AEROACOUSTIC ANALYSIS OF PERFORATED MUFFLER COMPONENTS

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Sullivan and Crocker's equations for two-duct perforated elements were solved recently by Jayaraman and Yam by means of a decoupling approach, which was valid only for equal mean-flow Mach numbers in the two interacting ducts. The present paper provides a generalized decoupling approach for the analysis of two-duct as well as three-duct muffler configurations consisting of perforated elements, taking into account the actual mean flow Mach numbers in the adjoining tubes. The theoretical results of transmission loss of typical muffler configurations are compared with the available measurements of Sullivan, and predictions of Jayaraman, Yam and Thawani. A comparison has been made with the TL values calculated by means of Sullivan's segmentation approach. The agreement between the two seems to be good.

1. INTRODUCTION

In order to attenuate internal combustion engine exhaust noise, various geometrical configurations of muffler elements have been developed. These include concentric tube resonators and cross-flow elements, consisting of a perforated tube enclosed in a concentric cylindrical chamber. These configurations may be of the straight through type and the reversed flow type. Generally, straight through mufflers offer low resistance to the flow of gases and consequently maintain low engine back pressures. These are however acoustically not always very effective. Mufflers with flow reversals, on the other hand, may offer increased attenuation but at the cost of higher back pressures. In practice, mufflers with cross flow have often been preferred in view of their greater effectiveness in noise attenuation with reasonable pressure drop.

A good amount of work has been done to predict the performance of mufflers consisting of simple area changes, extended tube elements and flow reversal resonator elements, by using plane wave acoustic theory, with and without mean flow [1–5]. An analysis for perforated element mufflers was presented in 1978 by Sullivan and Crocker [6], who provided an analytical approach to predict the transmission loss (TL) of concentric-tube resonators. They solved the coupled equations, writing the acoustic field in the annular cylindrical cavity as an infinite summation of natural modes satisfying the rigid wall boundary conditions at the two ends. Their predictions were corroborated by experimental observations, for the case of a stationary medium.

Sullivan [7, 8] followed this work up by presenting a segmentation procedure for modeling all types of perforated element mufflers. In this method, each segment is described by a separate transfer matrix. This method is wider in application. However, it requires the assumption that a perforated element would behave as if it were physically

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divided in several segments. This discretization of a uniformly perforated element is arbitrary, and for better prediction one has to go on increasing the number of segments. The predictions tallied well with experimental findings for different perforated elements.

Later, Jayaraman and Yam [9] introduced a decoupling approach to get closed form solutions. The method, as presented, is based on the assumption that the mean flow Mach numbers in the ducts are equal, which is of course not true in the case of an actual muffler system where the two Mach numbers would be in inverse proportion to the cross-sectional areas of the two ducts.

Recently Thawani and Jayaraman [10] extended the decoupling approach to concentric resonator mufflers, limiting their analysis to the case of zero mean flow.

The present paper, which deals essentially with alternative mathematically useful methods of solving the equations developed by Sullivan [6-8], describes a generalized decoupling approach, with unequal axial mean flow Mach numbers in the adjoining tubes. For convection of acoustic disturbances, the axial variation of mean flow Mach numbers has been neglected; instead, axially averaged (mean) values have been taken. It is also assumed that the flow transfer through the perforated portion is uniformly distributed over the length, and therefore the perforate impedance is constant along the length. Sullivan and Crocker's one-dimensional equations have been adopted. The radial particle velocity is assumed to be limited to the vicinity of the perforate, obviating the need for a three-dimensional analysis.

2. THE ANALYSIS OF TWO-DUCT PERFORATED ELEMENTS

For a concentric tube resonator (see Figure 1) as well as for cross-flow elements (see Figure 2), the mass continuity equation, momentum equations and energy equation can be combined for sinusoidal temporal variations to obtain the following two coupled differential equations, one for the inner tube and the other for the outer tube [6] (a list of notation is given in Appendix B):

$$\left[ \frac{d^2}{dz^2} - iM_i^2 \left( \frac{k_a^2 + k^2}{k} \right) \frac{d}{dz} \frac{k_a^2}{1 - M_i^2} \right] p_1(z) = - \left[ \frac{iM_i^2}{1 - M_i^2} \left( \frac{k_a^2 - k^2}{k} \right) \frac{d}{dz} \frac{k_a^2 - k^2}{1 - M_i^2} \right] p_2(z),$$

(1)

Figure 1. Partially perforated resonator configuration. (a) Partially perforated resonator; (b) concentric perforated section.
where

\( k = \frac{\omega}{c_0}, \quad M_1 = \frac{W_0}{c_0}, \quad M_2 = \frac{W_0}{c_0}, \)

\( k_2 = k^2 - i4k/d_1\zeta, \quad k_2' = k^2 - i4kd_2/(d_2^2 - d_1^2)\zeta. \)

In accordance with the assumption set out earlier, that the perforate impedance could be regarded as uniform as the mean flow would divide itself almost uniformly while moving across the perforated section, the perforate impedance \( \rho_0c_0\zeta \) here is taken to be uniform over the length. Thus, the radial particle velocity at the perforations is related to the pressure difference across the perforations by

\[
\frac{u(z)}{\rho_0c_0} = \frac{p_1(z) - p_2(z)}{(\rho_0c_0)^2}. \tag{4}
\]

Equations \(1\) and \(2\) may be written in matrix notation as

\[
\begin{bmatrix}
\frac{d^2}{dz^2} - \frac{iM_2}{1 - M_2^2} \left( \frac{k_2^2 + k_2'^2}{k} \right) \frac{d}{dz} + \frac{k_2}{1 - M_2^2} \frac{d}{dz} - \frac{k_2^2 - k_2'^2}{1 - M_2^2}
\end{bmatrix} p_2(z) = -\left[ \frac{iM_2}{1 - M_2^2} \left( \frac{k_2^2 - k_2'^2}{k} \right) \frac{d}{dz} - \frac{k_2^2 - k_2'^2}{1 - M_2^2} \right] p_1(z),
\]

where

\[
\begin{align*}
D &= \frac{d}{dz}, \\
\alpha_1 &= \alpha_2 = 1, \\
\alpha_3 &= \alpha_4 = 2, \\
\alpha_5 &= \alpha_6 = 3, \\
\alpha_7 &= \alpha_8 = 4.
\end{align*}
\]

Equations \(1\) and \(2\) can be rearranged as a set of four simultaneous first-order equations as

\[
\begin{bmatrix}
-1 & 0 & D & 0 & \{y_1\} \\
0 & -1 & 0 & D & \{y_2\} \\
D & 0 & \alpha_1 D + \alpha_2 & \alpha_3 D + \alpha_4 & \{y_3\} \\
0 & D & \alpha_5 D + \alpha_6 & \alpha_7 D + \alpha_8 & \{y_4\}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}, \tag{6a}
\]

or

\[
\begin{bmatrix}
\Delta \{y\} = \{0\}, \tag{6b}
\end{bmatrix}
\]

where

\[
y_1 = dp_1/dz, \quad y_2 = dp_2/dz, \quad y_3 = p_1, \quad y_4 = p_2. \tag{7}
\]
The characteristic polynomial of \([A]\) is the same as the characteristic polynomial of \([A]\).

Now, equations (6) can be decoupled by transforming them to the principal variables \(\Gamma_1, \Gamma_2, \Gamma_3\) and \(\Gamma_4\), related to the variables \(y_1, y_2, y_3\) and \(y_4\) through the eigenmatrix \([\psi]\) by

\[
\{y\} = [\psi]\{\Gamma\},
\]  

where

\[
\psi_{1,j} = 1 \text{ (say)}, \quad \psi_{2,j} = -\left(\beta_j^2 + \alpha_1\beta_j + \alpha_2\right)/(\alpha_1\beta_j + \alpha_2),
\]

\[
\psi_{3,j} = 1/\beta_j, \quad \psi_{4,j} = \psi_{2,j}/\beta_j = \psi_{2,j}\psi_{3,j}, \quad j = 1, 2, 3, 4.
\]  

The general solutions of the decoupled equations can be written as

\[
\Gamma_j(z) = C_je^{\beta_j z}, \quad j = 1, 2, 3, 4.
\]  

Substitution of expressions for \(dp/dz\) from equation (8) into the momentum equations gives

\[
\begin{bmatrix}
   p_1(z) \\
   p_2(z) \\
   p_3(z) \\
   p_4(z)
\end{bmatrix} = [A_{i,j}(z)]
\begin{bmatrix}
   C_1 \\
   C_2 \\
   C_3 \\
   C_4
\end{bmatrix},
\]  

where

\[
A_{1,j} = \psi_{3,j}e^{\beta_j z}, \quad A_{2,j} = \psi_{4,j}e^{\beta_j z},
\]

\[
A_{3,j} = -e^{\beta_j z}/(ik + M_1\beta_j), \quad A_{4,j} = -\psi_{2,j}e^{\beta_j z}/(ik + M_2\beta_j),
\]  

and \(j = 1, 2, 3\) and \(4\) for the respective columns of \(A_{i,j}(z)\). The state variables at \(z = 0\) can be related to the state variables at \(z = 1\) through the transfer matrix relation

\[
\begin{bmatrix}
   p_1(0) \\
   p_2(0) \\
   p_3(0) \\
   p_4(0)
\end{bmatrix} = [T_{i,j}]
\begin{bmatrix}
   p_1(1) \\
   p_2(1) \\
   p_3(1) \\
   p_4(1)
\end{bmatrix},
\]  

where

\[
[T_{i,j}] = [G_{i,j}]^{-1},
\]  

the elements in the matrices \([G_{i,j}]\) and \([H_{i,j}]\) being obtained by using the corresponding elements in the matrix \([A_{i,j}]\) evaluated at \(z = 0\) and \(z = 1\) respectively.

In the case of a simple concentric-tube resonator (see Figure 1), the rigid-end boundary conditions yield \([10]\)

\[
\begin{align*}
   z = 0: p_2(0)/-p_0c_0w_2(0) &= -i \cot(ka), \\
   z = 1: p_2(1)/p_0c_0w_2(1) &= -i \cot(kb).
\end{align*}
\]  

Similar equations can be obtained for cases of yielding end walls (finite impedances). Here, for algebraic simplicity, the analysis is restricted to rigid end boundary conditions—equations (15) and (16). By solving equations (14), (15) and (16) simultaneously, the upstream state variables can be related to the downstream variables through the transfer matrix relation

\[
\begin{bmatrix}
   p_1(0) \\
   p_2(0)
\end{bmatrix} = \begin{bmatrix}
   T_a & T_b \\
   T_c & T_d
\end{bmatrix}
\begin{bmatrix}
   p_1(1) \\
   p_2(1)
\end{bmatrix},
\]  

where

\[T_a = T_d.
\]
where

\[ T_a = T_{1,1} + A_1 A_2, \quad T_b = T_{1,3} + B_1 A_2, \quad T_c = T_{3,1} + A_1 B_2, \quad T_d = T_{3,3} + B_1 B_2, \]

\[ A_1 = (X_1 T_{2,1} - T_{4,1})/F_1, \quad B_1 = (X_1 T_{2,3} - T_{4,3})/F_1, \]

\[ A_2 = T_{1,2} + X_2 T_{1,4}, \quad B_2 = T_{3,2} + X_2 T_{3,4}, \]

\[ F_1 = T_{4,2} + X_2 T_{4,4} - X_1 (T_{2,2} + X_2 T_{2,4}), \]

\[ X_1 = -i \tan (k l_a), \quad X_2 = i \tan (k l_b). \] (18)

These expressions are exact solutions of equations (1) and (2). The transmission loss at any frequency may be computed from the four elements of the transfer matrix by making use of the relation [6]

\[ TL = 10 \log_{10} \left[ \frac{A_i}{A_o} \left( \frac{1 + M_i}{1 + M_o} \right)^2 \frac{T_a + T_b + T_c + T_d}{2} \right]^2, \] (19)

where \( A_i \) and \( A_o \) are inlet and outlet cross-sectional areas of elements of the muffler and \( M_i \) and \( M_o \) are the mean flow Mach numbers at the inlet and outlet respectively.

The equations governing other elements (Figure 2) are the same as those governing the concentric-tube resonator. There is, however, a change in the boundary conditions. These are as follows. For a cross flow expansion element (see Figure 2(a)),

\[ p_2(0)/-\rho_0 c_0 w_2(0) = -i \cot (k l_a), \] (20)

\[ p_1(1)/\rho_0 c_0 w_1(1) = -i \cot (k l_b); \] (21)

for a cross-flow contraction element (see Figure 2(b)),

\[ p_1(0)/-\rho_0 c_0 w_1(0) = -i \cot (k l_a), \] (22)

\[ p_2(1)/\rho_0 c_0 w_2(1) = -i \cot (k l_b). \] (23)

The four-pole parameters of these elements may also be derived explicitly [12]. These two elements are usually combined into a plug muffler (see Figure 3).

Figure 3. Cross-flow chamber (plug muffler). \( d_1 = 0.0493 \text{ m}, \ d_2 = 0.1016 \text{ m}, \ l = 0.1286 \text{ m}, \ l_a = l_b = 0.0, \ \sigma = 3.9\%, \ \text{temp} = 74^\circ \text{C}. \)

3. ANALYSIS OF THREE-DUCT PERFORATED ELEMENTS

In this section the decoupling approach is extended to three-duct muffler element configurations in which the inlet tube is offset from the outlet tube, and both tubes are enclosed in a cylindrical chamber. The three coupled partial differential equations for wave propagation are solved by using the generalized decoupling approach with and without mean flow, with the relevant boundary conditions, in order to derive explicit expressions for four-pole parameters of a cross flow expansion chamber as well as a reverse flow expansion chamber, as shown in Figure 4.
For the three-duct model as shown in Figure 5, the mass continuity, momentum and energy equations may be combined to obtain the following three coupled differential equations for ducts 1, 2 and 3 respectively [12]:

\[
\frac{d^2}{dz^2} \left[ \frac{iM_1}{1-M_1^2} \left( \frac{k_a^2+k_b^2}{k} \right) \frac{d}{dz} + \frac{k_a^2}{1-M_1^2} \right] p_1(z) + \left[ \frac{iM_1}{1-M_1^2} \left( \frac{k_a^2-k_b^2}{k} \right) \frac{d}{dz} - \frac{k_a^2-k_b^2}{1-M_1^2} \right] p_2(z) = 0,
\]

\[
\left[ \frac{iM_2}{1-M_2^2} \left( \frac{k_b^2-k_e^2}{k} \right) \frac{d}{dz} - \frac{k_b^2-k_e^2}{1-M_2^2} \right] p_1(z) + \left[ \frac{d^2}{dz^2} - \frac{iM_2}{1-M_2^2} \left( \frac{k_b^2+k_e^2}{k} \right) \frac{d}{dz} + \frac{k_b^2+k_e^2-k_e^2}{1-M_2^2} \right] p_2(z) + \left[ \frac{iM_2}{1-M_2^2} \left( \frac{k_b^2-k_e^2}{k} \right) \frac{d}{dz} - \frac{k_b^2-k_e^2}{1-M_2^2} \right] p_3(z) = 0,
\]

\[
\left[ \frac{iM_3}{1-M_3^2} \left( \frac{k_e^2-k_b^2}{k} \right) \frac{d}{dz} - \frac{k_e^2-k_b^2}{1-M_3^2} \right] p_2(z) + \left[ \frac{d^2}{dz^2} - \frac{iM_3}{1-M_3^2} \left( \frac{k_e^2+k_b^2}{k} \right) \frac{d}{dz} + \frac{k_e^2}{1-M_3^2} \right] p_3(z) = 0,
\]

Figure 5. The common three-duct section.
where
\[
\begin{align*}
    k &= \omega / c_0, \quad M_1 = W_0 / c_0, \quad M_2 = W_0 / c_0, \quad M_3 = W_0 / c_0, \quad k^2_d = k^2 - i4k/d_1, \\
    k^2_s &= k^2 - i4k d_1 / (d_2^2 - d_3^2 - d_4^2), \quad k^2_s = k^2 - i4k d_3 / (d_2^2 - d_3^2 - d_4^2), \\
    k^2 &= k^2_d - i4k / d_1. 
\end{align*}
\] (27)

Equation (24) is identical with equation (1).

The decoupling analysis runs exactly as for the two-duct perforated element mufflers, discussed in section 2.

Upon defining the new variables as
\[
y_1 = dp_1/dz, \quad y_2 = dp_2/dz, \quad y_3 = dp_3/dz, \quad y_4 = p_1, \quad y_5 = p_2, \quad y_6 = p_3, 
\] (28)
equations (24), (25) and (26) may be written in the form
\[
\begin{bmatrix}
-1 & 0 & 0 & D & 0 & 0 \\
0 & -1 & 0 & 0 & D & 0 \\
0 & 0 & -1 & 0 & 0 & D \\
D & 0 & 0 & \alpha_1 D + \alpha_2 & \alpha_3 D + \alpha_4 & 0 \\
0 & D & 0 & \alpha_5 D + \alpha_6 & \alpha_7 D + \alpha_8 & \alpha_9 D + \alpha_{10} \\
0 & 0 & D & 0 & \alpha_{11} D + \alpha_{12} & \alpha_{13} D + \alpha_{14} \\
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6 \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix},
\] (29)

where the \( \alpha \)'s are known constants.

The new variables, i.e., the principal co-ordinates \( \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5 \) and \( \Gamma_6 \), are related to variables \( y_1, y_2, y_3, y_4, y_5 \) and \( y_6 \), through the modal matrix \( [\psi] \), by
\[
\{y\} = [\psi]\{\Gamma\},
\] (30)

where
\[
\begin{align*}
    \psi_{1,j} &= 1.0 \text{ (say)}, \quad \psi_{2,j} = -(\beta_j + \alpha_1 \beta_j + \alpha_2)/(\alpha_3 \beta_j + \alpha_4), \\
    \psi_{3,j} &= [(\beta_j^2 + \alpha_1 \beta_j + \alpha_2)(\beta_j^2 + \alpha_3 \beta_j + \alpha_6) - (\alpha_3 \beta_j + \alpha_4)(\alpha_5 \beta_j + \alpha_6)]/(\alpha_3 \beta_j + \alpha_4)(\alpha_5 \beta_j + \alpha_{10}), \\
    \psi_{4,j} &= 1.0/\beta_j, \quad \psi_{5,j} = \psi_{2,j}/\beta_j, \quad \psi_{6,j} = \psi_{3,j}/\beta_j.
\end{align*}
\] (31)
The subscript \( j \) takes on the values 1, 2, \ldots, 6, and the \( \beta \)'s are the zeros of the determinant of the coefficient matrix of equations (29).

The general solutions of the first order uncoupled equations may be written in the form
\[
\Gamma_j(z) = C_j e^{\theta_j z},
\] (32)
where the subscript \( j \) takes on the values 1, 2, \ldots, 6. Next, the momentum equations may be used to obtain expressions for \( w_1(z), w_2(z) \) and \( w_3(z) \). Finally, one obtains
\[
\begin{bmatrix}
p_1(z) \\
p_2(z) \\
p_3(z) \\
\rho_0 c_0 w_1(z) \\
\rho_0 c_0 w_2(z) \\
\rho_0 c_0 w_3(z)
\end{bmatrix}
= [A]
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5 \\
C_6
\end{bmatrix},
\] (33)

where the elements \( A_{k,j} \) involving only \( M, k \) and \( l \) are given by the expressions in reference [12].
The pressure and velocity at \( z = 0 \) can then be related to the pressure and velocity at \( z = l \) by

\[
\begin{bmatrix}
    p_1(0) \\
    p_2(0) \\
    p_3(0) \\
    \rho_0c_0w_1(0) \\
    \rho_0c_0w_2(0) \\
    \rho_0c_0w_3(0)
\end{bmatrix} = [T]
\begin{bmatrix}
    p_1(l) \\
    p_2(l) \\
    p_3(l) \\
    \rho_0c_0w_1(l) \\
    \rho_0c_0w_2(l) \\
    \rho_0c_0w_3(l)
\end{bmatrix},
\]  

(34)

where the transfer matrix is given by

\[
[T] = [G][H]^{-1}.
\]  

(35)

Here the elements in matrices \( G_{ij} \) and \( H_{ij} \) are obtained by using the corresponding elements in the matrix \( [A] \) evaluated at \( z = 0 \) and \( z = l \), respectively.

To eliminate four of the six state variables in the foregoing general solution (equations (34)), the four available boundary conditions can be used. In the case of a cross flow expansion chamber with a rigid end termination, the boundary conditions are [10]

\[
z = 0: \quad \rho_0c_0w_2(0) = -i \tan (kla)p_2(0), \\
z = 0: \quad \rho_0c_0w_3(0) = -i \tan (kla)p_3(0), \\
z = l: \quad \rho_0c_0w_1(l) = i \tan (kla)p_1(l), \\
z = l: \quad \rho_0c_0w_2(l) = i \tan (kla)p_2(l).
\]  

(36-39)

After a little algebra, the final expressions for the upstream state variables and the downstream variables can be written as

\[
\begin{bmatrix}
    p_1(0) \\
    \rho_0c_0w_1(0)
\end{bmatrix} = \begin{bmatrix} T_a & T_b \\ T_c & T_d \end{bmatrix} \begin{bmatrix}
    p_1(l) \\
    \rho_0c_0w_1(l)
\end{bmatrix}.
\]  

(40)

The expressions for the \( T \)'s are given in reference [12].

The transmission loss at any frequency for an anechoic termination is simply related to the sum of all the four-pole parameters as per equation (19).

In the case of a three-duct flow-reversal element, the following changes have to be noted: (a) the mean flow in duct three is negative with respect to the reference direction; (b) the boundary conditions are different, being now as follows:

\[
z = 0: \quad \rho_0c_0w_2(0) = -i \tan (kla)p_2(0), \\
z = l: \quad \rho_0c_0w_1(l) = i \tan (kla)p_1(l), \\
z = l: \quad \rho_0c_0w_2(l) = i \tan (kla)p_2(l), \\
z = l: \quad \rho_0c_0w_3(l) = i \tan (kla)p_3(l).
\]  

(41-44)

The transmission matrix for this element is

\[
\begin{bmatrix}
    p_1(0) \\
    \rho_0c_0w_1(0)
\end{bmatrix} = \begin{bmatrix} T_a & T_b \\ T_c & T_d \end{bmatrix} \begin{bmatrix}
    p_1(0) \\
    \rho_0c_0w_1(0)
\end{bmatrix}.
\]  

(45)

and the resulting expressions for \( T_a, T_b, T_c \), and \( T_d \) are given in reference [12].

As indicated earlier, Sullivan [7] derived explicit expressions for four pole parameters of the transfer matrices of two-duct perforated elements, by means of his segmentation approach. For three-duct elements, however, he indicated only the governing equations.
and boundary conditions. These have been used to derive explicit expressions for the four-pole parameters of the transfer matrices of the two types of three-duct perforated elements [13].

4. TYPICAL RESULTS AND DISCUSSION

Transfer matrices derived for various perforated elements and concentric resonators may be used along with the transfer matrices of other elements constituting a muffler to predict the overall performance, say in the form of transmission loss (TL).

Sufficiently general FORTRAN programs have been developed for digital computation (on a DEC 10 computer) of TL and NR for concentric resonators of the type of Figure 1 and plug mufflers of the type of Figure 3, at various frequencies in the range 20 to 3500 Hz. Dimensions of the different muffler configurations have been so chosen that the predictions can be compared with those of Sullivan and Crocker [6], Sullivan [7, 8], and Jayaraman and Yam [9]. The calculations require appropriate values of the perforate impedance. These have been obtained directly from references [8] and [9] to facilitate later comparison. For the same reason, a similar perforate geometry, namely the same values of hole diameter d, plate thickness t and porosity ρ, has been adopted, the porosity being defined in the normal way as the ratio of open area to the total area of the perforate section. Thus, for zero flow, Sullivan's [8] empirical expression for the linear acoustic region has been employed, expressed as

\[ \zeta = \left(6 \times 10^{-3} + i4.8 \times 10^{-5}f\right)/\rho, \]  (46)

where \( f \) is the frequency. For cross flow the value given in reference [9] which was derived from reference [8] has been used. This is

\[ \zeta = (0.514 dM/l + i4.8 \times 10^{-5}f)/\rho, \]  (47)

where \( M \) is the upstream mean flow Mach number, and \( l \) is the length of the perforate section.

Notations used in the analysis are also as nearly as possible the same as used by Jayaraman and Yam, for the sake of easy comparison.

Figure 6. Transmission loss of the plug muffler of Figure 3 for \( M_t = 0.05 \). —, Prediction with estimated \( M_t \); --, prediction with \( M_t \) assumed equal to \( M_t \); ****, Sullivan's experimental observations.
Figure 6 shows transmission loss of the plug muffler of Figure 3 for the mean flow Mach number $M = 0.05$. With temperature gradients and the tube thickness neglected, $M_2$ would be given by

$$M_2 = M_1 \frac{d_1^2}{(d_2^2 - d_1^2)}.$$  \(48\)

The predictions are compared with Sullivan's measurements and with those for the simplified model having $M_2 = M_1$. Figures 7 and 8 show further comparisons with the simplified model for higher values of mean flow Mach number, i.e., $M_1 = 0.15$ and 0.2. The results for these three examples illustrate the discrepancies that exist between the predicted performance when the restrictive assumption that $M_2 = M_1$ is made and when the more realistic model is adopted.
The transmission loss predicted by the segmentation model for different Mach numbers for three-duct cross-flow element is compared with the $TL$ predicted by the distributed parameter approach in Figure 9. In both models the axial mean flow Mach number was assumed constant in accordance with the assumptions set out in the introduction. The results tally quite well.

![Figure 9. Transmission loss for the three-duct cross-flow element of Figure 4(a).](image)

$TL$ for different Mach numbers for a three-duct reverse flow element are compared with the $TL$ predicted by the distributed parameter approach in Figure 10. The $TL$ curves tally well, over the entire frequency range.

![Figure 10. Transmission loss for the three-duct reverse-flow element of Figure 4(b).](image)
The muffler element is a continuous system; however in the segmentation approach it is modeled as a discretized system. As the number of segments increases the results become comparable with those of the distributed parameter approach, as can be seen in Figure 11. Sullivan [8] also found that the predictions converged well with about 16 elements.

![Figure 11](image.png)

Figure 11. Effect of number of segments on transmission loss of the three-duct cross-flow element of Figure 4(a); $M = 0.2$. ——, 4 segments; ——, 8 segments; ——, 16 segments; ——, distributed parameter approach.

The phase angle versus frequency curve obtained for a three-duct crossflow element by using the segmentation model is compared with that obtained by using the distributed parameter model in Figure 12. The two curves seem to tally quite well up to about 2000 Hz.

![Figure 12](image.png)

Figure 12. Phase angle vs. frequency for the three-duct cross-flow element of Figure 4(a); $M = 0.2$. ——, Prediction by segmentation approach (16 segments); ——, prediction by distributed parameter approach.

Figure 13 shows curves of predicted transmission loss of the three-duct perforated element cross-flow muffler of Figure 4(a), calculated by means of the distributed parameter model, for mean flow Mach numbers $M = 0.0, 0.1$ and $0.2$ respectively. As the Mach
number increases, the resistive part of the perforated impedance also increases. This results in an increase in transmission loss. The positions of the maxima and minima remain the same. This is because the change in Mach number does not change the reactive part of the perforated impedance according to the empirical expression (equation (47)) used here for describing through flow perforated impedance. Sullivan [8, Figure 11] has investigated the results of changes in reactance. At the peaks there appears to be a little numerical instability. This is due to the fact that at some frequencies, the computed values of the $\beta$'s become very sensitive to the round-off errors in computation of coefficients of the polynomial.

The transmission loss of the reverse-flow three-duct perforated element muffler of Figure 4(b), calculated by means of the distributed parameter model, is shown in Figure 14. The
trend of the results is similar to that of the three-duct cross-flow perforated element mufflers.

To investigate theoretically the effect of mean flow wave convection on the predicted transmission loss for the case in Figure 13, computer runs were made with the mean flow velocity set equal to zero, and these results are shown along with those for the assumed mean flow velocity in the perforated pipe in Figure 15. The results show that, for this example, wave convection by the mean flow has no significant influence on the predictions, even at \( M = 0.2 \). The neglect of mean flow simplifies the analysis, so that roots of the sixth degree polynomial can be obtained effectively by inspection, as is indicated in Appendix A. The mathematical instability that appeared at peaks of the convective case can be completely eliminated, but the neglect of convection appears to over-estimate \( TL \) up to 5%. Similar observations have been made by Thawani and Jayaraman [10] for two-duct perforated element mufflers.

5. CONCLUSIONS

Like Sullivan’s segmentation approach, the distributed parameter approach described in this paper is general inasmuch as it can be applied to a perforated element with any number of tubes communicating with each other.

In the case of segmentation technique the number of segments needs to be high for reasonably good accuracy. Nevertheless, with easy accessibility of a digital computation facility, it is possible to predict the \( TL \) in a reasonable computation time.

Decoupling, or the distributed parameter approach, provides a closed form solution and is realistic insofar as the interaction among the perforated ducts is taken to be continuous in space. The mean flow Mach number in any duct, however, is taken as constant along the entire duct length, and this of course is not realistic as actually the Mach number does vary along the length of the element, because flow through the perforates is distributed along it. This fact can be taken into account easily in the segmentation technique. Thus both methods have strong and weak points.
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A paper dealing with another version of the decoupling approach, restricted to two-duct perforated elements, was presented as the First Prize Winning paper, at the Nelson Acoustics Conference, Madison, U.S.A. in July 1984, and has appeared in the proceedings of the conference.

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REFERENCES


APPENDIX A

Upon making the assumptions that (1) the inlet and outlet diameters of the three-duct muffler element are equal \((d_1 = d_3)\), (2) the porosity of the inlet tube is the same as that of the outlet tube, and (3) for convection purposes \(M_1 = M_2 = M_3 = 0\), the sixth degree polynomial of equation (29) becomes

\[
D^6 + [2(k_2^2 + k_3^2) - k_1^2]D^4 + [k_1^2 + 2k_2^2 k_3^2 + 2k_3^2 k_1^2 - 2k_2^4]D^2 + [2k_2^2 k_3^2 k_1^2 + k_2^2 k_3^2 k_1^2 - 2k_2^2 k_3^2] \Rightarrow 0,
\]

which is a cubic in \(D^2\). By inspection, one of the roots is obviously \(D_1^2 = -k_2^2\). By making use of this, the other two roots may be readily found to be \(D_2^2 = -k_3^2\) and \(D_3^2 = -(k_2^2 + 2k_3^2 - 2k_1^2)\). Thus, in this case, the roots are \(ik_2, -ik_2, ik_3, -ik_3, i(k_2^2 + 2k_3^2 - 2k_1^2)^{1/2}\) and \(-i(k_2^2 + 2k_3^2 - 2k_1^2)^{1/2}\).
APPENDIX B: NOTATION

\( A_i \)  
internal area of inlet pipe

\( A_o \)  
internal area of exit pipe

\( c_o \)  
velocity of wave propagation

\( d \)  
internal diameter of pipe

\( f \)  
frequency

\( i \)  
\( \sqrt{-1} \)

\( k \)  
wave number, \( \omega/c_0 \)

\( l \)  
length of a pipe

\( M \)  
Mach number, \( W_0/c_0 \)

\( M_i \)  
inlet Mach number

\( M_o \)  
exit Mach number

\( p_0 \)  
pressure of the undisturbed fluid

\( p \)  
fluctuating pressure

\( t \)  
time co-ordinate

\( temp \)  
temperature

\( u_{1,2} \)  
radial fluctuating velocity at 1, 2 interface of the control volume

\( u_{2,3} \)  
radial fluctuating velocity at 2, 3 interface of the control volume

\( W_0 \)  
velocity of the undisturbed fluid

\( w \)  
fluctuating velocity

\( z \)  
axial co-ordinate

\( \rho_0 \)  
density of undisturbed fluid

\( \rho \)  
fluctuation in density

\( \omega \)  
radian frequency

\( z_1 \)  
acoustical impedance at 1, 2 interface

\( z_2 \)  
acoustical impedance at 2, 3 interface