An investigation has been made of the packing of binary mixtures of either spherical or nonspherical particles. It is shown that this binary packing system can satisfactorily be described by the Westman equation. By use of the concept of "equivalent packing diameter," nonspherical particle packing may be related to spherical particle packing. The porosity of binary mixtures of nonspherical particles can then be predicted by means of a model developed for spherical particles. This approach is verified by the good agreement between the calculated and experimental results.

I. Introduction

Porosity is probably the simplest and most accessible parameter available in particle packing characterization. Therefore, it is very useful to develop a method to predict the porosity of a packing of particles. For example, such a method may be used in the optimum selection of particle mixtures for the property control of ceramic or composite products. For spherical particle packing, significant progress has been made in modeling the relationship between porosity and particle size distribution.\(^5\) For nonspherical particle packing, however, such a development is not yet available.

The packing behavior of nonspherical particles is similar to that of spherical particles.\(^2\) This suggests that a general model for predicting the porosity of spherical and/or nonspherical particle mixtures is possible. To facilitate the application of this approach, the concept of equivalent packing diameter has recently been introduced in particle characterization.\(^5\,6\) In this paper it will be shown that, by use of this newly developed concept, the porosity of binary mixtures of nonspherical particles can satisfactorily be predicted from an empirical model developed for spherical particles.

II. Proposed Method

In this paper, the specific volume \(V\), an alternative parameter to porosity,\(^5\,6\) of a binary mixture is considered to be a function of (1) volume fraction (or fractional solid volume) \(X_s\) or \(X_L\) (= 1 - \(X_s\)); (2) effective packing sizes \(d_s\) and \(d_L\); and (3) initial specific volumes \(V_s\) and \(V_L\). That is

\[
V = f(X_s, d_s, d_L, V_s, V_L)
\]

(1)

The subscripts \(S\) and \(L\) represent, respectively, the small and large components. For a spherical particle the effective packing size should be its diameter, while for a nonspherical particle the effective packing size should be dependent on its equivalent volume diameter and sphericity, as discussed later.

There are no simple theories which can successfully and satisfactorily be used to describe the relationship between porosity or specific volume and particle size distribution. For binary mixtures of spherical particles, it appears to be reasonable to model this relationship in terms of "the wall effect" as used by a number of investigators.\(^7\,10\) However, evaluation of the coefficients or parameters involved in the so-developed equations usually has to be made on purely empirical grounds. Consequently, it is unlikely that the model equations can directly be used in the porosity calculation of nonspherical particle mixtures. On the other hand, Westman\(^1\) proposed a simple conic equation to represent the relationship between the specific volume and the volume fractions of binary mixtures. This equation, referred to as the Westman equation here, has been found to be applicable to both spherical particles and nonspherical particles.\(^12\,14\) This equation will therefore be used in the present work.

The Westman equation was originally written as\(^1\)

\[
\left(\frac{V - V_sX_s}{V_s}\right)^2 + 2G\left(\frac{V - V_sX_s}{V_s}\right)\left(\frac{V - X_s - V_sX_s}{V_L - 1}\right)
\]

\[
+ \left(\frac{V - X_s - V_sX_s}{V_L - 1}\right)^2 = 1
\]

(2)

As given elsewhere,\(^14\) a more explicit form of Eq. (2) can be used in calculating \(V\). Referring to Eq. (1), it is evident that the coefficient, \(G\), is the only unknown parameter in the Westman equation. For spherical particles, \(G\) has been found to be dependent on the size ratio \(r = d_s/d_L\) and this dependence, i.e., the \(G-r\) relation, can be determined empirically. As shown in Fig. 1, the \(G-r\) relation for different sources of literature data may be slightly different, due mainly to the different experimental conditions.\(^8\,15\,16\) However, a general relation can be obtained, which is given by the following equation:\(^19\)

\[
\frac{1}{G} = \begin{cases} 
1.355 r^{1.56} & (r \leq 0.824) \\
1 & (r > 0.824)
\end{cases}
\]

(3)

It should be noted that a critical size ratio had been purposely introduced in formulating the above equation. Thus, if \(r\) is greater than this critical ratio, \(G\) is equal to unity, so that there is no specific volume variation for such a mixture. As reported in the literature,\(^14\,15\,16\) this critical size ratio, similar to the \(G-r\) relation, is not constant and depends on the experimental condition. In the present approach, it is equal to 0.824. Verification of the above approach has been detailed elsewhere.\(^19\) In the following, it will be shown that Eqs. (2) and (3) can also be used in the porosity calculation of nonspherical particles.

In applying Eq. (3) to nonspherical particles, one will immediately be faced with the question "what is the size of a nonspherical particle in the calculation of size ratio \(r\)?" Obviously, this size is only an assigned one, just like many other equivalent spherical diameters devised in the literature.\(^20\) Figure 2 schematically illustrates this idea. As recently discussed by Yu et al.,\(^6\) in order to accurately describe the particle-particle interaction in terms of porosity, a new concept—equivalent packing diameter—should be used in the particle characterization.
The equivalent packing diameter can be readily obtained from the similarity analysis between spherical and nonspherical particle packings.\textsuperscript{5,6} For spherical particle mixtures, it is known that the specific volume variation is equal to zero if the particles involved are of the same diameter. Applying this to nonspherical particles, the equivalent packing diameter of a particle may be defined as the diameter of a sphere which, when mixed with the particle, gives no specific volume variation. Detailed discussion of this concept can be found elsewhere.\textsuperscript{6} In actual application, the equivalent packing diameter $d_e$ of a nonspherical particle may be expressed as a function of its equivalent volume diameter $d_v$ and sphericity $\psi$, as given by\textsuperscript{6}

$$d_e = \left(3.1781 - 3.6821 \frac{1}{\psi} + 1.5040 \frac{1}{\psi^2}\right) d_v \tag{4}$$

$d_v$ of a particle is the diameter of a sphere having the same volume as the particle, and $\psi$ is defined as the ratio of the surface area of a sphere having the same volume as the particle to the actual surface area of the particle. It is well established that they both can be determined from the particle size analysis.\textsuperscript{20} In particular, for a regularly shaped particle, they can be calculated by definition. For example, for a cylindrically shaped particle of length $L$ and diameter $D$, its equivalent volume diameter and sphericity are, respectively,

$$d_v = 1.145 (L/D)^{0.3} D \tag{5}$$

and

$$\psi = \frac{2.621}{1 + 2(L/D)} \tag{6}$$

Therefore, the size ratio between two components $r$ as required in Eq. (3) can be evaluated by means of effective packing size, i.e., equivalent packing diameter. In this case, the specific volume or porosity of a binary mixture of nonspherical particles can be calculated by Eqs. (2) and (3), as mentioned. This approach will be validated in what follows.

### III. Validity of the Proposed Method

It should be noted that $V_1$ and $V_1$, as prescribed information in the Westman equation, should, respectively, correspond to small and large components. Since the size of a component can definitely be evaluated in terms of equivalent packing diameter, there is no ambiguity in identifying the large or small between any two components and hence their $V_1$ and $V_1$ values. For convenience, the applicability of the Westman equation in fitting the porosity data of binary mixtures of nonspherical particles was examined first. This was here made using the measurements of Milewski.\textsuperscript{12} Figures 3 and 4 show some typical results. They respectively correspond to his sphere-fiber and fiber-fiber binary packing. The good agreement between calculated and measured results shown in Figs. 3 and 4 clearly indicates that the Westman equation can be used to describe the packing of binary mixtures.

In order to find out the $G-r$ relation of nonspherical particles, calculation has been performed for all the measurements of Milewski, and Fig. 5 shows the result. It can be seen from Fig. 5...
Porosity Calculation of Binary Mixtures of Nonspherical Particles

that the $G-r$ relation for his binary packing systems can be well represented by Eq. (3), the generalized $G-r$ relation of spherical particles. This result indicates that the porosity of binary mixtures of nonspherical particles can be approximated by the result of spherical particles, i.e., Eqs. (2) and (3). Such calculations have been performed for the systems in Figs. 3 and 4. The results are also included in the two figures for comparison. It can be observed from Figs. 3 and 4 that the predictions are in good agreement with the measurements, though this agreement is generally not as good as that obtained from direct fit.

The equivalent packing diameter may provide a useful link between spherical and nonspherical particle packings, and theoretically it can be used generally. In other words, the proposed approach is applicable not only to cylindrical particles but also to any shaped particle. To demonstrate this, Eqs. (2) and (3) were also used to calculate the porosity of binary mixtures of other shaped particles including cubes, disks, and irregular particles. In this treatment, the equivalent volume and sphericity of a cube or disk, as a regularly shaped particle, were simply calculated by definition. And for irregular particles it was assumed, in line with the common engineering practice, that the sphericity does not vary with particle size so that, according to Eq. (4), the size ratios can be regarded as the same as those reported. The results are shown in Figs. 6–8. It is evident that the predictions are all in reasonably good agreement with the measurements.

![Fig. 4. Porosities of binary mixtures of short fiber ($L/D = 3.91$) and (long) fibers of various length-to-diameter ratios, $D = 2.09$ mm for all fibers: points, measurements of Milewski; solid lines, fitted results; broken lines, predicted results.](image)

![Fig. 5. Dependence of coefficient $G$ on size ratio $r$ for nonspherical particles.](image)

![Fig. 6. Porosity of large cube (length = 4 mm) and other shaped particles: points, measurements of Iannella; solid lines, predictions.](image)

![Fig. 7. Porosity of disk (diameter = 19.374, length = 1.384 mm) and cylinders: points, measurements of Yu et al.; solid lines, predictions.](image)
Therefore, the proposed method can be used to simulate the packing of binary mixtures of any nonspherical particles. Since Eq. (4) was to a great degree obtained on the basis of sphere–fiber packing results, the method should definitely be applicable to sphere–fiber binary packing. Such an investigation of this important packing system has been carried out and detailed elsewhere. For other binary packing systems, as shown in Figs. 6–8, the simulated results can at least be used as a general guide in practical problem solving. Methodologically, the proposed approach is also applicable to multicomponent mixtures of nonspherical particles, though such a mathematical model is not yet available. In this aspect, as pointed out by Yu and Standish, further experimental and theoretical work is necessary.

IV. Conclusions

1. The packing of binary mixtures of particles, no matter if they are spherical or nonspherical, can satisfactorily be described by the Westman equation. The coefficient \( G \) in this equation is dependent only on particle size ratio \( r \), and the \( G-r \) relation can be empirically developed for the purpose of prediction.

2. The equivalent packing diameter is a useful concept in connecting nonspherical particle packing with spherical particle packing. The present results show that, by use of this concept, the porosity of binary mixtures of nonspherical particles can satisfactorily be predicted from the Westman equation and the generalized \( G-r \) relation of spherical particles.

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References


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