Effective viscosity prediction of crude oil-water mixtures with high water fraction

Jiangbo Wen\textsuperscript{a,b}, Jinjun Zhang\textsuperscript{a,⁎}, Min Wei\textsuperscript{b}

\textsuperscript{a} National Engineering Laboratory for Pipeline Safety/ME Key Laboratory of Petroleum Engineering/Beijing Key Laboratory of Urban Oil & Gas Distribution Technology, China University of Petroleum, Beijing 102249, China
\textsuperscript{b} College of Petroleum Engineering, Liaoning Shihua University, Fushun 113001, China

A R T I C L E   I N F O

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A B S T R A C T

In the petroleum industry, a flowing crude oil-water system with high water fraction usually does not exist as a stable and homogeneous emulsion, but in the form of a mixture of oil and free water with some water being emulsified into the oil and thus, hydrodynamically, acting as part of the oil. Therefore, the existing viscosity models developed for stable and homogeneous emulsions cannot accurately predict viscosity of this kind of liquid-liquid mixtures. In the present work, the effective viscosity of crude oil-water mixtures with high water fraction was measured by using a stirring method. It was found that the effective viscosity of the mixtures decreased with increasing shear rate, water fraction and temperature. Based on the Taylor viscosity model and combined with our previous study on the emulsification behaviors of crude oil-water systems, a viscosity model was developed for crude oil-water mixtures with high water fraction, which is characterized in covering the effects of shear rate, emulsified water fraction and crude oil compositions. Validation was done by using 40 data points which were obtained from mixtures of two crude oils and not used for model development, and the results showed that the proposed model gave accurate predictions of the mixture viscosity, with an average relative deviation of 7.4%. Approach is proposed to application of this model in the prediction of the frictional loss of oil and water two phase flow in pipes.

1. Introduction

The crude oil-water two-phase or crude oil-gas-water multiphase flow is quite common in the petroleum industry. Due to the natural surfactants such as resins, asphaltenes and wax particles containing in the crude oil, crude oil and water emulsify easily in the process of production. The emulsification of crude oil and water has significant influences on multiphase flow (Grassi et al., 2008; Nädler and Mewes, 1997; Wang et al., 2011; Yusuf et al., 2012). Actually, a flowing crude oil-water system usually does not exist as a stable and homogeneous emulsion, but in the form of an unstable oil-water mixture composed of W/O emulsion and free water, significantly differing from a stable emulsion in dispersity and static stability.

A large number of viscosity models for stable emulsions have been developed in the previous studies. In 1906, Einstein (1906) derived an equation for the viscosity of dilute dispersed systems.

\[ \eta_i = (1 + 2.5k_d \phi_k) \]  

(1)

where \( \eta_i \) is the relative viscosity, defined as the ratio between emulsion viscosity (\( \eta_d \)) and continuous phase viscosity (\( \eta_c \)), i.e., \( \eta_i = \eta_d/\eta_c \); \( \phi_k \) is the volume fraction of the dispersed phase.

Later researchers (Schramm, 1992) added more terms into the Einstein’s equation to extend its applicability to emulsions, with the general form as follows.

\[ \eta_i = 1 + k_1 \phi_k + k_2 \phi_k^2 + k_3 \phi_k^3 + \ldots \]  

(2)

where \( k_1, k_2, k_3 \) are parameters associated with particular emulsion systems.

Brinkman (1952) proposed a model for the emulsion with spherical droplets.

\[ \eta_i = (1 - \phi_k)^{-2.5} \]  

(3)

Richardson (1933) firstly developed an exponential relationship between the relative viscosity and the volume fraction of the dispersed phase.

\[ \eta_i = e^{k_0 \phi_k} \]  

(4)

where \( k_0 \) is an undetermined parameter, if \( \phi_k < 0.74, k_0 = 7 \), if \( \phi_k > 0.74, k_0 = 8 \). The Richardson equation has been found suitable for many emulsion systems. And it has been modified by many researchers to...
adapt more emulsion systems (Broughton and Squires, 1938; Marsden, 1977; Mooney, 1951).

The above-mentioned viscosity models correlate the relative viscosity with the volume fraction of the dispersed phase only, with the effect of other factors implied in the undetermined parameters. While Taylor (1932) assumed that the dispersed phase was extremely small spherical droplets, and derived an equation for dilute emulsions which contains the dispersed phase viscosity \( \eta_d \)

\[
\eta_d = 1 + \left( \frac{\eta_o + 2.5 \eta_i}{\eta_o + \eta_i} \right) \phi_d
\]  
(5)

Based on the Taylor model and the continuum theory, Phan-Thien and Pham (1997) developed an equation for emulsion viscosity.

\[
\eta_e = \left[ \frac{2 \eta_o + 5 \eta_i}{2 + 5 K} \right]^{1/5} = (1 - \phi_d)^{-1}
\]  
(6)

where \( K \) is the ratio of dispersed phase viscosity and the of continuous phase viscosity, i.e., \( K = \eta_o/\eta_i \).

After Phan-Thien and Pham, Pal (2000) proposed an improved model by considering the effect of surfactants on the viscosity.

\[
\eta_e = \left[ \frac{2 \eta_o + 5 \eta_i}{2 + 5 K} \right]^{1/5} = 1 - K \phi_d
\]  
(7)

where \( K_\text{fi} \) is a coefficient associated with surfactants, which needs to be determined by fitting experimental data.

Emulsions often exhibit non-Newtonian flow behaviors, such as the shear-rate-dependence of viscosity. Therefore, some works had been done to cover this non-Newtonian property.

Based on the Brinkman’s equation (Brinkman, 1952), Pal and Rhodes (1989) developed a model for predicting the shear-rate-dependence of emulsion viscosity.

\[
\eta_e = [1 - K_\text{fl} K_\text{fl}(\gamma)] \phi_w^{1.5}
\]  
(8)

where \( K_\text{fl}, K_\text{fl}(\gamma) \) are defined as the non-Newtonian coefficients; \( K_\text{fl} \) represents the influence of hydration, which associates with the emulsion; \( K_\text{fl}(\gamma) \) represents the influence of droplets coalescence, which associates with shear rate.

Based on the Richardson’s equation (Richardson, 1933), Ronningsen (1995) proposed an empirical equation which correlates the emulsion viscosity with shear rate and temperature.

\[
\ln \eta_e = k_1 + k_2 T + k_3 \phi_w + k_4 T \phi_w
\]  
(9)

where \( T \) is temperature; \( k_1, k_2, k_3, k_4 \) are the parameters associated with shear rate, which can use the following empirical values: \( k_1 = 0.0412, k_2 = 0.002605, k_3 = 0.03841, k_4 = 0.0002497 \).

After Pal and Rhodes (1989), Dou and Gong (2006) developed an equation for water/heavy crude oil (W/O) emulsion, which includes the effect of the water fraction on the non-Newtonian coefficient.

\[
\eta_e = [1 - K_\text{fl}(\gamma) K_\text{fl}(\phi_w)] \phi_w^{1.5}
\]  
(10)

\[
K_\text{fl}(\phi_w) = \frac{1 - \eta_o^{0.4} \phi_w^{0.4}}{\eta_o^{10.4} \phi_w^{10.4}}
\]  
(11)

where \( K_\text{fl}(\phi_w) \) is the coefficient of water fraction, representing the effect of water fraction on the non-Newtonian rheological behavior; \( \phi_w^{\text{max}} \) is maximum water fraction of the emulsion.

It has been known that the droplet size and size distribution of the dispersed phase has significant effect on emulsion viscosity. Wang (2009) incorporated the effect of droplet size into the Dou’s equation and derived the following viscosity model.

\[
\eta_e^{-\frac{3}{5}} = \left( \frac{\eta_o + 2.5 \eta_i}{1 + 2.5 K} \right)^{3/5} = [1 - K_\text{fl}(\phi) K_\text{fl}(N_{\text{Re,p}}) \phi_i]^{-1}
\]  
(12)
2. Experimental section

2.1. Materials

Ten crude oils produced in different oilfields in China were used in experiments, which were named as oils #1–#10. The physical properties of these oils are listed in Table 1, and the measurements are carried out on stabilized stock tank oils. Besides the crude oil compositions, the brine salinity also affects the emulsification tendency. However, this work focuses on the effect of oil compositions on the viscosity of crude oil-water mixture, so the water used in experiments was ultrapure water. The experimental data of oils #1–#8 are used to develop the viscosity model, and the experimental data of oils #9 and #10 are used to validate the model.

The pour point of crude oil was tested according to ASTM D5853-11. The wax appearance temperature (WAT) and the concentration of the precipitated wax of crude oil were determined according to ASTM D4419-90. The viscosity of crude oil was determined according to ASTM D445-09. The contents of asphaltenes, resins, and wax of crude oil were determined according to ASTM D4124-09. The total acid number of crude oil was determined according to ASTM D664-11a. The content of the mechanical impurities of crude oil was determined according to ASTM D473-07. The carbon number distributions of crude oil were determined using the gas chromatograph Agilent 6890 according to SY/T 5779-2008, and the average carbon number was calculated by using the data of carbon number distributions and the mass fraction of alkanes, which can be learned in the literature (Yi and Zhang, 2011) for details.

Take oils #9 and #10 as examples. The viscosity and the concentration of precipitated wax vs. temperature curves are shown in Figs. 1 and 2.

Table 1

<table>
<thead>
<tr>
<th>oils</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
<th>#10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density at 20 °C (kg/m³)</td>
<td>879.5</td>
<td>863.8</td>
<td>839.4</td>
<td>872.0</td>
<td>864.9</td>
<td>856.6</td>
<td>835.9</td>
<td>818.7</td>
<td>864.3</td>
<td>868.9</td>
</tr>
<tr>
<td>Pour point (°C)</td>
<td>−7</td>
<td>36</td>
<td>−1</td>
<td>−9</td>
<td>1</td>
<td>29</td>
<td>18</td>
<td>17</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Wax appearance temperature (°C)</td>
<td>20</td>
<td>45</td>
<td>20</td>
<td>34</td>
<td>23</td>
<td>45</td>
<td>33</td>
<td>22</td>
<td>33</td>
<td>25</td>
</tr>
<tr>
<td>Viscosity at 40°C (mPa·s)</td>
<td>71.25</td>
<td>24.97</td>
<td>3.40</td>
<td>11.91</td>
<td>17.26</td>
<td>16.30</td>
<td>7.65</td>
<td>3.06</td>
<td>17.00</td>
<td>28.72</td>
</tr>
<tr>
<td>Asphaltenes (wt%)</td>
<td>3.72</td>
<td>2.92</td>
<td>0.81</td>
<td>3.96</td>
<td>2.18</td>
<td>1.30</td>
<td>6.79</td>
<td>5.11</td>
<td>2.30</td>
<td>3.10</td>
</tr>
<tr>
<td>Resins (wt%)</td>
<td>7.98</td>
<td>7.94</td>
<td>0.69</td>
<td>1.61</td>
<td>5.75</td>
<td>8.04</td>
<td>1.46</td>
<td>0.11</td>
<td>9.45</td>
<td>10.20</td>
</tr>
<tr>
<td>Wax (wt%)</td>
<td>3.50</td>
<td>22.01</td>
<td>6.52</td>
<td>3.78</td>
<td>7.53</td>
<td>12.60</td>
<td>10.63</td>
<td>9.39</td>
<td>14.13</td>
<td>14.29</td>
</tr>
<tr>
<td>Total acid number (mg KOH/g)</td>
<td>3.32</td>
<td>0.24</td>
<td>0.27</td>
<td>0.15</td>
<td>0.38</td>
<td>0.07</td>
<td>0.64</td>
<td>0.17</td>
<td>0.30</td>
<td>0.26</td>
</tr>
<tr>
<td>Mechanical impurities (wt%)</td>
<td>0.25</td>
<td>0.24</td>
<td>0.30</td>
<td>0.41</td>
<td>0.34</td>
<td>0.05</td>
<td>0.19</td>
<td>0.03</td>
<td>0.28</td>
<td>0.23</td>
</tr>
<tr>
<td>The average carbon number of crude oil</td>
<td>18.8</td>
<td>20.3</td>
<td>17.8</td>
<td>16.8</td>
<td>18.2</td>
<td>19.0</td>
<td>18.3</td>
<td>17.9</td>
<td>18.5</td>
<td>17.6</td>
</tr>
</tbody>
</table>

Fig. 1. Viscosity vs. temperature of oils #9 and #10.

Fig. 2. Concentration of precipitated wax vs. temperature of oils #9 and #10.
2.2. Viscosity measurement apparatus for crude oil-water mixtures

A stirring apparatus is used to measure the effective viscosity of crude oil-water mixtures. As shown in Fig. 3, the experimental apparatus is mainly composed of three parts: (1) A flat impeller, 75 mm in height and 75 mm in diameter, and with six diversion holes on it; A cylindrical vessel, 120 mm in diameter and 180 mm in height, and connected with a circulating water bath for temperature control. A total of 1 L of oil-water mixture was used once in the experiment. (2) A circulating water bath (HAAKE model AC200), with the effective temperature ranging from 5 to 90 °C, and temperature control precision of 0.1 °C. (3) A stirring power system composed of a stirrer, a tachometer, and a torque meter (IKA), with the effective stirring speed ranging from 10 to 1600 rpm, and the accuracy of the torque measurement of 0.01 N cm.

2.3. Stirring method for determining the viscosity of crude oil-water mixture

The crude oil-water mixture with high water fraction contains both emulsified water, as a W/O emulsion, and free water. Because of the inhomogeneity of the system and the tendency of phase separation under static conditions, the traditional viscometers, such as coaxial cylinder viscometer and cone-plate viscometer, cannot used to measure the effective viscosity of the mixture. However, the mixture can keep homogeneous under the condition of sufficient stirring; and the shaft torque of the stirrer should be related to the effective viscosity of the mixture. Based on this consideration, Yu et al. (2013) developed a method for determining the effective viscosity of unstable oil-water mixtures. The basic principle and main procedures are as follows:

In the flow field of a stirred vessel, the relationship between fluid viscosity $\eta$ and shaft torque $M$ of the stirrer at a certain stirring speed is

![Fig. 3. Schematic of the viscosity measurement apparatus for crude oil-water mixture.](image)

![Table 2](image)

<table>
<thead>
<tr>
<th>Stirring speed (r/min)</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>$\eta = 414.62M^{0.546}$</td>
</tr>
<tr>
<td>250</td>
<td>$\eta = 154.75M^{0.332}$</td>
</tr>
<tr>
<td>300</td>
<td>$\eta = 43.37M^{0.414}$</td>
</tr>
<tr>
<td>350</td>
<td>$\eta = 17.51M^{0.331}$</td>
</tr>
<tr>
<td>400</td>
<td>$\eta = 7.32M^{0.728}$</td>
</tr>
</tbody>
</table>

Note: the unit of $\eta$ is mPa s.; the unit of $M$ is N·cm.

![Fig. 4. Effective viscosity of crude oil-water mixture vs. shear rate.](image)
\[ \eta = a M^b \]  

(18)  

where \( a \) and \( b \) are fitting parameters.

Eighteen Newtonian fluids with the viscosities ranging from 5 to 1024 mPa·s were used to obtain the shaft torques at specific speeds. Then, the values of \( a \) and \( b \) were determined by fitting the torque vs. viscosity data at each stirring speed, and correlations shown in Table 2 were thus obtained.

The effective viscosity of crude oil-water mixtures with high water fraction is usually shear-dependent. In order to characterize this shear-dependence, the shear rate in the stirred vessel at a specific speed has to be determined. It should be noted that the fluid at different positions of the vessel experiences different shear rates. For example, the shear rate near the impeller must be higher than that away from the impeller. Taking advantage of the relationship between the energy dissipation rate and the shear rate, Zhang et al. (2003) presented a method for estimating the mean shear rate in a stirred vessel, as shown in Eq. (19).

\[ \bar{\gamma} = \frac{\sqrt{2\pi MN \eta V}}{2} \]  

(19)

where \( M \) is the shaft torque of the stirrer, N·m; \( N \) is the stirring speed, r/s; \( \eta \) is fluid viscosity, Pa·s; \( V \) is fluid volume, m³.

For a particular stirring system (stirred vessel, impeller, and volume of fluid), the correlation relating the mean shear rate to the stirring speed and viscosity were determined by using fluids with known viscosities and measuring the shaft torques at various stirring speeds. For the stirring system used in this study, the correlation thus obtained for the mean shear rate is as follows.

\[ \bar{\gamma} = 0.0775 - 0.3618 \eta^{0.4616} \]  

(20)

Based on the above principle, following procedures are carried out. Firstly, the oil-water mixture is stirred at a chosen speed, and the shaft torques is recorded; then a correlation corresponding to that stirring speed listed in Table 2 is used to obtain the viscosity; and further, Eq. (20) is used to determine the corresponding mean shear rate. Consequently, one set of effective viscosity vs mean shear rate data is determined. By duplicating the above procedures at different stirring speeds, effective viscosities at different mean shear rates are thus obtained.

3. Experimental results

3.1. Effect of shear rate on the effective viscosity of crude oil-water mixtures

The measured effective viscosity of crude oil-water mixtures at various shear rates is shown in Fig. 4.

It can be seen that the effective viscosity decreases with the increase of shear rate, i.e. taking on shear thinning behavior. Moreover, the shear thinning behavior weakens with the increase of the water fraction (see oils #1 and #2), and the increase of temperature (see oils #3 and #4).

When a crude oil-water mixture is stirred at low shear rate, the emulsification degree of oil and water is low, and the oil phase is not entirely dispersed. Therefore, the oil phase has greater influence on the effective viscosity of the mixture than the water phase does, leading to a higher viscosity. With the increase of shear rate, the emulsification degree becomes higher and the oil phase is dispersed to a greater extent, forming a dispersion system with the oil droplets containing emulsified water are disperse phase, while the free water is continuous phase. Therefore, the free water has more effect on the effective viscosity than the oil, leading to a lower viscosity.

With the increase of water fraction, the effect of oil viscosity on the effective viscosity of mixture becomes weaker, while the effect of water
on the effective viscosity increases. Therefore, although the crude oil-water mixture still shows shear thinning behavior under a higher water fraction, this shear thinning behavior is weaker than that at lower water fraction.

Similarly, with the increase of temperature, the effect of oil viscosity on the effective viscosity weakens due to the decreasing oil viscosity. Therefore, the shear thinning behavior of a crude oil-water mixture at higher temperature is weaker than that at lower temperature.

3.2. Effect of water fraction on the effective viscosity of crude oil-water mixtures

The measured effective viscosity of crude oil-water mixtures at various water fractions is shown in Fig. 5.

As expected, the effective viscosity decreases with the increase of water fraction. Moreover, the trend of the viscosity varying with water fraction becomes gentle with the increase of temperature.

3.3. Effect of temperature on the effective viscosity of crude oil-water mixtures

The measured effective viscosity of crude oil-water mixtures at various temperatures is shown in Fig. 6.

It can be seen that the effective viscosity decreases with the increase of temperature. Moreover, the trend of the viscosity variation with temperature becomes gentle with the increase of water fraction.

4. Viscosity model of crude oil-water mixtures with high water fraction

4.1. Model development

A flowing crude oil-water system may exist in different patterns dependent on the fraction and properties of the oil and water phase, and particularly on the shear intensity. There are two extreme mixing patterns for a high water fraction system, i.e., the complete stratification of oil and water and the complete dispersion of oil and water (O/W), as shown in Fig. 7. The other mixing patterns must be fallen
between these two extreme patterns.

Let $\eta_{SF}$ denotes the effective viscosity of the complete stratification of flow, while $\eta_{DF}$ represents the effective viscosity of the complete dispersion flow. When a crude oil-water mixture flows in any pattern between the two extreme cases, assume that the effective viscosity of the oil-water mixture can be expressed as the weighted average of the viscosity of the two extreme patterns.

$$\eta_c = \eta_{SF}(1 - cm) + \eta_{DF}cm$$  \hspace{1cm} (21)

where $cm$ is defined as the mixing coefficient with the value between 0 and 1, with $cm=0$ representing the complete stratified flow, and $cm=1$ representing complete dispersion flow.

For the complete stratified flow, assume that the effective viscosity of crude oil-water mixture ($\eta_{SF}$) could be represented by the weighted average rule.

$$\eta_{SF} = \eta_o(1 - \phi_w) + \eta_w\phi_w$$  \hspace{1cm} (22)

where $\eta_o$ and $\eta_w$ are the oil and water viscosity in mPa·s, respectively; $\phi_w$ is the volume fraction of water.

For the complete dispersion flow, the oil and water are mixed uniformly, and the mixture can be regarded as a homogeneous emulsion. Therefore, in this case, the effective viscosity of the crude oil-water mixture ($\eta_{DF}$) can be calculated by using viscosity model developed for stable emulsions. We’ll use the Taylor model for further derivation:
\[ \eta_{\text{eff}} = \eta_c \left[ 1 + \left( \frac{\eta_d + 2.5\eta_c}{\eta_c + \eta_d} \right) \phi_d \right] \]

where \( \eta_c \) and \( \eta_d \) are the viscosity of the continuous phase and dispersed phase respectively, mPa s; \( \phi_d \) is the volume fraction of the dispersed phase.

As stated above, most of the emulsion viscosity models were developed for stable emulsions, and the crude oil-water mixture with high water fraction is an unstable and inhomogeneous system, containing both emulsified water and free water. In this system, the emulsified water acts as part of the oil phase hydrodynamically. Therefore, the effective volume fraction of the oil phase should be actually greater than the real volume fraction of the pure oil. The volume fraction of the "effective oil phase" is actually the sum of the volume fraction of pure oil and the volume fraction of the emulsified water.

Given the above, for the crude oil-water mixture with high water fraction, water is the continuous phase and the "effective oil phase" is the dispersed phase. Based on this fact, we modify Eq. (23) by using the volume fraction of the "effective oil phase", and obtain the following equation:

\[ \eta_{\text{eff}} = \eta_c \left[ 1 + \left( \frac{\eta_c + 2.5\eta_d}{\eta_c + \eta_d} \right) \phi_c \right] \]

where \( \phi_c \) is the volume fraction of crude oil; \( \phi_e \) is the emulsified water fraction.

Furthermore, we substitute Eqs. (22) and (24) into Eq. (21), and obtain a new model for calculating the effective viscosity of crude oil-water mixture with high water fraction:

\[ \eta_{\text{eff}} = \eta_c \left[ 1 + \left( \frac{\eta_c + 2.5\eta_d}{\eta_c + \eta_d} \right) \phi_c \right] \eta_w \left[ 1 + \left( \frac{\eta_d + 2.5\eta_c}{\eta_d + \eta_c} \right) \phi_d \right] + \eta_w \left[ \eta_c \eta_d \phi_c \phi_d \right] \]

where \( \eta_c \) and \( \eta_d \) are the viscosity of the continuous phase and dispersed phase respectively, mPa s; \( \phi_d \) is the volume fraction of the dispersed phase.

In application of Eq. (25), it is necessary to determine the emulsified water fraction (\( \phi_E \)) and the mixing coefficient (\( c_m \)).

4.2. Calculation of emulsified water fraction (\( \phi_E \))

In our previous work (Wen et al., 2014), it was found that the emulsified water fraction under flowing conditions can be calculated by the following correlation.

\[ \phi_E = k_1 \phi_g^{k_2} \]

where \( S_\phi \) is the entropy production rate of viscous flow, W/(kg K); \( k_1, k_2 \) are the parameters related to the composition of crude oil.

According to thermodynamics, the entropy production rate is a measure of the irreversibility of a thermodynamic process, and so the entropy production rate of viscous flow describes the irreversibility of the viscous flow (Jou et al., 2010). The entropy production rate of viscous flow can be calculated as follows.

\[ S_\phi = \frac{\phi}{\rho T_k} \]

where \( \phi \) is the energy dissipation rate per unit volume of fluid (White, 2011), W/m³; \( \rho \) is the density of fluid, kg/m³; \( T_k \) is the absolute temperature of the fluid, K.

The mean energy dissipation rate (\( \varphi \)) in a stirred vessel can be calculated according to the shaft power of stirring:

\[ \varphi = \frac{2\pi N M}{V} \]
The mean energy dissipation rate of pipe flow can be calculated as follows (Zhang and Yan, 2002):

\[ \phi = \frac{\lambda \rho v}{d} \]

(29)

where \( \lambda \) is the hydraulic friction factor (Darcy friction factor); \( v \) is sectional average velocity of the fluid, m/s; \( d \) is the pipe diameter, m.

According to our previous work (Wen et al., 2016), \( k_1 \) and \( k_2 \) are related to crude oil compositions and can be calculated by using the following empirical correlations:

\[ k_1 = 0.031 c_{as+r}^{0.125} c_{w+m}^{0.017} TAN^{0.030} CN_{oil}^{0.246} \]

(30)

\[ k_2 = 0.998 c_{as+r}^{0.036} c_{w+m}^{0.024} TAN^{-0.027} CN_{oil}^{-0.351} \]

(31)

where \( c_{as+r} \) is the content of asphaltenes and resins, wt%; \( c_{w+m} \) is the sum of the concentration of precipitated wax and the content of mechanical impurities, wt%; \( TAN \) is the total acid number of the crude oil, mg KOH/g; \( CN_{oil} \) is the average carbon number of the crude oil.

4.3. Calculation of mixing coefficient \( (c_m) \)

By rearranging Eq. (21), the mixing coefficient \( c_m \) can be obtained as follows:

\[ c_m = \frac{\eta_{mix} - \eta_{SF}}{\eta_{mix} - \eta_{DF}} \]

(32)

By using effective viscosity data \( \eta_{mix} \) of the mixtures of the eight crude oils, the values of \( c_m \) can be obtained by Eq. (32), where \( \eta_{SF} \) and \( \eta_{DF} \) are calculated by using Eq. (22) and Eq. (24), respectively.

As mentioned above, the mixing degree of the oil and water, i.e. the mixing coefficient \( c_m \), depends on flow, too. Therefore, it is a reasonable assumption that \( c_m \) is a function of the entropy production rate of viscous flow, \( S_g \). Fig. 8 shows how \( c_m \) correlates with the \( S_g \).

According to the tendency of the curves in Fig. 8, we use a linear equation to correlate \( c_m \) and \( S_g \):

\[ c_m = a S_g + b \]

(33)

where \( a \) and \( b \) may be determined by fitting experimental data.

As to the determination of \( a \) and \( b \), we assume that they are related to water fraction, temperature and crude oil composition. By fitting 160 sets of experimental data from the mixtures of eight crude oils (oil #1 to oil # 8), correlations were obtained between \( a \), \( b \) and water fraction, temperature and crude oil compositions.
5. Model validation

The measured effective viscosities of crude oil-water mixtures of oils #9 and #10 were used to validate the proposed viscosity model. Fig. 11 and Fig. 12 show the comparison between the experimental results and the predicted values.

For the forty sets of experimental data used for model validation, the proposed model gave prediction of an average relative deviation of 7.0% and 15.2%, as shown in Fig. 9 and Fig. 10...

According to the conditions under which this model was developed, we recommend that it is better to use the model for: (1) light to medium crude oil; (2) water fraction ranging from 0.70 to 0.90; (3) temperature above the pour point of crude oil.

6. Approach to model application

Because the viscosity model developed in this study contains the

\[ a = 0.6377^{0.202} \left( \frac{V_w}{V_o} \right)^{0.665} \epsilon^{0.041} C^0.006 \tan^{-0.073} C^0.207 \]  

\[ b = 0.0417^{0.072} \left( \frac{V_w}{V_o} \right)^{0.344} \epsilon^{0.417} C^{0.235} \tan^{0.229} C^{0.545} \]

The average relative deviations of Eq. (34) and Eq. (35) are respectively 7.0% and 15.2%, as shown in Fig. 9 and Fig. 10...

According to the conditions under which this model was developed, we recommend that it is better to use the model for: (1) light to medium crude oil; (2) water fraction ranging from 0.70 to 0.90; (3) temperature above the pour point of crude oil.

7. Conclusion

The effective viscosity of crude oil-water mixtures with high water fraction was measured by using the stirring method, and the effect of shear rate, water fraction and temperature on the effective viscosity of the mixtures was studied. Combined with our previous study on the emulsification behaviors of crude oil-water systems with high water fraction, an effective viscosity model was developed. The following conclusions could be drawn from the present study:

(1) The effective viscosity of crude oil-water mixtures with high water fraction decreases with the increase of shear rate, water fraction, and temperature.

(2) A model was developed for predicting the effective viscosity of crude oil-water mixtures with high water fraction. This model is characterized in three aspects: First, it may describe the shear-
rate-dependence of the effective viscosity of the crude oil-water mixtures with high water fraction. Second, it takes the effect of emulsified water into account. Third, it associates with the composition of crude oil.

(3) Forty sets of the effective viscosity data from mixtures of two crude oils, which were not used in development of the model, were used to validate the model. The results show that the proposed model gives accurate prediction, with an average relative deviation of 7.4%.

Conflict of interest

The authors declare no competing financial interest.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.petrol.2016.09.052.

References
