Simple Relations Derived from a Phased-Array
Antenna Made of an Infinite Current Sheet

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Abstract—The simplest concept of a phased array is an infinite planar current sheet backed by a reflecting boundary. The electric current sheet, or resistance sheet, is the limiting case of many small electric dipoles, closely spaced, and backed by an open-circuit boundary. If the array is viewed as a receiver, a plane wave incident on the array at some angle \( \theta \) meets a boundary resistance varying in proportion to \( \cos \theta \) for angles in the \( H \) plane, and \( 1/\cos \theta \) for angles in the \( E \) plane. If the array is matched at broadside \( (\theta = 0) \), the corresponding reflection coefficient has the magnitude \( \tan \theta \).

While the electric current sheet is realizable, the open-circuit boundary is not. However, a magnetic current sheet can be simulated by a conductive sheet with holes utilized as magnetic dipoles, such a sheet providing the backing equivalent to a short-circuit boundary. The latter case is related to the former by electromagnetic inversion or duality. Therefore, an incident plane wave meets a boundary conductance varying in proportion to \( \cos \theta \) for angles in the \( E \) plane, and \( 1/\cos \theta \) for angles in the \( H \) plane. The predicted behavior is verified qualitatively by tests of such a model with elements of a practical size.

The derivation is based on the principle of dividing the space in front of the array into parallel tubes or waveguides, one for each element cell in the sheet or array. This is one of the principles published by the author in 1948. A related principle enables the simulation of an infinite array by imaging a few elements in the walls of a waveguide. This latter principle is utilized for making tests of the array.

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I. INTRODUCTION

A PLANAR ARRAY of many radiating elements is used to develop a narrow pencil beam. The connections between the elements and the sending or receiving circuit are phased to control the beam angle of deflection from broadside (the "scan angle"). This is the type of antenna commonly called a "phased array," implying beam steering by electrical phasing without mechanical motion of the array.

This type of antenna introduces an unusual problem of design: the variation of element impedance with scan angle. This variation is caused by the inherent coupling of the elements, which contributes to the apparent impedance of each element in a manner dependent on the phasing required for the scan angle. With phasing for wide-angle scanning, the impedance variation from this cause becomes substantial and should be taken into account, along with the variation from other causes such as the frequency bandwidth.

There have been various studies of the element impedance variation with scan angle. Some are based on laborious computation of an array of many elements [5], [7], [12], [13]. Others are based on the concept of an infinite array [2], [6], [8], [14], [17], [18], [21]. This latter concept has been found by the author to offer the best basis for the design of an element for use.
at all locations in a finite array of many elements. Moreover, it offers the only evaluation that is independent of the number of elements. Therefore, there is a particular interest in the behavior of an element in an infinite array.

In general, the variation of element impedance with angle depends on the “fine structure” of the element, and on the arrangement of elements in the array. However, there is a hypothetical array which avoids any question of fine structure. This is the array that is formed of an infinite current sheet, representing elements so small and so closely spaced that the fine structure is not resolved. On the basis of such an array, there is found a simple derivation for impedance variation with angle, which is inherent in arrays fitting into a certain pattern. Furthermore, it is found that some practical types do fall into this pattern, so the conclusions from this hypothesis are useful in practice.

It is the purpose of this paper to describe the hypothetical array formed of an infinite current sheet, to derive its impedance variation with angle, and to introduce some corollary concepts, such as the “ideal element pattern,” which theoretically would avoid the implement variation.

The scope of this treatment is limited to the infinite planar array, having a well-defined planar face (ground plane), which separates the radiation region (free space, in front) from the associated circuits (shielded, behind). A practical example is an array of radiating holes in a sheet of conducting material (metal).

It is noted that the apparent impedance of the element, as formulated here, is the “active impedance” that would be seen looking out of one element with all the other elements excited in the phase required for the scan angle.

II. Symbols

MKS rationalized units.

- \( I \) = effective length of small electric dipole.
- \( a \) = element spacing in square array.
- \( A = a^2 \) = area of element cell in square array.
- \( \theta \) = scan angle from broadside.
- \( \lambda \) = free-space wavelength.
- \( \epsilon \) = electrivity (electric permittivity)
- \( \mu \) = magnetivity (magnetic permeability)
- \( H \) = magnetic field intensity (A/m).
- \( E \) = electric field intensity (V/m).
- \( I \) = current in electric dipole.
- \( H_e \) = magnetic field intensity adjacent to electric current sheet (A/m).
- \( I_e \) = electric current density in sheet (A/m).
- \( F(\theta) \) = relative field intensity in radiation pattern of isolated element.
- \( R \) = radiation resistance of dipole in active array phased for some angle.
- \( R_e = 377 \) ohms = wave resistance across square area of wavefront in free space.

\( G \) = radiation conductance of magnetic dipole (across a slot) in active array phased for some angle.

\( \rho \) = voltage-reflection coefficient at face of array.

\( \text{sub-}c = \text{free space} \)
\( \text{sub-o} = \text{broadside} (\theta = 0) \)
\( \text{sub-s} = \text{sheet} \)
\( \text{sub-m} = \text{plane of magnetic field} (H) \)
\( \text{sub-e} = \text{plane of electric field} (E) \)
\( \text{sub-i} = \text{ideal} \)
\( \text{sub-h} = \text{Huygens source} \)

III. Theoretical Background

The concept of the infinite array was introduced by the author in 1948 [2], long before the recent intensive work on phased arrays with beam steering. That publication showed the opportunity for simple computation of some properties of a radiating element in an infinite array. This approach had been suggested by the theory of such an element in a waveguide. Now it turns out that the same principle is applicable to phased arrays. Moreover, it is found that the experimental design of an element for a phased array is facilitated by imaging a few elements in waveguide walls to simulate an infinite array [17], [20], [23], [24].

As an introduction to the present discussion, Figs. 1 and 2 are reproduced from the 1948 publication. Figure 1 shows an infinite planar array of electric dipoles in a square arrangement in a plane with free space in front and “open circuit behind.” This is the same basis to be used for the infinite current sheet. Fig. 2 shows a modification of the same to form an array phased for an oblique angle of radiation. This is a special case of a phased array, in which adjacent elements are phased in opposition; a condition that is most simply simulated by the metal walls of a rectangular waveguide. (The same publication also describes the simulation of an infinite array in a rectangular waveguide, with oblique angles of radiation.)

Among the formulas in that publication, there are two that form the background for the present discussion. Referring to Fig. 1, the radiation resistance of each element is given as

\[ R_s = R_e \frac{\theta^2}{a^2} = R_e \frac{\theta^2}{A}. \]  \hspace{1cm} (1)

Referring to Fig. 2, for oblique radiation at an angle \( \theta \) from broadside, in the \( H \) plane, the radiation resistance happens to be given for a pair of directions (±\( \theta \)). Taking one-half of this for one direction,

\[ R = R_e \frac{\theta^2}{\cos \theta \ a^2}. \]  \hspace{1cm} (2)

This forms an introduction to the angle variation of impedance that is to be derived. In this case, the radiation resistance increases with angle from broadside.

In the 1948 publication, some reliance is placed on
earlier derivations of the radiation resistance of an electric dipole in a rectangular waveguide. The completed derivation can be described briefly with reference to the tubes of radiation in front of the array, as shown in Figs. 1 and 2. In the broadside direction ($\theta = 0$), shown in Fig. 1, the tubes are square and are bounded by hypothetical image planes. The wave resistance between top and bottom of each square cell ($R_s$) is that of free space ($R_c$). The electric dipole (of effective length $l$) is coupled to this resistance by an effective transformation ratio ($Z/u$), so its radiation resistance is proportional to the square of this ratio, as appears in formula (1). In an oblique direction ($\theta$), as shown in Fig. 2, each tube is decreased in width by the projection cosine, so the resistance of the tube and of the dipole is increased by the factor $1/\cos \theta$.

It is noted that this simple computation for an infinite array is based on real power, and, therefore, only the resistance component is given. In general, there is also a variation of the reactance component, so that the impedance variation is the resultant of both components.

Further theoretical background is found in more recent publications, which will be reviewed in a later section of this paper.

IV. THE INFINITE CURRENT SHEET

The present discussion is to be based on an infinite current sheet. There are two kinds of current sheet that may be assumed, electric and magnetic. The former is conceptually the easier, because an electric conductor is within our experience. Therefore it will be utilized for the derivations. On the other hand, the magnetic current sheet (and not the electric) is found to be realizable in the required environment of the array, so it will be relied on for experimental verification.

Figure 3 shows the basis for a planar phased array made of an electric current sheet. The baseline in Fig. 3(a) is the scan-plane projection of the plane of the array. Above the line is represented the radiation region (free space) in front of the array. Below the line is assumed to be an "open-circuit" reflector behind the array ($\epsilon = 0, \mu = \infty$). The latter region is not physically realizable, but is conceptually simple; it is carried over from Figs. 1 and 2.

The electric current sheet is in the plane of the array. It has a current density $I_z$ of uniform amplitude, but phased for radiating a plane wave in a direction at an angle $\theta$ from broadside. The current density is associated with a magnetic intensity $H_z$ of equal amplitude and

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![Fig. 1. An infinite planar array at the boundary of half-infinite space. (a) Plan. (b) Elevation.](image)

![Fig. 2. The same array phased for an oblique angle of radiation.](image)

![Fig. 3. An idealized model of a planar array, made of an electric current sheet.](image)
phase, but crossed in space. In the radiation region, the \( H \) plane and \( E \) plane are determined to include, respectively, the \( H_1 \) and \( I_1 \) vectors. The scan angle \( \theta \) may be in either of these planes, as shown.

Figure 3(b) shows the face of the array, the infinite current sheet being exemplified by a square cell of area \( A = a^2 \). For some purposes, there will be used also the illustrated concept of a small electric current element, or electric dipole, equivalent to a small cell. These are said to be equivalent if they have equal moment. If the moment of the latter is \( II \), as indicated, the condition for equivalence is

\[
II = a(aI_1) = AI_1. \tag{3}
\]

For the broadside direction of radiation (\( \theta = 0 \)), obtained by uniform phase over the array, the radiation resistance \( R_0 \) of this element is given by formula (1).

The current sheet may have any value of uniform surface resistance \( R_s \) (the resistance across any square of the surface). For present purposes, it is assumed to be “matched” to free space for broadside radiation, so \( R_s = R_o \) (377 ohms, the wave resistance across a square area of wavefront in free space).

\section*{V. The Variation of Impedance with Scan Angle}

Since the receiving viewpoint may be conceptually simpler, it will be taken first for deriving the variation of impedance with scan angle. This derivation will be based on Fig. 3(a), showing the two principal planes of scan angle, the \( H \) plane and the \( E \) plane. Any reflection from the face of the array, as will be derived, appears in the form of a beam radiated at the opposite angle from broadside.

Referring to an oblique angle of scanning in the \( H \) plane, we have a situation similar to Fig. 2. The incoming wave sees a section of current sheet which is wider than the section of wavefront, so the wave appears to be reflected by a resistance that is lesser in the ratio, \( \cos \theta \). The resulting voltage-reflection coefficient is negative:

\[
\rho_m = \frac{\cos \theta - 1}{\cos \theta + 1} = - (\tan \frac{1}{2} \theta)^2. \tag{4}
\]

Referring instead to an oblique angle of scanning in the \( E \) plane, we find an opposite situation. The incoming wave sees a section of current sheet which is longer than the section of wavefront, so the wave appears to be reflected by a resistance that is greater in the ratio, \( 1/\cos \theta \). The resulting voltage-reflection coefficient is positive:

\[
\rho_e = \frac{1/\cos \theta - 1}{1/\cos \theta + 1} = 1 - \cos \theta \quad 1 + \cos \theta = (\tan \frac{1}{2} \theta)^2. \tag{5}
\]

Changing over to the sending viewpoint, we may derive the same relations, but naturally the reflection coefficient will be reversed. Here we refer also to the equivalent current elements in Fig. 3(b).

The method is given in the author’s early paper on this subject [2], here represented by Figs. 1 and 2. The space in front of the array is partitioned by hypothetical waveguide walls and the radiation resistance of each element is formulated. Referring to Fig. 3(a), these walls are located by projecting the element cell in the beam direction determined by the array phasing. The nature of the walls is determined by the requirement for imaging the fields of a uniform plane wave: the walls parallel to the \( H \) field are “electric” walls, or “short-circuit” boundaries \( (\varepsilon = \infty, \mu = 0) \); and the walls parallel to the \( E \) field are “magnetic” walls, or “open-circuit” boundaries \( (\varepsilon = 0, \mu = \infty) \).

From (1), the radiation resistance of each element is known for broadside phasing \( (R_0 \) for \( \theta = 0 \)). Changing to the phasing for an oblique angle modifies the situation in two respects. First, the area of wavefront is decreased by the factor \( \cos \theta \), increasing the resistance by the factor \( 1/\cos \theta \), as in (2). Second, the radiation pattern of the element may affect the relative amount of radiation in the beam direction. The latter effect is to be formulated here.

The resulting variation of element resistance with angle is given by the following formulas, which are essentially similar to some derived by Stark [6].

\[
H \text{ plane: } R/R_0 = \frac{1}{\cos \theta} |F_m(\theta)|^2 = 1/\cos \theta \tag{8}
\]

\[
E \text{ plane: } R/R_0 = \frac{1}{\cos \theta} |F_e(\theta)|^2 = \cos \theta. \tag{9}
\]

As would be expected from the principle of reciprocity, these ratios are the inverse of those derived from the receiving viewpoint. Each of the corresponding reflection coefficients is merely reversed.

The factor \( 1/\cos \theta \), which appears in (8) and (9), may be inferred from another viewpoint in transmission. Let us consider a finite planar array of many elements, so large that the beam width is a small fraction of one radian. In any particular plane of scan, the aperture width projected in the beam direction is proportional to \( \cos \theta \), so the beam width is inversely proportional to this ratio. The power in the beam is proportional to the
factor \( P^2 \) in the beam direction, and also to the beam width. The latter introduces the factor \( 1/\cos \theta \) in the radiation resistance.

From the sending viewpoint, the reflected power is returned to the generator, or may be absorbed in an intervening “isolator.” In the case of a generator matched to the element broadside resistance \( R_0 \), the net result of the reflection is to decrease the radiated power below the “available” power by the factor \( (1 - |\rho|^2) \).

Figure 4 shows the variation of element resistance and reflection coefficient with scan angle in the \( H \) and \( E \) planes. The polarity of \( \rho \), in Fig. 4(a), is given for the electric dipole, as in Fig. 3(b).

![Fig. 4](image)

In general terms, the same derivation is applicable to the inverse case of a magnetic current sheet, with transformation by the principles of duality. This means interchanging \( H \) and \( E \), interchanging open-circuit and short-circuit reflector, interchanging resistance \( R \) and conductance \( G \) (or inverting \( R \)), and interchanging voltage and current-reflection coefficients (or reversing the sign of the voltage-reflection coefficient). The radiation pattern of a small magnetic dipole (or slot in a metal sheet) is as follows,

\[
E \text{ plane: } E/E_0 = F_e(\theta) = 1 \\
H \text{ plane: } E/E_0 = F_m(\theta) = \cos \theta.
\]

The resulting variation of element conductance is as follows,

\[
E \text{ plane: } G/G_0 = \frac{1}{\cos \theta} [F_e(\theta)]^2 = 1/\cos \theta \\
H \text{ plane: } G/G_0 = \frac{1}{\cos \theta} [F_m(\theta)]^2 = \cos \theta.
\]

VI. THE IDEAL ELEMENT PATTERN

The formulas (8) and (9) give a basis for describing an “ideal” element pattern that would yield an element resistance invariant with scan angle, likewise in any scan plane (or in any linear polarization relative to the scan plane). This pattern is \( F'_o(\theta) \), defined as follows.

\[
R/R_0 = \frac{1}{\cos \theta} [F_o(\theta)]^2 = 1; \quad F'_o(\theta) = \sqrt{\cos \theta}.
\]

Figure 5 shows the three patterns mentioned here. In either plane, the ideal pattern \( (E'H') \) is a change from the dipole pattern \( (H \text{ to } H' \text{ or } E \text{ to } E') \).

![Fig. 5](image)

The element pattern, as used here, is that of an isolated element, and the ideal pattern is derived on the same basis. This same concept might have been inferred from previous articles, though not explicitly stated [5], [6]. An ideal element pattern has been implied or described also on a different basis, the element being located in the environment of an infinite array of passive elements [18]. Both viewpoints happen to yield the same shape for the ideal element pattern, but with different significance.

It is noted in passing that there is a kind of simple radiator that offers a theoretical attraction for use as
an element in an array. It is the “electromagnetic dipole” or “Huygens source” [1], [3], [4], [9], [11]. It is formed by crossed superposition of an electric dipole (or current element) and a magnetic dipole (or current loop) radiating equal (maximum) fields in one direction (the axis). This gives the same amplitude of field in all planes through the axis, namely,

\[ |F_\theta(\theta)| = \frac{1}{2}(1 + \cos \theta) = (\cos \frac{\theta}{2})^2. \]  

(15)

This has a cardioid shape which is a close approximation to the ideal shape (14). Aside from its favorable pattern, it has some peculiarities that require further study before it can be used as an element in an array. 1) It is difficult to realize in a physical structure. 2) It is a two-port for each linear polarization, as distinguished from the usual one-port. 3) Its pattern would be destroyed by any reflector behind the element. In the meantime, this type of radiator offers some intriguing opportunities for theoretical studies.

VII. A Practical Structure

As previously mentioned, the relations derived here are applicable to a practical structure simulating a magnetic current sheet backed by a short-circuit reflector. Figure 6 shows such a structure. The array is formed of a metal sheet with the elements radiating through holes in the sheet. It has some properties in common with the arrays of slots, as exemplified in the studies by Rabinowitz, Edelberg, and Oliner [5], [8].

There is a periodic array of holes in either of the two forms that are useful. Figure 6(a) shows a square array, which is the simplest form. Figure 6(b) shows a triangular array, which is still simple and offers some practical advantages. Also it gives the closer approximation to a current sheet, for the same separation of adjacent elements. Figure 6(c) shows a cross section of either form through a “cardinal” plane (C-C).

The resistance sheet in this presentation is isotropic. Following this concept here, each element hole is made circular, and is backed by a circular waveguide. Simulation of the current sheet requires that the hole diameter and center-to-center spacing be sufficiently small in terms of the wavelength, so that this “fine structure” cannot be “resolved” by the wave. This condition is nearly satisfied if these dimensions are much less than one-half wavelength, whereas dimensions of this order are required in practice. In the circular waveguide, the TE-11 mode is to be utilized. Its propagation requires dielectric loading in this diameter. Such loading may be provided by spaced circular disks of high dielectric constant, to avoid proportionate loading of the undesired TM-01 mode [19].

This structure will handle any polarization. In practice, it requires impedance matching over the required frequency band, which can be approximated by inserting obstacles in the circular waveguide. There remains the impedance variation with angle, which is the subject of this monograph.

VIII. Experimental Verification

The author’s laboratory has made tests of a practical array utilizing such a structure. An infinite array has been simulated by imaging a few elements in the plane walls of a waveguide [17], [20], [23], [24]. Then the reflection coefficient has been measured from the point of reception.

The experimental structure is a triangular array, as shown in Fig. 6(b). The hole diameter is 0.45 wavelength, and the center-to-center spacing is 0.60 wavelength. The dielectric loading is sufficient to lower the TE-11 cutoff frequency down to about 1/\sqrt{2} or 0.7 of the operating frequency. Figure 7 shows on the reflection chart the voltage-reflection coefficients for several scan angles. Each point is observed in an individual waveguide to simulate the particular angle in an infinite array.

In this experiment, there is no attempt to provide impedance matching at the array face for broadside phasing, which is assumed in the theoretical derivation. Each waveguide is provided with a matched termination to assure a pure traveling wave in reception.

On the reflection chart, Fig. 7, the two extreme points represent a large scan angle of 56° in the cardinal plane, C-C in Fig. 6(b), with E or H vector in this plane as indicated. The intermediate point represents a small scan angle of 29°, so it is not much different from 0°.
Fig. 7. The coefficient of reflection from the face of an array, at various angles, from the receiving viewpoint.

(broadside); this angle happens to be in the inter-cardinal plane (30° from C-C) with \( H \) in this plane. These angles are well within the limits out to which the array would develop a single beam free of grating lobes.

On this chart, the scale of normalized resistance is marked in the region of interest.

The relation of the three points on the reflection chart is, roughly, what would be expected from the simple theory based on a magnetic current sheet, off-center, though as not matched for broadside. 1) The points are near the real axis of impedance and reflection coefficient. 2) The resistance is higher for \( E \) in the scan plane. 3) The extreme points are separated slightly more than the predicted closest resistance ratio, the lowest value being roughly 0.22 times the highest value, as compared with \((\cos 56°)² = 0.31\). 4) The intermediate point is not far from the ratio mean of the extreme value of resistance.

Regarding the experimental points more critically, it is reasonable to say that all the discrepancies may be caused by the size of the element, specifically the hole diameter and the spacing. The resistance variation is close to the predicted amount and there is little variation of reactance. At least, there is qualitative agreement and a fair approximation to quantitative agreement.\(^1\) It is remarkable that it has been possible to measure the behavior of a practical element in an environment simulating an infinite array.

\(^1\) More recent theoretical and experimental studies have evaluated the reflection coefficient from the transmitting viewpoint. From this viewpoint, the phase of reflection varies with scan angle and polarization, as affected mainly by element spacing.

IX. Historical Evolution of the Concepts

Having developed a simple picture of the array reflection, as it varies with scan angle, we shall review the evolution of ideas that has led to this result. We shall pass over the historic studies of the coupling between elementary antennas, which enabled the detailed computation of finite arrays, since this method becomes unduly laborious for a large number of elements. The present objective is to evaluate the typical or average behavior of an element in a very large planar array, the number of elements being in the range from 10 by 10 = 100, up to 100 by 100 = 10,000.

The viewpoint of the infinite array was introduced by the author's publication of 1948 [2]. In a unified presentation, it showed the identity of behavior of a radiating element in a waveguide with plane walls and in an infinite array. It then applied the waveguide viewpoint for computation of the element radiation resistance in closed form, avoiding the summation of a doubly infinite series. Inherent in this presentation were some more specific principles whose significance was not appreciated until later.

The infinite array which received most attention in that monograph was the planar array phased for a broadside pencil beam, Fig. 1 here. For theoretical purposes, this was identified with a nonphysical rectangular waveguide, so the element behavior in the array was found to be different from what had previously been computed for practical waveguides. On the other hand, the infinite array which was shown phased for oblique radiation, Fig. 2, was intended only for its theoretical interest in relation to the well-known wave behavior in physical rectangular waveguides. At that time, the author did not envision the future opportunities for an array with electrical phasing for beam steering without mechanical motion. However, the behavior of such a phased array was inherent in the theoretical derivation for waveguides, as seen here (2).

About ten years later, the growing interest in phased arrays began to be reflected in the published literature. There were three principal sources of publications related to the present subject, namely, S. Edelberg and A. A. Oliner [8], P. S. Carter, Jr. [7], and J. L. Allen [13]. Of special significance, also, were the reports of S. J. Rabinowitz [5], J. L. Allen [12], and L. Stark [6]. All of these sources were concerned with the variation of element impedance with scan angle.

Among the recent publications, the one most closely related to the present subject is that of Edelberg and Oliner [8]. They extend the waveguide analogy to a rectangular array of holes in a metal sheet, and to any scan angle. While their presentation is complicated by the finite dimensions of the element cell and the hole, one can glean from it the simple relations that are
The concept of an "ideal" element pattern has not yet been published in clear and explicit terms. In the formulas of Rabinowitz [5], and in those of Stark [6], the radiation resistance of an element is seen to depend on the radiation pattern of an isolated element (including its reflector). From these formulas, one might infer that an element with a particular pattern (14) would cause the radiation resistance to be invariant with scan angle. An attempt is made here to present this topic clearly. (This concept of isolated element pattern should not be confused with the different concept of array element pattern, the latter being effective in the array environment [18]. It happens that the ideal pattern has the same shape in both cases.)

While beyond the scope of this report, it is natural to speculate on the significance of the impedance variation with a current sheet, and how it might be decreased in practice. The current sheet is a thin array, having no depth in the transition face between the circuits and the radiation region. One approach for decreasing the impedance variation involves structural depth with dimensional controls, such as projections in front, or the spacing between dipoles and reflector, or the addition of interconnecting circuits behind the array [15], [22]. We may infer a principle that the simple relations are associated with a thin array of small elements, and that element size or structural depth offers some opportunity for departure from these relations.

The problems of evaluating an array are those of computation, on one hand, and of measurement, on the other. Since the computations were greatly simplified by the analogy between an infinite array and a waveguide, it should not be surprising that a similar approach has been found invaluable in measurements. This application is naturally restricted to the cases where the hypothetical waveguide is physically reliable. Since 1960, the author and his associates have been exploring the discrete cases where a physical waveguide is available for simulating the behavior of an infinite array [17]. [20], [23]. [24]. Rectangular waveguides have been utilized for square and triangular arrays, and there are some cases of the latter type where a triangular waveguide is particularly useful.

The concept of an element in an infinite array has been acquiring stature also as the sound basis for designing a practical element for use in a large array of many elements. The element impedance in an infinite array appears to be centered in the range of element impedance at all locations in the finite array. This practical utility excites further interest in the concept.

It is common experience in science, that the ultimate simplification in concepts comes as a culmination of more complicated studies. The present topic appears to be an example of this principle, and an attempt has been made to reduce it to the simplest terms.

X. CONCLUSION

The current sheet is a highly simplified conceptual model of a phased array. It is used as the basis for deriving some simple relations. One is the variation of element impedance and reflection coefficient with scan angle. Another is the ideal isolated-element pattern that would make the element radiation resistance invariant with angle.

The derivation relies on the principle that an element in an infinite array behaves like one in a waveguide with properly defined walls, as presented in the author's monograph of 1948.

In this model, the reflection coefficient varies with the scan angle, oppositely for the $H$ or $E$ vector in the scan plane. The predicted variation has been verified qualitatively by tests of an array approximately simulating a magnetic current sheet.

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Geodesic Lens Antennas for Low-Angle Radiation

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Abstract—Previous unit-index geodesic lens antenna designs have not been able to produce a lens with good radiation characteristics in or near the plane of the lens rim. This paper extends previous work to permit the design of a geodesic lens for angles within approximately 20° of the plane of the lens rim. This is accomplished by requiring that less than the full semicircular aperture be exactly focused, and also by dividing the outer annulus into two or more constant-slope sections. The capability of these lenses for beam elevation positioning is also discussed.

Introduction

A general analysis of geodesic Luneberg lenses has been considered previously by Rudduck and Walter.1 In this class of lenses the index of refraction is arbitrary and the necessary contour is obtained for radiation at an arbitrary elevation angle β. Unfortunately, the unit-index designs described by Rudduck and Walter2 are unable to produce a lens with good radiation characteristics for small values of β, that is, for β less than approximately 20°. This region is of particular interest if a lens were to be flush mounted—on the fuselage of an aircraft for instance. The unit-index characteristic is important because it eliminates the need for dielectric materials in a lens, thus allowing high power handling capabilities to be achieved.

Rinehart2 has derived the geodesic solution for the rim-fed Luneberg lens antenna which radiates in the plane of the lens rim. Rinehart’s lens, however, has a vertical slope at the rim. This vertical slope gives rise to an aperture element pattern generally in the β = 90° direction, whereas the phase distribution is for β = 0° radiation, which is not compatible with the element pattern. To overcome this, Kunz,3 Warren and Pinnell,4 Scheggi and di Francia,5 and others have attacked the

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