Asymmetric multifractal cross-correlations and time varying features between Latin-American stock market indices and crude oil market

Gabriel Gajardo, Werner Kristjanpoller*

Departamento de Industrias, Universidad Técnica Federico Santa María. Avenida España 1680, Casilla 110-V, Valparaíso, Chile

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A B S T R A C T

We apply MF-ADCCA to analyze the presence and asymmetry of the cross-correlations between Latin-American and US stock market indices and crude oil market. We find that multifractality exists in this cross-correlations, and that there is asymmetry on its behavior. The asymmetry degree changes accordingly to the series considered for the trend behavior. We find that fluctuation sizes greatly influence the asymmetry in the cross-correlation exponent, increasing for large fluctuations when we consider the trend of the crude oil price. We also find no clear differences in the exponents with different scales under different trends of the WTI, contrary to other studies in asymmetric scaling behavior. When we examine the time varying features of the asymmetry degree we find that the US indices show a consistent behavior in time for both trends, where the cross-correlation exponents tend to be larger for downward trends. On the other hand, given the more heterogeneous individual properties of Latin-American indices, the asymmetry behavior varies more depending on the trend considered.

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1. Introduction

Despite the growth of alternative energy sources, oil still is and will be a key resource for the industrialized nations, playing an important role in almost all productive activities. For this reason, industrialized countries have felt the big impact of the oil price variations in the last decades, giving rise to a great number of investigations of the quantitative and qualitative effects of this variations in economic growth, exchange rates and stock markets, making essential the understanding of the dynamics in the cross-correlations.

Since the works by Hurst [1,2] and Mandelbrot [3] were published, a lot of work has been done identifying the power-law characteristics in varied real world series with the use of the Rescaled Range Analysis (R/S), such as mining [4], traffic flow [5], air pollutants [6], and economics [7–9]. However, this method presented sensitivity to the short term auto-correlation and the non-stationarities, which can lead to a biased estimation of long memory parameters [10]. With the publication of Peters Fractal Market Hypothesis [11], a lot of work build on this hypothesis and tried to address the issues of the R/S, such as the Wavelet Transform Modulus Maxima (WTMM) method [12], the Fluctuation Analysis (FA) [13], the Detrended Fluctuation Analysis (DFA) [14], the Detrended Moving Average analysis (DMA) [15], its Multifractal ex- tension (MF-DMA) [16] and the Multifractal Detrended Fluctuation Analysis (MF-DFA) [17] which has been extensively used in the analysis of financial series [18–23], and has given a new way to investigate market efficiency [23,24].

With the identification of the cross-correlations in many financial time series [25–32], other methods were developed, such as the Detrended Cross-Correlation Analysis (DCCA) [33], the Multifractal Detrending Moving Average Cross-Correlation Analysis (MF-X-DMA) [34], and the Multifractal Detrended Cross-Correlation Analysis [35] (MF-DCCA) which integrates DCCA into the MF-DFA framework. Since then, the MF-DCCA method has been widely used in analysis of financial series [36–41]. However, the methods mentioned above do not consider the asymmetric market responses to different economic news [42–45]. In this study we are interested in studying the cross-correlations under different trends of one market. To this end, we apply the Multifractal Asymmetric Detrended Cross-Correlation Analysis method (hereinafter MF-ADCCA) proposed by Cao et al. [46]. This method is a combination of the MF-DCCA method, and the asymmetric version of the DFA, the A-DFA proposed by Alvarez-Ramirez et al. [47]. It has revealed interesting features of stock markets [50], commodities [49] and energy markets [48]. By using this method, we can study the multifractal properties in cross-correlations without falling in the subjective sample division between bull and bear markets.

This study focuses on six Latin-American stock market indices, on which the major portion of the Latin-American economy de-
pends, and are also the major recipients of foreign investment in the last decades. Of the six countries, Mexico, Brazil, Argentina and Colombia are oil producers, making Latin-America a net exporter in the global oil market. On the other hand, Chile and Peru are net importers, so countries with different relationships with the oil market are studied. Understanding the dynamics of this relationship is pivotal for the government, multinational organizations and investors in this region to prepare for potential adverse movements in the economy.

To the best of our knowledge, this is the first study of the multifractal and asymmetric properties of the cross-correlations between the crude oil price and the Latin-American stock market indices. We also incorporate the rolling window method to investigate the time varying dynamics of the asymmetry degree.

The remaining of this paper is organized as follows. Section 2 describes the method used, Section 3 describes the data used in the analysis. Section 4 provides the results obtained. Finally Section 5 concludes the paper.

2. Multifractal asymmetric detrended cross-correlation analysis method

The MF-ADCCA [46] is based on the A-DFA and the MF-DCCA, and it allows to examine the asymmetric multifractal characteristics of two cross-correlated time series. The method achieves great performance even with highly non-stationary series, and its steps are similar to the steps in the MF-DCCA method.

Assuming two time series $x_i$ and $y_i$, $i = 1, \ldots, N$, where $N$ is the length of the series, the method can be summarized as follows.

- **Step 1:** Construct the profile

  \[ X(i) = \sum_{t=1}^{i}(x_t - \bar{x}). \quad Y(i) = \sum_{t=1}^{i}(y_t - \bar{y}), \quad i = 1, \ldots, N \quad (1) \]

  Where $\bar{x}$ and $\bar{y}$ represent the average of the series in the whole period.

- **Step 2:** Divide the profiles $X(i)$ and $Y(i)$ into $N_q = \lfloor N/s \rfloor$ non-overlapping windows of equal length $s$. Since the length of the series $N$ is not necessarily a multiple of the time scale $s$, some part of the profile can remain at the end. In order to not discard this part, the same procedure is applied starting from the end of the series. This means that we obtain $2N_q$ segments.

- **Step 3:** The trends, $X^u(i)$ and $Y^u(i)$ for each one of the $2N_q$ segments, are estimated by means of a linear regression where $X^u(i) = a_X^u + b_X^u \cdot i$ and $Y^u(i) = a_Y^u + b_Y^u \cdot i$. This precedes the determination of the detrended covariance, calculated as follows

  \[ F(v,s) = \frac{1}{s} \sum_{i=1}^{s} |X[(v-1)s+i] - X^u(i)| \cdot |Y[(v-1)s+i] - Y^u(i)| \quad (2) \]

  for each segment $v$, $v = 1, \ldots, N_q$ and

  \[ F(v,s) = \frac{1}{\lfloor N/(v-N_q) \rfloor s+1} \]

  \[ -X^u(i) \cdot |Y[(N - (v-N_q))s+i] - Y^u(i)| \quad (3) \]

  for each segment $v$, $v = N_q + 1, \ldots, N_q$.

- **Step 4:** By means of averaging over all segments, the $q$th order of fluctuation function can be obtained as follows for the different behavior of the trends in time series $x_i$

  \[ F_q^+(s) = \left( \frac{1}{M^+} \sum_{v=1}^{2N_q} \frac{\text{sign}(b_X^v)}{2} \frac{F(v,s)^q}{2} \right)^{1/q} \quad (4) \]

  when $q \neq 0$, and

  \[ F_q^+(s) = \exp \left( \frac{1}{2M^+} \sum_{v=1}^{2N_q} \frac{\text{sign}(b_X^v) + 1}{2} F(v,s)^q \right)^{1/q/2} \quad (5) \]

  \[ F_q^-(s) = \exp \left( \frac{1}{2M^-} \sum_{v=1}^{2N_q} \frac{\text{sign}(b_X^v) - 1}{2} F(v,s)^q \right)^{1/q} \quad (6) \]

  for $q = 0$. $M^+ = \sum_{v=1}^{2N_q} [\text{sign}(b_X^v) + 1]$, and $M^- = \sum_{v=1}^{2N_q} [\text{sign}(b_X^v) - 1]$, are the number of subtime series with positive and negative trends. We assume $b_X^v \neq 0$ for all $v = 1, \ldots, 2N_q$, such that $M^+ + M^- = 2N_q$.

The traditional MF-DCCA is implemented by computing the average fluctuation function for $q \neq 0$

\[ F_q(s) = \left( \frac{1}{2N^q} \sum_{v=1}^{2N_q} F(v,s)^q \right)^{1/q} \quad (8) \]

and as follows when $q = 0$

\[ F_0(s) = \exp \left( \frac{1}{4N} \sum_{v=1}^{2N_q} \ln F(v,s) \right) \quad (9) \]

- **Step 5:** The scaling behavior of the fluctuations is analyzed by observing the log-log plots of $F_q(s)$ versus $s$ for each value of $q$. In the case where the two series are long-range cross-correlated, $F_q(s)$ will increase for large values of $s$ as a power law

\[ F_q(s) \sim s^{H_q(q)} \quad (10) \]

\[ F_q^+(s) \sim s^{H_q^+(q)} \quad (11) \]

\[ F_q^-(s) \sim s^{H_q^-(q)} \quad (12) \]

The scaling exponent $H_q(q)$ is known as the generalized cross-correlation exponent, and describes the power-law relationship between two series. It can be obtained by calculating the slope of the log-log plots of $F_q(s)$ versus $s$ through the method of Ordinary Least Squares (OLS).

In the case of $q = 2$, the generalized cross-correlation exponent has similar properties and interpretation as the univariate Hurst exponent calculated by the DFA. If $H_{qy}(2) > 0.5$, the series are cross-persistent, so a positive (negative) change in one price is more statistically probable to be followed by a positive (negative) value of the other price. In the case where $H_{qy}(2) < 0$ the series are cross-antipersistent, which means that a positive (negative) change in one price is more statistically probable to be followed by a negative (positive) change on the other price. For $H_{qy}(2) = 0.5$ only short-range cross-correlations (or no correlations at all) are present in the relationship between the series.

To measure the asymmetric degree of the cross-correlations we can calculate, for every $q$, the following metric

\[ \Delta H_q(q) = H_q^+(q) - H_q^-(q) \quad (13) \]

The greater the absolute value, the greater the asymmetric behavior. If $\Delta H_q(q)$ is equal or close to zero, then the cross-correlations are symmetric for different trends of time series $x_i$. If the value of $\Delta H_q(q)$ is positive, it means that the cross-correlation exponent is higher when the time series $x_i$ has a positive trend than when it is negative. If it is negative, the cross-correlation exponent is lower...
when the time series $x_t$ has a positive trend that when it is negative.

If the value of the generalized cross-correlation exponent $H_{xy}(q)$ depends on the value of $q$, the cross-correlation between the two time series is multifractal. Just like in the MF-DCCA, for $q > 0$, $H_{xy}(q)$ and $H_{xy}(q)$ describe the scaling behavior of large fluctuations, and for $q < 0$, describe the scaling behavior of small fluctuations. In this paper, we study the multifractal behavior under different trends of both the WTI and the stock market indices.

### 3. Data

For this study, we choose the daily closing price of the West Texas Intermediate (WTI), six Latin-American indices and two US indices, namely, the IBOVESPA (Brazil), IPSA (Chile), MERVAL (Argentina), IPC (Mexico), COLCAP (Colombia), SPBV (Peru), S&P500 and Dow Jones Industrial Average (both from the US). The objective of including US indices is to compare stock markets with different sizes and maturity.

The sample interval is from the January 28th, 2005, to December 29th, 2016, which after cleansing leaves us with 2545 observations. All data was taken from Economática.

To analyze the relationship between the indices and the WTI crude oil, we calculate the daily return, $r_t$, as the logarithmic difference between consecutive prices, $r_t = \log(P_t) - \log(P_{t-1})$.

The descriptive statistics for the nine time series are shown in Table 1. All of the return series present large fluctuations, but their mean is close to zero. From the third and fourth moments we can see a deviation from the normal distribution, which is confirmed by the Jarque-Bera statistic, rejecting Gaussian distribution at the 1% significance level for all series.

### 4. Empirical results

In this section we first analyze the cross correlations for changes in trend of the WTI, then on the stock market indices, and finally the time-varying dynamics of the asymmetry in the cross-correlations.

#### 4.1. Asymmetric behavior in the cross-correlations for different trends of crude oil prices

As can be observed in the log-log plots of $F_z(n)$, $F_y(n)$ and $F_y(n)$ versus $n$ (Fig. 2), there is a clear slope difference between $F_z(n)$ and $F_y(n)$, which shows the presence of asymmetry in the cross-correlations. Specifically the generalized cross-correlation exponent, $H_{xy}(2)$, tends to be higher when the WTI is downwards. This also apply in the multifractal analysis as can be observed in Fig. 3, where only the IPC and IPSA show higher $H_{xy}(q)$ when the WTI market is up. The US indices show almost identical behavior, indicating a similar relationship with the WTI. Moreover, as can be observed in Fig. 1, the asymmetry tends to be higher for large fluctuations ($q > 0$), where all series, except for IPSA, show higher cross-correlation exponent when the WTI market is downwards. All Latin-American indices, except for IPSA, show strongest multifractal behavior when the WTI market is upwards.

As shown in Table 2, all stock markets present long-range persistence in the overall cross-correlation and show similar asymmetry degree. When we observe the cross-correlation exponent $H_{xy}(2)$ under different trends we see that only the IPSA show an increment in cross-persistence when the WTI is upwards. The other series present varied behavior in that case. Ibovespa, IPC, DJIA and SP500 show anti-persistent behavior, and MERVAL reports random cross-correlations with the WTI. When the crude oil market is downwards, all series present cross-persistence.

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**Table 1**

The descriptive statistics of return series. J-B represents the Jarque-Bera statistic, ADF denotes the Augmented Dickey-Fuller, Q(10) denotes the value of Ljung-Box-Pierce Q statistic with 10 lags and the ARCH(10) is the Engle’s ARCH test with 10 lags.

<table>
<thead>
<tr>
<th></th>
<th>WTI</th>
<th>IBOVESPA</th>
<th>IPSA</th>
<th>IPC</th>
<th>MERVAL</th>
<th>SPBV</th>
<th>COLCAP</th>
<th>DOW JONES</th>
<th>SP500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>-0.073</td>
<td>0.036</td>
<td>0.033</td>
<td>0.049</td>
<td>0.098</td>
<td>0.053</td>
<td>0.032</td>
<td>0.025</td>
<td>0.026</td>
</tr>
<tr>
<td>S.D. (%)</td>
<td>4.194</td>
<td>1.904</td>
<td>1.131</td>
<td>1.362</td>
<td>2.185</td>
<td>1.882</td>
<td>1.748</td>
<td>1.203</td>
<td>1.301</td>
</tr>
<tr>
<td>Skew.</td>
<td>0.183</td>
<td>0.092</td>
<td>0.302</td>
<td>0.139</td>
<td>-0.349</td>
<td>0.671</td>
<td>-0.109</td>
<td>-0.241</td>
<td>-0.418</td>
</tr>
<tr>
<td>J-B</td>
<td>1996.737</td>
<td>3306.243</td>
<td>29590.145</td>
<td>4264.528</td>
<td>2315.469</td>
<td>39551.769</td>
<td>3117.362</td>
<td>10702.726</td>
<td>10555.938</td>
</tr>
<tr>
<td>ADF</td>
<td>-49.889</td>
<td>-51.491</td>
<td>-44.559</td>
<td>-46.998</td>
<td>-48.706</td>
<td>-44.825</td>
<td>-43.452</td>
<td>-56.001</td>
<td>-56.202</td>
</tr>
<tr>
<td>Q(10)</td>
<td>18.302</td>
<td>34.387</td>
<td>59.469</td>
<td>71.855</td>
<td>12.499</td>
<td>56.228</td>
<td>103.277</td>
<td>65.070</td>
<td>70.559</td>
</tr>
<tr>
<td>ARCH(10)</td>
<td>268.259</td>
<td>687.662</td>
<td>298.446</td>
<td>572.424</td>
<td>339.299</td>
<td>288.269</td>
<td>570.148</td>
<td>784.484</td>
<td>803.852</td>
</tr>
<tr>
<td>Obs.</td>
<td>2545</td>
<td>2545</td>
<td>2545</td>
<td>2545</td>
<td>2545</td>
<td>2545</td>
<td>2545</td>
<td>2545</td>
<td>2545</td>
</tr>
</tbody>
</table>

**Fig. 1.** Asymmetry degree $\Delta H_{xy}(q)$ for the bivariate series when WTI has different trends.

**Table 2**

$H_{xy}(2)$, $H_{xy}(2)$ and $H_{xy}(2)$ for the bivariate series when the WTI has different trends.

<table>
<thead>
<tr>
<th></th>
<th>WTI-IBOVESA</th>
<th>WTI-IPSIA</th>
<th>WTI-MERVAL</th>
<th>WTI-SPBV</th>
<th>WTI-COLCAP</th>
<th>WTI-DJIA</th>
<th>WTI-SP500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.522</td>
<td>0.463</td>
<td>0.546</td>
<td>-0.083</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.521</td>
<td>0.525</td>
<td>0.513</td>
<td>0.012</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.518</td>
<td>0.482</td>
<td>0.527</td>
<td>-0.044</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.539</td>
<td>0.500</td>
<td>0.555</td>
<td>-0.055</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.597</td>
<td>0.533</td>
<td>0.617</td>
<td>-0.064</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.571</td>
<td>0.527</td>
<td>0.594</td>
<td>-0.067</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.519</td>
<td>0.473</td>
<td>0.530</td>
<td>-0.058</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{xy}(2)$</td>
<td>0.519</td>
<td>0.471</td>
<td>0.531</td>
<td>-0.060</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.2. Asymmetric behavior in the cross-correlations for different trends of stock market index

As can be seen in Fig. 5, when we consider different trends in the stock market index, the asymmetry is not so clear for Latin-American indices. In Table 3 we can see that all Latin-American indices, except for IPSA, show a decrease in asymmetry in the cross-correlations for $q = 2$. This contrasts with the findings in the US indices, which almost double the asymmetry degree presented in Section 4.1. Both indices and IPSA show a clear change in the direction of the cross-persistence under different trends. In the case of the US indices, they go from a cross-antipersistent behavior to a cross-persistent behavior, contrary to the movement of the IPSA.

As can be seen in Fig. 6, Ibovespa, IPC, MERVAL and SPBVL present the strongest multifractal behavior of the cross-correlation when the stock market is upwards, showing higher cross-persistent behavior for small fluctuations ($q < 0$), and higher cross-antipersistent behavior for large fluctuations ($q > 0$).

The US indices again show similar behavior, presenting a consistently higher cross-correlation exponent when the stock market is downwards. According to Fig. 4, the behavior of the multifractal asymmetry degree is more heterogeneous, meaning that the direction of the trend of each stock market index greatly influences its cross-correlations with the WTI. We can see that COLCAP has a consistent asymmetry degree across different values of $q$, and that MERVAL, Ibovespa, SPBVL, IPC change the asymmetric behavior for large fluctuations, where the cross-correlation exponent is just slightly greater for downward trends than upwards.
Fig. 4. Asymmetry degree $\Delta H(q)$ for the bivariate series when the stock market index has different trends.

Fig. 5. Log-log plots of $F_2(n), F^+_2(n), F^-_2(n)$ versus $n$ when the stock market index has different trends.

Fig. 6. Plots of the Generalized Cross Correlation Exponent $H_0(q), H^+_q(q)$ and $H^-_q(q)$ under different trends of the stock market index.

trends, contrasting with what happens for small fluctuations. On the other hand, IPSA shows an increase in the asymmetry degree for large fluctuations, meaning that under such conditions the cross-correlations are higher under upward trends of the index.

For the US indices, the asymmetry is clear across the size of fluctuations, showing an identical degree under large fluctuations. For this indices, the cross-correlation exponent is always larger when the index is downwards.

4.3. Time varying features of the asymmetry degree

We use the rolling window method to study how the asymmetry in cross-correlations changes over the period of analysis. We follow the work by Vasilescu et al. [51], who suggested that singular measure algorithms can yield reliable results for window lengths of more than 800 points, so we consider a window of 1000 points for our analysis (approximately four years).

In Fig. 7 we can observe the time varying features of the asymmetry degree for the different trends of the two series. Under different trends of the WTI, IPSA and MERVAL show similar move-
Table 4

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>S.D.</th>
<th>Max.</th>
<th>Min.</th>
<th>(\Delta H_y(2) \geq 0) (%)</th>
<th>(\Delta H_y(2) &lt; 0) (%)</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI-IPSA</td>
<td>0.017</td>
<td>0.020</td>
<td>0.047</td>
<td>0.134</td>
<td>-0.131</td>
<td>64.932</td>
<td>35.068</td>
<td>4</td>
</tr>
<tr>
<td>WTI-IPC</td>
<td>0.001</td>
<td>0.003</td>
<td>0.071</td>
<td>0.187</td>
<td>-0.165</td>
<td>52.282</td>
<td>47.718</td>
<td>1</td>
</tr>
<tr>
<td>WTI-MERVAL</td>
<td>-0.004</td>
<td>-0.003</td>
<td>0.051</td>
<td>0.149</td>
<td>-0.137</td>
<td>47.464</td>
<td>52.536</td>
<td>2</td>
</tr>
<tr>
<td>WTI-SPBVL</td>
<td>-0.036</td>
<td>0.043</td>
<td>0.071</td>
<td>0.223</td>
<td>-0.147</td>
<td>72.644</td>
<td>27.356</td>
<td>6</td>
</tr>
<tr>
<td>WTI-COLCAP</td>
<td>0.028</td>
<td>0.024</td>
<td>0.060</td>
<td>0.212</td>
<td>-0.165</td>
<td>65.020</td>
<td>34.980</td>
<td>5</td>
</tr>
<tr>
<td>WTI-DJIA</td>
<td>-0.052</td>
<td>-0.051</td>
<td>0.062</td>
<td>0.110</td>
<td>-0.222</td>
<td>21.530</td>
<td>78.470</td>
<td>7</td>
</tr>
<tr>
<td>WTI-SP500</td>
<td>-0.058</td>
<td>-0.061</td>
<td>0.056</td>
<td>0.117</td>
<td>-0.217</td>
<td>13.029</td>
<td>86.971</td>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 7. Evolution of the asymmetry degree for different trends for both series using a rolling window fixed at 1000 trading days.

The IPSA shows a predominant positive value in the asymmetry degree in contrast with the balanced degree in MERVAL. Ibovespa, IPC, SPBVL and COLCAP show similar behavior after mid-2013, where the four-year rolling window is capturing most of the consequences of the subprime crisis. When considering the whole period, IPSA, SPBVL and COLCAP tend to show a higher cross-correlation exponent when the crude oil market is upwards. DJIA and S&P 500 show a consistently higher cross-correlation exponent for downward trends of the oil price, as the \(\Delta H_y(2)\) is mostly negative for the period in analysis.

We find that, when considering the absolute value of the mean asymmetry degree, the IPC show the most symmetric behavior, and that the US indices are the most asymmetric ones, showing a clear difference for both trends.

When we consider the asymmetry under different trends of the index, we can also observe similar direction changes (Fig. 7) and absolute asymmetry degree over time for IPSA and MERVAL. However, MERVAL is more prone to negative degrees, as can be observed in Table 5, in contrast with IPSA.

The DJIA show similar behavior for the different trends of both series. In the case of the S&P 500, this behavior holds until mid-2013, were the behavior experience significant differences accordingly to the series used for the trend analysis. Around the same period, for Ibovespa, SPBVL and COLCAP, we can observe a decrease in the asymmetry degree (increase in the absolute value), in contrast with the rise under different trends of crude oil price. This behavior changes after 2014, becoming more similar to the asymmetry degree for different crude oil trends.

All series, except for IPSA, present higher cross-correlation exponent under downward trends for most of the period analyzed.

5. Conclusion

We applied the asymmetric version of the MF-DCCA, the MF-ADCCA, in the study of the cross-correlations between Latin-American stock market indices and the WTI, to compare how it behaves under different trends of one of the series at a time. We also applied this methodology to the US stock market indices, namely the Dow Jones Industrial Average and the S&P 500, to compare the scaling behavior in markets with different sizes and maturity.

Our study resulted in the following findings:

- We find that multifractality exists in the cross-correlations between crude oil market and Latin-American indices, and that there is an asymmetry on the cross-correlation exponent depending on the trend of one series, reporting different asymmetry degrees according to the trend considered.
- For most of the series, the generalized cross-correlation exponent is greater under downward trends of crude oil price across fluctuation sizes, becoming more asymmetric with large fluctuations. However, this behavior becomes more heterogeneous when we consider the trends of the stock market indices. In this case we find that all Latin-American indices show a greater cross-correlation exponent when their trends are upwards for small fluctuations and that the US indices have identical asymmetric behavior for large fluctuations.
- Unlike other studies in asymmetric scaling behavior [46, 54], we find that there is no clear difference in asymmetry under differ-
ent trends of the WTI for different scales in the series studied. This can be attributed to a more stable behavior in the cross-correlations, where despite the longer period of analysis, the characteristics of the cross-correlations will hold.

We do find differences, as the other studies, under different trends of the stock market indices. Actually, for smaller scales, the asymmetric behavior is clearly visible in countries like Chile, Argentina and Colombia, which can be interpreted as a more homogeneous behavior that becomes more heterogeneous as we increase the period analyzed. This behavior should be considered when making long term plans, as the asymmetry will not necessarily be present. It is important to consider the fact that not all studies use the same scale range, so differences in the behavior could be attributed to this range. For example, in [36], the maximum scale is approximately 250 days, while in [46], the maximum scale is approximately of 375, and in [54] is 1300. While the analysis may differ on their objectives, these differences are observed in other multifractal studies for autocorrelations and cross-correlations [36,52,53], and present different conclusions based on the adequacy of the linear adjustment under different scales. Future research should focus in determining appropriate length of the series for meaningful insights.

• The US indices present a clear asymmetry in scaling behavior for both trends, where the cross-correlation exponent is higher for downward trends than for upward trends. This results are also consistent in time. This behavior can be interpreted as a closer relationship, where the potential bidirectional influence is stronger, making it more relevant in the dynamics of the cross-correlations. For the US, this finding poses interesting and complex subjects in hedging. When considering applications in investing in certain economic sectors or firms, the inclusion of information about the WTI can greatly improve the allocation of assets in the future. As the WTI is more predictable in terms of potential trends given by global factors, using the WTI and having information about the specific behavior of the cross-correlations can shed light for future allocations.

• Due to the fact that for the US indices the asymmetry is always negative, the focus of analysis should be in the stressful periods and its predictability. The insights about low probability events such as extremely small fluctuations, and extremely large fluctuations are considered when taking $q = -10$ and $q = 10$ respectively. The correlations under these schemes should be considered when trying to predict indices behavior given past behavior of the WTI. In fact, future research should consider studying how the autocorrelations can relate to the cross-correlations empirically.

• The asymmetry in cross-correlation exponents possess time-varying features, and experience high changes in behavior in mid-2013, which can be attributed to the changes in relationship caused by the effects of the sub-prime crisis. Moreover, the asymmetry under different trends is more heterogeneous for Latin-American countries, while US indices show a more similar behavior despite of the trend considered.

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