution is then updated using the second set of measurements through Bayesian updating, and this procedure is continued. The corresponding PDFs are shown in Figure 3. For each Bayesian update, the system model was evaluated 100 times to solve the integral in Equation (22) using the adaptive quadrature technique. Hence, 300 evaluations of the system model were necessary for damage quantification; in contrast, MCMC simulation to evaluate the integral in Equation (22) would have taken at least 20,000 evaluations of the system model.

From Figure 3, it can be seen that (i) the MLE of \( k_1 \) matches closely with the true damage value, and (ii) the uncertainty in the estimate of \( k_1 \) decreases with acquisition of more measurements. Note that this PDF is conditioned on two events: (i) damage is true and (ii) \( k_1 \) is the true damage parameter. In our previous work [22], the conditional PDF was approximated based on multiple confidence intervals; in this paper, the conditional PDF has been calculated directly using Bayes theorem.

The unconditional PDF, which is a measure of the overall uncertainty in diagnosis, can be calculated based on Equation (25). (In this example, the probability that the damage is true and the probability that the \( k_1 \) is the true damage parameter are both equal to 100% and hence, \( P_d = P_I = 1 \); therefore, the unconditional PDF is the same as the conditional PDF in Figure 3.)

### Table IV. Damage detection and uncertainty quantification.

| Time (s) | Bayes factor | \( P_d = P \text{(Damage|Data)} (%) \) | \( P \text{(No damage|Data)} (%) \) |
|---------|--------------|-------------------------------------|----------------------------------|
| 5.00    | 1.6          | 61                                  | 39                               |
| 5.01    | 16.9         | 94.4                                | 5.6                              |
| 5.02    | 76.5         | 98.7                                | 1.3                              |
| 5.03    | 369.5        | 99.7                                | 0.3                              |
| 5.04    | 1669.2       | 99.94                               | 0.06                             |
| 5.05    | 5863.4       | 99.98                               | 0.02                             |

### Table V. Damage localization and uncertainty quantification.

<table>
<thead>
<tr>
<th>Damage parameter</th>
<th>Measurements</th>
<th>( k_1 ) (%)</th>
<th>( D_1 ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior probability that the parameter is damaged</td>
<td>I set</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td>II set</td>
<td>99.12</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>III set</td>
<td>100.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Posterior probability that the parameter is damaged</td>
<td>I set</td>
<td>99.12</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>II set</td>
<td>100.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>III set</td>
<td>100.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
6.2. Extension to multi-story frames

As mentioned in Section 3, the bond graph for a two-story frame can be easily extended to multi-story frames. For example, a five-story frame with 10 possible damage parameters, two for each story—stiffness and damping coefficient—is considered. The nominal values of these parameters and the system inputs are tabulated in Table VI.

The measurements made are the displacements at the five different floors. After damage is detected, the TCG is able to localize the damage to the respective floor. The uncertainty in isolation between the stiffness and the damping factor of that particular floor is quantified using the methods developed in Section 3.

Consider a typical damage scenario; say for example, the value of the stiffness of the second floor is reduced to 70% of its nominal value at 5 s. Sensor noise is simulated by adding Gaussian white noise to the measurements.

Damage detection is done through Bayesian hypothesis testing, as explained in Section 2. A moving window is used to calculate the mean of the residuals, and this mean is tested to detect damage. The Bayes factor is calculated, and the uncertainty in damage detection is calculated until an acceptable level of probability is achieved. The results are tabulated in Table VII.

Table VI. Five-story frame: nominal values.

<table>
<thead>
<tr>
<th>Story no.</th>
<th>Stiffness (N/m)</th>
<th>Damping (Ns/m)</th>
<th>Mass (kg)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24,000</td>
<td>550</td>
<td>75</td>
<td>75 sin(t)</td>
</tr>
<tr>
<td>2</td>
<td>22,000</td>
<td>850</td>
<td>65</td>
<td>80 sin(t)</td>
</tr>
<tr>
<td>3</td>
<td>21,000</td>
<td>450</td>
<td>65</td>
<td>85 sin(t)</td>
</tr>
<tr>
<td>4</td>
<td>19,500</td>
<td>500</td>
<td>60</td>
<td>90 sin(t)</td>
</tr>
<tr>
<td>5</td>
<td>18,000</td>
<td>650</td>
<td>75</td>
<td>95 sin(t)</td>
</tr>
</tbody>
</table>

Table VII. Damage detection and uncertainty quantification.

| Time (s) | Bayes factor | $P_d = P$ (Damage|Data) | $P$ (No damage|Data) |
|----------|--------------|-----------------|----------|-----------|
| 5.00     | 0.6          | 0.38            | 0.62     |
| 5.01     | 3.2          | 0.66            | 0.34     |
| 5.02     | 20.4         | 0.98            | 0.02     |
| 5.03     | 141.0        | 1.00            | 0.00     |
| 5.04     | 900.8        | 1.00            | 0.00     |

Figure 3. Conditional PDF of first story stiffness $k_1$. 

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DOI: 10.1002/stc
From Table VII, there is no evidence that there is damage at $t=5$, and this evidence slowly increases with time; the Bayes factor at $t=5.04$ s decisively supports the conclusion that there is damage. This probability increases steadily and asymptotically reaches 100% at 5.05 s.

After damage detection, the next steps are damage localization and damage quantification and assessment of uncertainty in each of them. For the purpose of illustration, three sets of measurements are considered—(i) from 5 to 5.05 s; (ii) from 5.05 to 5.1 s; and (iii) from 5.1 to 5.15 s.

The qualitative damage localization results, as given in Table III, suggest that either the stiffness term ($k$) or the damping term ($D$) in the second floor could be faulty, thereby reducing the number of candidate damage parameters from 10 to two. Similar to the two-story frame, three sets of successive measurements are used in damage localization and uncertainty quantification. The corresponding results are shown in Table VIII.

From Table VIII, there is approximately 100% (up to six decimal places) probability that $k$ is the damage parameter. The damage quantification procedure is then performed, and the probability distribution of the damage parameter $k$ is calculated using the first set of measurements. This distribution is then updated using the second set of measurements through Bayesian updating, and this procedure is continued. The corresponding PDFs are shown in Figure 3. For each Bayesian update, the system model was evaluated 100 times to solve the integral in Equation (22) using the adaptive quadrature technique. Hence, 300 evaluations of the system model were necessary for damage quantification; in contrast, MCMC simulation to evaluate the integral in Equation (22) would have taken at least 20,000 evaluations of the system model.

From Figure 4, it can be seen that (i) the MLE of $k$ matches closely with the true damage value, and (ii) the uncertainty in the estimate of $k$ decreases with acquisition of more measurements. Note that this PDF is conditioned on two events: (i) damage is true and (ii) $k$ is the true damage parameter. The unconditional PDF, which is a measure of the overall uncertainty in diagnosis, can be calculated based on Equation (25). (In this example, the probability that the damage is true and the probability that the $k$ is the true damage parameter are both equal to 100%, and hence, $P_d=P_i=1$; therefore, the unconditional PDF is the same as the conditional PDF in Figure 4.)

<table>
<thead>
<tr>
<th>Damage parameter</th>
<th>Measurements</th>
<th>$k$ (%)</th>
<th>$D$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior probability that the parameter is damaged</td>
<td>I set</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td>II set</td>
<td>90.9</td>
<td>9.1</td>
</tr>
<tr>
<td></td>
<td>III set</td>
<td>99.8</td>
<td>0.2</td>
</tr>
<tr>
<td>Posterior probability that the parameter is damaged</td>
<td>I set</td>
<td>90.9</td>
<td>9.1</td>
</tr>
<tr>
<td></td>
<td>II set</td>
<td>99.8</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>III set</td>
<td>100.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 4. Conditional PDF of stiffness $k$. 

7. HYDRAULIC ACTUATION SYSTEM

The proposed Bayesian approach for uncertainty quantification is now illustrated for a hydraulic actuation system. The pump–actuator–load system shown in Figure 5 is modeled using bond graphs to aid in damage isolation. The system is studied in detail by Sankararaman et al. [38], and only the data and results pertaining to diagnosis uncertainty quantification are provided in this section.

A pump (powered by a voltage source $V$) drives a hydraulic actuator. The pump fills the first chamber of the actuator with fluid, and the increase in pressure causes the piston to move. As a result, the fluid in the second chamber is also compressed, and the excess fluid flows out into the reservoir. The piston is connected to the load through a rigid rod, and hence, the movement of the piston guides the load. The bond graph model for this system was constructed by the authors earlier and was used to derive the cause–effect relationships between the various parameters of the entire model and, hence, the TCG of the pump actuation system, thereby aiding in rapid qualitative damage isolation. Note that the bond graph model is used only for damage isolation to derive damage signatures and not for uncertainty quantification in diagnosis.

The model shown in Figure 5 consists of three components, a pump, an actuator chamber, and a load (control surface). Individual bond graph models were constructed for each of these components, viz. pump [39], actuator, and control surface. Because bond graphs model the flow of energy, the three models are easily connected to represent the entire system. The various symbols in the model in Figure 5 are explained in Table IX.

Six different fault candidates are considered: control surface damping (+), control surface compliance (−), valve resistance (+), mass of motor assembly (−), area of pump vanes (−), and resistance of pump (+). Both abrupt faults (the value of the parameter changes abruptly) and incipient faults (the value of the parameter increases/decreases slowly and steadily) can be diagnosed using the proposed approach. For example, the area of the vanes could either decrease suddenly by chipping (abrupt damage) or could degrade over time due to corrosion (incipient damage). In the case of abrupt

![Figure 5. Hydraulic actuation system.](image)

Table IX. Parameters of the pump actuation system.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Physical parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Voltage source</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Mass of motor</td>
</tr>
<tr>
<td>$m_2$</td>
<td>Mass of vanes (pump)</td>
</tr>
<tr>
<td>$A$</td>
<td>Area of vanes (pump)</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance of pump</td>
</tr>
<tr>
<td>$R_v$</td>
<td>Valve resistance</td>
</tr>
<tr>
<td>$C_1$, $C_2$</td>
<td>Capacity of 1st and 2nd chamber</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass (control surface)</td>
</tr>
<tr>
<td>$B$</td>
<td>Damping (control surface)</td>
</tr>
<tr>
<td>$C$</td>
<td>Compliance (control surface)</td>
</tr>
<tr>
<td>$k = 1/C$</td>
<td>Stiffness (control surface)</td>
</tr>
</tbody>
</table>
faults, the fault parameter is estimated directly; in the case of incipient faults, the parameters (such as the rate of material removal) that govern the evolution of the damage with time are estimated.

Five response quantities are measured during the system operation. These are pressure in the first actuation chamber, pressure in the second actuation chamber, speed of mass $M$, pump output pressure, and speed of pump vanes.

The damage signatures corresponding to the six fault candidates and the five measurements are shown in Table X.

Several faults were introduced in the system at $t=150$ s and were diagnosed successfully. Measurements are collected at the rate of $2/s$. The uncertainties in damage detection, isolation, and quantification were also quantified as discussed in the following paragraphs.

7.1. Bayesian damage diagnosis—increase in $C$

The value of $C$, the load compliance, is suddenly doubled at 150 s. Similar to the previous section, measurements with noise are simulated by adding Gaussian white noise to the signals.

Bayesian hypothesis testing-based damage detection is done similar to the previous example, and the results are tabulated in Table XI.

As seen in Table XI, the probability that the damage is true (which is a measure of confidence in detection) increases with time and reaches 100% (up to four decimal places) after 155 s.

For the purpose of illustration, three different measurement sets are considered: (i) from 150 to 155 s; (ii) from 155 to 160 s; and (iii) from 160 to 170 s. The uncertainties in damage localization and damage quantification are quantified and updated continuously.

In the damage localization stage, it is possible to qualitatively isolate the damage parameters to compliance $C$ and damping $B$, thereby reducing the number of candidate damage parameters from 4 to 2. The results of uncertainty quantification in damage localization are tabulated in Table XII.

From Table XII, it is clear the load compliance $C$ is the true damage parameter based on the uncertainty in isolation. The damage quantification procedure is used to calculate the PDF of the damage parameter $C$, as shown in Figure 5.

Similar to the structural frame example, each Bayesian update that required 100 evaluations of the system model was evaluated 100 times to solve the integral in Equation (22) using the adaptive quadrature technique. Hence, 300 evaluations of the system model were necessary for damage quantification; in contrast, MCMC simulation to evaluate the integral in Equation (22) would have taken at least 20,000 evaluations of the system model.

<table>
<thead>
<tr>
<th>Fault</th>
<th>Pressure in 1st cylinder</th>
<th>Pressure in 2nd cylinder</th>
<th>Speed of load mass</th>
<th>Pump output pressure</th>
<th>Vane speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$ (−)</td>
<td>0−</td>
<td>0+</td>
<td>0+</td>
<td>0−</td>
<td>0−</td>
</tr>
<tr>
<td>$C=1/k$ (+)</td>
<td>0−</td>
<td>0+</td>
<td>0+</td>
<td>0−</td>
<td>0−</td>
</tr>
<tr>
<td>$R_v$ (+)</td>
<td>0−</td>
<td>0−</td>
<td>0−</td>
<td>0+</td>
<td>0+</td>
</tr>
<tr>
<td>$m_1$ (−)</td>
<td>0+</td>
<td>0+</td>
<td>0+</td>
<td>++</td>
<td>0+</td>
</tr>
<tr>
<td>$R$ (+)</td>
<td>0+</td>
<td>0+</td>
<td>0+</td>
<td>0+</td>
<td>0−</td>
</tr>
<tr>
<td>$A$ (−)</td>
<td>0+</td>
<td>0+</td>
<td>−+</td>
<td>−+</td>
<td>0+</td>
</tr>
</tbody>
</table>

Table X. List of damage signatures for the actuation system.

<table>
<thead>
<tr>
<th>Damage introduced at $T=150$ s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (seconds)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>150.5</td>
</tr>
<tr>
<td>151</td>
</tr>
<tr>
<td>151.5</td>
</tr>
<tr>
<td>152.0</td>
</tr>
</tbody>
</table>

Table XI. Damage detection and uncertainty quantification.
From Figure 6, it can be observed that (i) the MLE of damage is reasonably equal to the true damage value and (ii) the uncertainty associated with the PDF decreases with acquisition of more measurements. Note that this PDF is conditioned on two events: (i) damage is true and (ii) \( C \) is the true damage parameter. The unconditional PDF needs to be estimated using Equation (25). However, as the probability that the damage is true and the probability that \( C \) is the true damage parameter are both equal to 100%, that is, \( P_d = P_i = 1 \), the unconditional PDF is exactly the same as the conditional PDF shown in Figure 6. The unconditional PDF is representative of the overall uncertainty in diagnosis as it includes the uncertainty from all the stages, that is, detection, localization, and quantification.

8. CONCLUSION

This paper developed methods to quantify the uncertainty in the three steps of damage diagnosis, that is, damage detection, damage localization, and damage quantification. A Bayesian approach has been developed for each of these processes. Damage detection is based on Bayesian hypothesis testing, and the uncertainty associated is quantified and continuously updated using a Bayesian metric. Isolation is done qualitatively based on fault signatures derived from a temporal causal graph. When several fault candidates have the same signature, causing ambiguity in isolation, a quantitative approach, again based on Bayesian inference, is adopted to quantify the uncertainty in isolation. Finally, a probability distribution for the damaged parameter is evaluated through Bayesian calibration. The assumption of single faults reduces damage quantification to a set of one-dimensional parameter estimation problems, which can be parallelized, resulting in further reduction of computational time. Also, this helps in replacing expensive time-consuming MCMC sampling techniques to be replaced with an efficient numerical integration approach. Finally, the uncertainties in the three different steps are combined to determine the PDF of the damage parameter.

Hence, it is clear that this paper improves the existing methods for assessing the uncertainty in diagnosis in all the three stages, that is, detection, localization, and quantification. The proposed
The approach is not only convenient to implement but also yields results that are easy to interpret, that is, (i) calculating uncertainty in damage detection does not require a choice of significance level and directly calculates the likelihood of presence of fault; (ii) the uncertainty in localization is quantified using a robust measure as against the previous heuristic measure; (iii) damage quantification is done using a maximum likelihood approach where multiple measurements are aggregated efficiently using the principle of probability; (iv) the uncertainty in damage quantification is expressed using a probability distribution rather than confidence intervals; (v) the Bayesian updating procedure in the damage quantification stage is carried out using an advanced integration technique as against conventional MCMC sampling; and (iv) the Bayesian approach provides a suitable framework for continuously updating the uncertainty with acquisition of more measurements, thereby making the health monitoring system more robust and enabling better decision-making.

There are several directions for future work. In certain applications, system parameters may drift due to changes in environmental conditions, temperature, and others [40]; the diagnosis methods must be able to quantify not only the extent of drift but also the uncertainty in the drift. This paper considered only single faults; in realistic problems, several faults may occur in the system. A multivariate approach may be required to address uncertainty in diagnosis when there are multiple faults. Daigle [41] developed methods for damage detection and quantitative damage isolation of multiple simultaneous faults, and the uncertainty quantification methods proposed in this paper need to be extended to such multiple-fault scenarios in the future. Also, methods need to be developed to incorporate the diagnosis uncertainty into prognosis, system maintenance, and control.

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