Intelligent Optimal Control of Single-Link Flexible Robot Arm
Rong-Jong Wai, Member, IEEE, and Meng-Chang Lee

Abstract—This paper addresses the design and properties of an intelligent optimal control for a nonlinear flexible robot arm that is driven by a permanent-magnet synchronous servo motor. First, the dynamic model of a flexible robot arm system with a tip mass is introduced. When the tip mass of the flexible robot arm is a rigid body, not only bending vibration but also torsional vibration are occurred. In this paper, the vibration states of the nonlinear system are assumed to be unmeasurable, i.e., only the actuator position can be acquired to feed into a suitable control system for stabilizing the vibration states indirectly. Then, an intelligent optimal control system is proposed to control the motor-mechanism coupling system for periodic motion. In the intelligent optimal control system a fuzzy neural network controller is used to learn a nonlinear function in the optimal control law, and a robust controller is designed to compensate the approximation error. Moreover, a simple adaptive algorithm is proposed to adjust the uncertain bound in the robust controller avoiding the chattering phenomena. The control laws of the intelligent optimal control system are derived in the sense of optimal control technique and Lyapunov stability analysis, so that system-tracking stability can be guaranteed in the closed-loop system. In addition, numerical simulation and experimental results are given to verify the effectiveness of the proposed control scheme.

Index Terms—Flexible robot arm, fuzzy neural network (FNN), intelligent control, optimal control, permanent-magnet (PM) synchronous servo motor.

I. INTRODUCTION

In order to achieve the high performance requirements such as high-speed operation, increased accuracy in positioning, lower energy consumption, less weight, and safer operation due to reduced inertia, great attention has been paid to the dynamics and control of flexible robot arms in the recent years [1]–[5]. However, the designed control system of the flexible robot arm must be able to control the motion of the rigid-body mode of the arm and to suppress the vibration modes of the arm due to the flexibility of the flexible robot arm [1]. Therefore, complex modeling procedures and complicated control techniques are usually required [2], [3]. These model-based control systems, originally designed for the demands of high performance, may not be so easy to implement in flexible arm control practice. It is due to uncertainties in design models and large variations of loads at robot hand, to the ignored high-frequency dynamics, and to the high order of the designed control system. Although many modern control techniques [6] have been designed to overcome the mentioned difficulties by using complex control laws, these techniques require an exact knowledge of the nonlinear terms, knowledge of bounds on uncertainties, or knowledge of a nonlinear regression matrix of robot functions. In practice, it is very difficult to have such a priori knowledge of the arm dynamics.

In recent years the fuzzy-neural-network (FNN)-based control techniques, which combine the capability of fuzzy reasoning in handling uncertain information [7]–[9] and the capability of artificial neural networks in learning from processes [10]–[12], have represented an alternative method to deal with uncertainties of the control system [13]–[15]. The characteristics of fault-tolerance, parallelism, easy understandability and learning suggest that they may be good candidates for implementing real-time adaptive control for nonlinear dynamical systems. It has been proven that FNN can approximate a wide range of nonlinear functions to any desired degree of accuracy under certain condition [15]. It is generally understood that the selection of the FNN-training algorithm plays an important role for most neural network applications. In the conventional gradient-decent-type approach, the sensitivity of the controlled system should be required in the online training process [13], [14]. However, it is difficult to acquire for the unknown system dynamics or highly nonlinear dynamics. In addition, the local minimum of the performance index remains to be challenged [15]. In practical control applications, it is desirable to have systematic method of ensuring the stability, robustness, and performance properties of the overall system. Recently, the adaptive control schemes based on the Lyapunov stability theorem that incorporate the techniques of FNNs have also grown rapidly [16]–[19]. One main advantage of these control schemes is that the adaptive laws were derived based on the Lyapunov synthesis method, therefore, guarantee the stability of the control system. However, some constrained conditions should be assumed in the control process, e.g., the approximation error, optimal parameter vectors or higher order terms in Taylor series are bounded [16]–[18]. In addition, the prior knowledge of the controlled system may be required, e.g., the external disturbance is bounded or all states of the controlled system are measurable [19]. These requirements are not easy to satisfy in practical control applications. The motivation of this study is to design a FNN control scheme not only guarantee the stability of the controlled system but also the constrained conditions and prior knowledge of the...
controlled system are not necessary in the design process such that it can easily extend to other nonlinear mechanism. On the other hand, the optimal control technique is one of the effective nonlinear robust control approaches since it provides system dynamics with an invariance property to uncertainties [20], [21]. Therefore, a FNN control scheme based on optimal control technique is proposed in this study to accomplish the mentioned motivation.

This paper is organized as follows. Section II presents the dynamic model of the flexible robot arm system with a tip mass briefly. In Section III, an intelligent optimal control system is proposed to control the motor-mechanism coupling system for periodic motion. The design procedures of the proposed intelligent optimal control system are described in detail. The control laws of the intelligent optimal control system are derived in the sense of optimal control technique and Lyapunov stability analysis, so that system-tracking stability can be guaranteed in the closed-loop system. Numerical simulation and experimental results of the nonlinear motor-mechanism coupling system are provided to demonstrate the robust control performance and learning ability of the proposed intelligent optimal control system in Section IV. Conclusions are drawn in Section V. The merits of the proposed control scheme are that not only the stability of the controlled system can be guaranteed

Fig. 1. (a) Field-oriented PM synchronous servo motor drive system. (b) Simplified position control system.
but also the strict constrained conditions and prior knowledge of the controlled plant are not necessary in the design process such that it can easily extend to other nonlinear mechanism.

II. DYNAMIC ANALYSIS

A. Field-Oriented Permanent-Magnet (PM) Synchronous Motor Drive

The configuration of a general field-oriented PM synchronous motor drive system is depicted in Fig. 1(a) [22], which consists of a PM synchronous motor coupled with a mechanism, a ramp comparison current-controlled pulsedwidth-modulation (PWM) voltage-source inverter (VSI), a unit vector generator (where \( \theta_r \) is the position of rotor flux), a coordinate translator, a speed control loop, and a position control loop. The PM synchronous motor used in this drive system is a three-phase four-pole 750-W 3.47-A 3000-r/min type. The current-controlled VSI is implemented by insulated gate bipolar transistor (IGBT) switching components with a switching frequency of 5 kHz.

With the implementation of the field-oriented control, the PM synchronous servo motor drive system can be simplified to a control system block diagram, as shown in Fig. 1(b) [22], in which

\[
\begin{align*}
\tau_e &= K_T i_q^* \\
K_T &= \frac{3}{2} P_l L_{md} I_{fd} \\
H_p(s) &= \frac{1}{(J_m s + B_m)} \\
\tau_e &= \tau_m + B_m \dot{\theta}_r + J_m \ddot{\theta}_r
\end{align*}
\]

where \( \tau_e \) is the electric torque; \( K_T \) is the torque constant; \( i_q^* \) is the torque current command; \( P_l \) is the number of pole pairs; \( L_{md} \) is the \( d \)-axis mutual inductance; \( I_{fd} \) is the equivalent \( d \)-axis magnetizing current; \( s \) is the Laplace operator; \( \tau_m \) is the load torque; \( B_m \) is the torsional damping coefficient; \( J_m \) is the inertia of rotor and gear; and \( \theta_r \) is the rotor position. Moreover, in Fig. 1(b) \( \theta_r^* \) and \( \omega_r^* \) are the rotor position and speed commands; \( \omega_r \) is the rotor speed.

B. Mathematical Model of Motor-Mechanism Coupling System

Fig. 2 shows a PM synchronous servo motor system, which is applied to a flexible robot arm, including a geared speed reducer with a gear ratio

\[
g_r = \frac{n_a}{n_b} = \frac{\tau_a}{\tau_m} = \frac{\dot{\theta}_r}{\dot{\theta}}
\]

where \( g_r \) is the gear ratio; \( \dot{\theta} \) is the rotation angle of the flexible arm; and \( n_a \) and \( n_b \) are the gear numbers. Substituting (1) and (5) into (4), the following applied torque can be obtained:

\[
\tau_a = g_r (\tau_e - J_m \dot{\theta}_r - B_m \dot{\theta}_r) = g_r (K_T i_q^* - g_r J_m \dot{\theta} - g_r B_m \ddot{\theta})
\]

where \( \tau_a \) is the applying torque.

Fig. 3 [23] represents a slender flexible arm rotating in a horizontal plane. The physical model is similar to that in [24]. The flexible arm of length \( L \), cross-sectional area \( A \), and uniform mass per unit length \( \rho \), is clamped on a vertical shaft of a PM synchronous servo motor at one end, and has a tip mass attached at the free end. When the tip mass of the flexible arm is a rigid body, not only bending vibration but also torsional vibration will occur. In Fig. 3, \((XYZ)\) designate inertial Cartesian coordinate axes, where \( X \) and \( Y \) axes span a horizontal plane, and \( Z \) axis is taken so that it coincides with the vertical rotation shaft of the motor; \((xyz)\) denotes a rotating coordinate; \( H \) denotes the mass center of the rigid tip mass; \( P \) denotes the intersection of the arm’s tip tangent with a perpendicular plane passing through the mass center \( H \); and \( c \) denotes a small distance between the arm’s tip point and the point \( P \). It is assumed that the points \( P \) and \( H \) never coincide and lie on the same vertical line in the equilibrium state. Moreover, \( e \) denotes the distance between \( P \) and \( H \); \( R_M \) and \( R_H \) represent the position vector of point \( A' \) and \( H \).

Now, \( v(x, t) \) and \( \phi(x, t) \) denote the transverse displacement of the arm in the rotating frame and the angle of twist of the arm, respectively, with any position \( x \) \((0 < x < L)\) at time \( t \). Hamilton’s principle and integral by parts are adopted to derive the nonlinear govern equations of transverse vibration, torsional vibration and rigid-body motion in the Appendix [23]. Substituting (6) into (A17), the dynamic motion equation of the motor-mechanism coupling system can be represented as

\[
\begin{align*}
\ddot{\theta} \left( I_b + g_r^2 J_m + \rho A \int_0^L v^2 dx + \rho I \int_0^L \phi^2 dx \right) + g_r^2 B_m \dot{\theta} + 2 \dot{\theta} \rho A \int_0^L v dx + \rho A \int_0^L \ddot{\phi} dx \\
+ 2 \dot{\theta} \rho I \int_0^L \phi dx + m \ddot{v}(L, t) + c^2 v_x(L, t) + c^2 \phi^2(L, t) + 2 v_x(L, t) \dot{v}(L, t) + 2 \dot{\phi}(L, t) \phi(L, t) \\
- 2 c v_x(L, t) \phi(L, t) \\
+ 2 \ddot{v}(L, t) v_x(L, t) + c^2 v_x(L, t) v_x(L, t) + c^2 \phi(L, t) \phi(L, t) + c v_x(L, t) \phi(L, t) + c \dot{v}(L, t) \phi(L, t) \\
- c \phi(L, t) \phi(L, t) - c c v_x(L, t) \phi(L, t) - c c \dot{v}(L, t) \phi(L, t) \\
+ (L + c) \ddot{\theta} \right) = g_r K_T \theta^*_q.
\end{align*}
\]
Rearranging (7), the dynamic equation can be rewritten as
\[
M(X; t)\ddot{\theta} + D(X; t)\dot{\theta} + F(X; t) = BU(t)
\]  
where \( X = [v \phi]^T; B = g_r K_T > 0 \) is nonsingular; \( U(t) = i_q^* \) is the control input; and

\[
M(X; t) = I_b + \rho A \int_0^L v^2 dx + \rho I \int_0^L \phi^2 dx + g_r^2 J_m + m \left\{ v^2(L, t) + \phi^2(L, t) + 2v \phi v_L(t) + 2v \phi \phi_L(t) \right\} + 2c_h \phi v_L(t) v_L(t) - 2c_h \phi \phi_L(t) \phi_L(t) - 2c_e \phi \phi_L(t) \phi_L(t) + (L + c)^2 \}
\]

\[
D(X; t) = g_r^2 B_m + 2\rho A \int_0^L v \phi dx + 2\rho I \int_0^L \phi \phi dx + 2m \left\{ v(L, t) \phi(L, t) + \phi v_L(t) \phi_L(t) + v \phi v_L(t) + \phi \phi_L(t) \phi_L(t) \right\} + c_h \phi v_L(t) v_L(t) + c_h \phi \phi_L(t) v_L(t) - c_h \phi \phi_L(t) \phi_L(t) - c_e \phi \phi_L(t) \phi_L(t)
\]

\[
F(X; t) = \rho A \int_0^L x v dx + m(L + c) \cdot \left\{ \dot{v}(L, t) + \phi_L(t) \phi_L(t) \right\}.
\]  

Since all elements of \( M(X; t) \) shown in (9) are positive values, it can be obtained that \( M(X; t) > 0 \) is nonsingular. In this paper, the vibration state of the flexible arm is assumed to be unmeasurable. When only the rotation angle of the flexible robot arm is measurable, an intelligent optimal control system is proposed to control the nonlinear motor-mechanism coupling system in the following section.

III. INTELLIGENT OPTIMAL CONTROL SYSTEM

In order to control the position of the flexible robot arm effectively, an intelligent optimal control system is proposed in this section. The configuration of the proposed intelligent optimal control system is depicted in Fig. 4, in which the reference model is chosen according to the prescribed time-domain control specifications. The control problem is to find a control law so that the state \( \theta(t) \) can track the desired command. To achieve the control objective, define the tracking error \( e_{\theta}(t) = \theta_m(t) - \theta(t) \), in which \( \theta_m(t) \) represents the reference trajectory specified by a reference model. Reformulating (8), the dynamic equation of the motor-mechanism coupling system can be represented as follows:

\[
U(t) = f(X, t) \ddot{\theta}(t) + G(X, t) \dot{\theta}(t) + d(X, t)
\]

where \( f(X, t) = B^{-1} M(X, t) > 0; G(X, t) = B^{-1} D(X, t) \) and \( d(X, t) = B^{-1} F(X, t) \). Now, assume that all the states and parameters of the system are well known, and rewrite (12) as

\[
U(t) = f_n(X, t) \ddot{\theta}(t) + G_n(X, t) \dot{\theta}(t) + d_n(X, t)
\]

where \( f_n(X, t), G_n(X, t) \) and \( d_n(X, t) \) are the nominal values of \( f(X, t), G(X, t) \) and \( d(X, t) \). If the uncertainties occur, the dynamic equation of the coupling systems can be modified as

\[
U(t) = (f_n(X, t) + \Delta f) \ddot{\theta}(t) + (G_n(X, t) + \Delta G) \dot{\theta}(t) + (d_n(X, t) + \Delta d)
\]

where \( \Delta f, \Delta G \) and \( \Delta d \) denote the uncertainties; \( W(X, t) \) is called the lumped uncertainty and defined as

\[
W(X, t) = \Delta f \ddot{\theta}(t) + \Delta G \dot{\theta}(t) + \Delta d
\]

Now, a filtered-tracking error is defined as

\[
r(t) = \left( \frac{d}{dt} + \lambda \right)^2 \int_0^t e_{\theta}(\tau) d\tau = e_{\theta}(t) + 2\lambda e_{\theta}(t) + \lambda^2 \int_0^t e_{\theta}(\tau) d\tau
\]

where \( \lambda > 0 \) is a positive constant. The filtered-tracking error (16) is a proportional–integral–derivative (PID)-type performance measure. For simplicity, only one tuning parameter \( \lambda \) is introduced in (16) for reducing the complexity of the selection of control parameters in practical applications. Differentiating \( r(t) \) with respect to time, it can be obtained that

\[
\ddot{r}(t) = -r(t) + \ddot{\theta}_m(t) + 2\lambda \dot{e}_{\theta}(t) + \lambda^2 e_{\theta}(t).
\]
According to (17), the motor-mechanism coupling system shown in (14) can be represented as

\[
f_n(X, t) \ddot{r}(t) = -G_n(X, t) r(t) + h(X, t) - U(t)
\]

where the nonlinear function \( h(X, t) \) is defined as

\[
h(X, t) = f_n(X, t)[\ddot{\theta}_n(t) + 2 \lambda \dot{\theta}_n(t) + \lambda^2 \theta_n(t)] + G_n(X, t) \left[ \dot{\theta}_n(t) + 2 \lambda \dot{\theta}_n(t) + \lambda^2 \int_0^t \dot{\theta}_n(t) dt \right] + d_n(X, t) + W(X, t).
\]

(19)

A. Optimal Control Design

In the optimal control design, the optimal control law is assumed to take the following form [11]:

\[
U(t) = h(X, t) - u(t)
\]

(20)
in which \( u(t) \) is an auxiliary control input to be optimized. Substituting (20) into (18), the closed-loop system becomes

\[
f_n(X, t) \ddot{r}(t) = -G_n(X, t) r(t) + u(t).
\]

(21)

For optimizing the auxiliary control input, a new error state is defined as follows:

\[
\tilde{z}(t) = \left[ 0 \quad 1 \quad 0 \right]^T.
\]

(22)

Differentiating \( \tilde{z}(t) \) with respect to time, the following error dynamic equation using (16) and (22) can be obtained:

\[
\dot{\tilde{z}}(t) = \begin{bmatrix}
  0 & 1 & 0 \\
  -\lambda^2 & -2\lambda & 1 \\
  0 & 0 & -\int_n^{-1}(X, t)G_n(X, t)
\end{bmatrix} \begin{bmatrix}
  \dot{\theta}_n(t) \\
  \dot{\theta}_n(t) \\
  r(t)
\end{bmatrix} + \begin{bmatrix}
  0 \\
  0 \\
  \int_n^{-1}(X, t)
\end{bmatrix} u(t)
\]

\[
\equiv A_{u}(X, t) \tilde{z}(t) + B_{u}(X, t) u(t)
\]

(23)

where \( A_{u}(X, t) = \begin{bmatrix} 0 & 1 & 0 \\ -\lambda^2 & -2\lambda & 1 \\ 0 & 0 & -\int_n^{-1}(X, t)G_n(X, t) \end{bmatrix} \) and \( B_{u}(X, t) = \begin{bmatrix} 0 \\ 0 \\ \int_n^{-1}(X, t) \end{bmatrix} \). According to the optimal control technique [20], [21], a quadratic performance index \( J(\tilde{z}(t), t) \) is defined as

\[
J(\tilde{z}(t), t) = \Phi(\tilde{z}(t), t_f) + \int_t^{t_f} L(\tilde{z}, u) dt
\]

(24)

with the Lagrangian

\[
L(\tilde{z}, u) = \frac{1}{2} \tilde{z}(t)^T T \tilde{z}(t) + \frac{1}{2} u^T(t) Ru(t)
\]

(25)
in which \( T \in R^{3 \times 3} \) is a positive-definite matrix and \( R \) is a positive value. The control objective is to design the auxiliary control input \( u(t) \) that minimizes the performance index \( J(\tilde{z}(t), t) \) according to the error dynamic equation shown in (23). The minimum performance index \( J^*(\tilde{z}(t), t) \) is defined as

\[
J^*(\tilde{z}(t), t) = \min_u \left\{ \Phi(\tilde{z}(t), t_f) + \int_t^{t_f} L(\tilde{z}, u) dt \right\}
\]

(26)

Clearly, in the time \( t = t_f \), \( J^*(\tilde{z}(t), t) \) must satisfy the boundary condition

\[
J^*(\tilde{z}(t), t_f) = \Phi(\tilde{z}(t_f), t_f).
\]

(27)

Suppose that \( t \) represents the current time and \( t + \Delta t \) denotes a future time close to \( t \), then (26) can be rewritten as

\[
J^*(\tilde{z}(t), t) = \min_u \left\{ \int_t^{t+\Delta t} L(\tilde{z}, u) dt + \min_{u+\Delta t} \left[ \Phi(\tilde{z}(t), t_f) + \int_t^{t_f} L(\tilde{z}, u) dt \right] \right\}
\]

\[
\cong \min_u \left\{ L(\tilde{z}, u) \Delta t + J^*(\tilde{z}(t+\Delta t), t+\Delta t) \right\}
\]

(28)
Expanding $J^*(\z(t + \Delta t), t + \Delta t)$ by Taylor series, (28) can be represented as

$$J^*(\z(t), t) = \min_u \{ J(\z, u)\Delta t + J^*(\z(t), t)$

$$+ \left[ \frac{\partial J^*(\z(t), t)}{\partial \z} \Delta t + \left[ \frac{\partial J^*(\z(t), t)}{\partial \z} \right] \Delta t + O_{\Delta t} \right \} \tag{29}$$

where $O_{\Delta t}$ is a high-order term and is neglected in the following derivation. Rewriting (29), the Hamilton–Jacobi equation $H(\z, u, \partial J^*(\z(t), t)/\partial u, t)$ can be obtained as [20], [21]

$$- \frac{\partial J^*(\z(t), t)}{\partial t} = L(\z, u) + \frac{\partial J^*(\z(t), t)}{\partial \z} \z \equiv H(\z, u, \frac{\partial J^*(\z(t), t)}{\partial t}, t). \tag{30}$$

It implies that $J^*(\z, t)$ must satisfy the Hamilton–Jacobi equation shown in (30). According to (27) and (30), the value function $\Phi(\z, t)$ is defined as

$$\Phi(\z, t) = J^*(\z, t) = \frac{1}{2} \z(t)^T P(X, t) \z(t)$$

$$= \frac{1}{2} \z(t)^T \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & f_n(X, t) \end{bmatrix} \z(t) \tag{31}$$

where $K_1$ and $K_2$ are real value. Differentiating $J^*(\z, t)$ with respect to $t$, then

$$\frac{dJ^*(\z, t)}{dt} = \frac{\partial J^*(\z(t), t)}{\partial t} + \frac{\partial J^*(\z(t), t)}{\partial \z} \z. \tag{32}$$

Substituting (23), (25), and (31) into (30), the following equation can be obtained:

$$H(\z, u, \frac{\partial J^*(\z(t), t)}{\partial t}, t)$$

$$= \frac{\partial J^*(\z(t), t)}{\partial \z} \z + L(\z, u)$$

$$= \z(t)^T P A_n \z + P B_n u + \frac{1}{2} \z(t)^T T_2 \z + \frac{1}{2} u^T R u. \tag{33}$$

According to $\partial H/\partial t = 0$, the optimal auxiliary control input $u^*(t)$ can be derived as

$$u^*(t) = - R^{-1} B_n^T P \z. \tag{34}$$

Substituting (32) into (30) and using (31), one can obtain

$$\frac{1}{2} \z(t)^T P \z + \z(t)^T P \z + L(\z, u) + 0, \tag{35}$$

Inserting (23), (25), and (34) into (35) gives

$$\frac{1}{2} \z(t)^T P \z + \z(t)^T P [A_n \z + B_n u^*] + \left[ \frac{1}{2} \z(t)^T T_2 \z + \frac{1}{2} u^T R u \right] = 0. \tag{36}$$

Substituting (34) into (36), it can be concluded that

$$\frac{1}{2} \z(t)^T [P + 2P A_n - P B_n R^{-1} B_n^T P + T] \z = 0. \tag{37}$$

Therefore, the Riccati differential equation can be derived as [21]

$$2PA_n - PB_n R^{-1} B_n^T P + \dot{P} + T = 0_{3 \times 3}. \tag{38}$$

Remark 1: Generally speaking, the linear quadratic method is an easy way to design a suitable control law for achieving the optimal performance requirement [21]. To find this control law, the Riccati equation must be solved, then an optimal gain will be obtained naturally and it will lead to a minimum performance index. However, once the uncertainties occur, the control performance may be destroyed. Thus, introducing PID-type control into the linear quadratic design is aimed to reduce the effect induced by unpredictable uncertainties.

B. Intelligent Control Design

From (19), the nonlinear function $h(X, t)$ contains the system nominal parameters ($f_n(X, t)$, $G_n(X, t)$ and $d_n(X, t)$) and the lumped uncertainty $W(X, t)$. Since the system state $X$ is assumed to be unmeasured and the lumped uncertainty $W(X, t)$ is unknown in practical applications, the optimal control law shown in (20) can not be implemented practically. Therefore, an intelligent controller is proposed to approximate the nonlinear function $h(X, t)$; moreover, the intelligent optimal control system is designed as

$$U(t) = \hat{h}(X, t) - u^*(t) \tag{39}$$

where the intelligent controller $\hat{h}(X, t)$ is used to learn the nonlinear function $h(X, t)$ and defined as

$$\hat{h}(X, t) = \hat{h}_{FNN} + U_n \tag{40}$$

in which $\hat{h}_{FNN}$ is a FNN controller and $U_n$ is a robust controller. The proposition of the FNN control $\hat{h}_{FNN}$ is used to learn the nonlinear equation due to the uncertain system dynamics, and the robust control $U_n$ is designed to compensate the difference between $h(X, t)$ and $\hat{h}_{FNN}$.

A four-layer FNN as shown in Fig. 5 [13], [14], [19], which comprises the input (the $i$ layer), membership (the $j$ layer), rule (the $k$ layer) and output layer (the $o$ layer), is adopted to implement the FNN controller in this paper. The inputs of the FNN are $r$ and $r(1 - z^{-1})$, in which $z^{-1}$ is a time delay; the output of
Fig. 6. Simulated results of intelligent optimal control system due to periodic step command. (a) Tracking response at Case 1. (b) Tracking error at Case 1. (c) Control effort at Case 1. (d) FNN control at Case 1. (e) Bending vibration at Case 1. (f) Torsional vibration at Case 1.
Fig. 6. (Continued.) Simulated results of intelligent optimal control system due to periodic step command. (g) Tracking response at Case 2. (h) Tracking error at Case 2. (i) Control effort at Case 2. (j) FNN control at Case 2. (k) Bending vibration at Case 2. (l) Torsional vibration at Case 2.
Fig. 7. Simulated results of intelligent optimal control system due to periodic sinusoidal command. (a) Tracking response at Case 1. (b) Tracking error at Case 1. (c) Control effort at Case 1. (d) FNN control at Case 1. (e) Bending vibration at Case 1. (f) Torsional vibration at Case 1.
Fig. 7. (Continued.) Simulated results of intelligent optimal control system due to periodic sinusoidal command. (g) Tracking response at Case 2. (h) Tracking error at Case 2. (i) Control effort at Case 2. (j) FNN control at Case 2. (k) Bending vibration at Case 2. (l) Torsional vibration at Case 2.
Fig. 8. Simulated results of optimal control system due to periodic step command. (a) Tracking response at Case 1. (b) Tracking error at Case 1. (c) Control effort at Case 1. (d) Tracking response at Case 2. (e) Tracking error at Case 2. (f) Control effort at Case 2.
Fig. 9. Simulated results of optimal control system due to periodic sinusoidal command. (a) Tracking response at Case 1. (b) Tracking error at Case 1. (c) Control effort at Case 1. (d) Tracking response at Case 2. (e) Tracking error at Case 2. (f) Control effort at Case 2.
the FNN is $h_{FNN}$. The signal propagation and the basic function in each layer are introduced in the following paragraph.

For every node $i$ in the input layer, the net input and the net output are represented as

$$net_i^1 = x_i^1, \quad y_i^1 = f_i^1(net_i^1) = net_i^1, \quad i = 1, 2 \quad (41)$$

where $x_i^1 = r$ and $x_2^1 = r(1 - x_i^{-1})$. Moreover, each node performs a membership function in the membership layer. The Gaussian function is adopted as the membership function. For the $j$th node

$$net_j^2 = -\frac{(x_j^2 - m_{ij})^2}{(\sigma_{ij})^2} \quad y_j^2 = f_j^1(net_j^2) = \exp(net_j^2), \quad j = 1, \ldots, n \quad (42)$$

where $m_{ij}$ and $\sigma_{ij}$ are, respectively, the mean and the standard deviation of the Gaussian function in the $j$th term of the $i$th input linguistic variable $x_j^2$ to the node of membership layer, and $n$ is the total number of the linguistic variables with respect to the input nodes. In addition, each node $k$ in the rule layer is denoted by $\prod$, which multiplies the input signals and outputs the result of the product. For the $k$th rule node

$$net_k^3 = \prod_j w_{jk}^3 x_j^3, \quad y_k^3 = f_k^3(net_k^3) = net_k^3, \quad k = 1, \ldots, l \quad (43)$$

where $x_j^3$ represents the $j$th input to the node of rule layer; $w_{jk}^3$, the weights between the membership layer and the rule layer, are assumed to be unity; $l = (n/i)^i$ is the number of rules with complete rule connection if each input node has the same linguistic variables. Furthermore, the single node $o$ in the output layer is labeled as $\Sigma$, which computes the overall output as the summation of all input signals

$$net_o^4 = \sum_k w_{ko}^4 x_k^4, \quad y_o^4 = f_o^4(net_o^4) = net_o^4, \quad o = 1 \quad (44)$$

where the connecting weight $w_{ko}^4$ is the output action strength of the $o$th output associated with the $k$th rule; $x_k^4$ represents the $k$th input to the node of output layer, and $y_o^4 = h_{FNN}$. For ease of notation, define vector $m$ and $S$ collecting all mean and standard deviation vectors of Gaussian functions as

$$m = [m_{11} m_{12} \cdots m_{2n}]^T \quad (45)$$

$$S = [\sigma_{11} \sigma_{12} \cdots \sigma_{2n}]^T. \quad (46)$$

Moreover, the output of a FNN can be represented as [13]

$$h_{FNN}(r,W, m, S) = W\Gamma \quad (47)$$

where $W = [w_{11}^4 \ w_{21}^4 \cdots \ w_{l1}^4]$ and $\Gamma = [x_1^4 \ x_2^4 \cdots x_l^4]^T$, in which $x_k^4$ is determined by the selected membership functions and $0 \le x_k^4 \le 1$. Thus, an optimal FNN controller $h_{FNN}^*$ is designed to learn the nonlinear function $h(X, t)$ such that

$$h(X, t) = h_{FNN}^*(r, W^*, m^*, S^*) + \varepsilon \equiv W^*\Gamma^* + \varepsilon \quad (48)$$

where $\varepsilon$ is a minimum reconstructed error; $W^*$, $m^*$ and $S^*$ are optimal parameters of $W$, $m$ and $S$ in the FNN. If there exist $l$ (the number of rules) and constant optimal parameters $(W^*, m^*, S^*)$ so that $\varepsilon = 0$ for all $X$, it can say that $h(X, t)$ is in the functional range of the FNN [15]. In general, given a constant real number $\varepsilon_b$, one can obtain that $h'(X, t)$ is within $\varepsilon_b$ of the

![Computer control system for single-link flexible robot arm.](image-url)
and constant parameters so that (48) holds with $|e| < \varepsilon_t$ for all $X$. Rewriting (40), one can obtain [9]

$$\hat{h}(X_1, t) = \hat{h}_{FNN}(r, \hat{W}, \hat{m}, \hat{S}) + U_s = \hat{W}T + U_s \quad (49)$$

where $\hat{W}$, $\hat{m}$ and $\hat{S}$ are some estimates of the optimal parameters, as provided by tuning algorithms to be introduced. Subtracting (48) from (49), an approximation error $\hat{h}$ is defined as

$$\hat{h} = h - \hat{h} = W^* \Gamma^* + e - \hat{W}T - U_s = \hat{W}T + e - U_s \quad (50)$$

where $\hat{W} = W^* - W$; $\hat{T} = \Gamma^* - \bar{T}$, in which $\Gamma^*$ is an optimal parameter of $\Gamma$, and $\bar{T}$ is the estimated value of the optimal parameter $\Gamma^*$ in the FNN. In this study, a control methodology is proposed to guarantee the closed-loop stability and the tracking performance. The linearization technique is employed to transform the membership functions into partially linear form so that the expansion of $\hat{T}$ in Taylor series to obtain

$$\hat{T} = \left[ \begin{array}{c} 2 \bar{x}_{1}^4 \\ 2 \bar{x}_{2}^2 \\ \vdots \\ 2 \bar{x}_{l}^4 \end{array} \right] = \left[ \begin{array}{cccc} \frac{\partial x_1^4}{\partial \bar{m}} & \frac{\partial x_2^4}{\partial \bar{m}} & \cdots & \frac{\partial x_l^4}{\partial \bar{m}} \end{array} \right]^T \bar{m}$$

$$= \Gamma_m \hat{m} + \Gamma_s \hat{S} + \bar{O}_m \quad (51)$$

where $\hat{m} = m^* - \bar{m}$; $\hat{S} = S^* - \bar{S}$; $\bar{O}_m$ is a vector of higher order terms: $\Gamma_m = \left[ \frac{\partial x_1^4}{\partial \bar{m}} \frac{\partial x_2^4}{\partial \bar{m}} \cdots \frac{\partial x_l^4}{\partial \bar{m}} \right]^T$.

Fig. 11. Experimental results of intelligent optimal control system due to periodic step command. (a) Tracking response at nominal condition. (b) Tracking error at nominal condition. (c) Control effort at nominal condition. (d) Tracking response at parameter variation condition. (e) Tracking error at parameter variation condition. (f) Control effort at parameter variation condition.
Substituting (54) into (50), it is revealed that

\[ \Gamma_{\delta} \equiv \left[ \frac{\partial x_{1}^{4}}{\partial S} \ \frac{\partial x_{2}^{4}}{\partial S} \ \cdots \ \frac{\partial x_{k}^{4}}{\partial S} \right]^T; \ \frac{\partial x_{k}^{4}}{\partial m} \] and

\[ \frac{\partial x_{k}^{4}}{\partial S} = \left[ \frac{\partial x_{1}^{4}}{\partial m_{11}} \ \frac{\partial x_{1}^{4}}{\partial m_{12}} \ \cdots \ \frac{\partial x_{1}^{4}}{\partial m_{n1}} \right]^T \ (52) \]

\[ \frac{\partial x_{k}^{4}}{\partial S} = \left[ \frac{\partial x_{2}^{4}}{\partial m_{11}} \ \frac{\partial x_{2}^{4}}{\partial m_{12}} \ \cdots \ \frac{\partial x_{2}^{4}}{\partial m_{n2}} \right]^T \ (53) \]

Rewriting (51), it can be obtained that

\[ \Gamma^* = \Gamma + \Gamma_{\delta} \tilde{m} + \Gamma_{\delta} \tilde{S} + O_{nv} \] (54)

Substituting (54) into (50), it is revealed that

\[ \tilde{h} = W^* \Gamma^* + \varepsilon - \tilde{W} \Gamma - U_{s} \]

\[ = W^* [\tilde{\Gamma} + \Gamma_{\delta} \tilde{m} + \Gamma_{\delta} \tilde{S} + O_{nv}] + \varepsilon - \tilde{W} \tilde{\Gamma} - U_{s} \]

\[ = (W^* - \tilde{W}) \tilde{\Gamma} + (\tilde{W} + \tilde{W}) \Gamma_{\delta} \tilde{m} \]

\[ + (\tilde{W} + \tilde{W}) \Gamma_{\delta} \tilde{S} + \varepsilon - U_{s} + W^* O_{nv} \]

\[ = \tilde{W} \tilde{\Gamma} + WT_{\delta} \tilde{m} + WT_{\delta} \tilde{S} - U_{s} \]

\[ + \tilde{W} T_{\delta} \tilde{m} + \tilde{W} T_{\delta} \tilde{S} + W^* O_{nv} + \varepsilon \]

\[ = \tilde{W} \tilde{\Gamma} + WT_{\delta} \tilde{m} + WT_{\delta} \tilde{S} - U_{s} + E \] (55)

where the uncertain term is defined as

\[ E = \tilde{W} T_{\delta} \tilde{m} + \tilde{W} T_{\delta} \tilde{S} + W^* O_{nv} + \varepsilon \]

and is assumed to be a bounded function by \(|E| < \psi\). Substituting (39), (49) and (55) into (18), one can obtain

\[ f_{\delta}(X; t) = -G_{\delta}(X, t) r(t) + h(X, t) - U(t) \]

\[ = -G_{\delta}(X, t) r(t) + \tilde{h}(X, t) + u^*(t) \]

\[ = -G_{\delta}(X, t) r(t) + \tilde{W} \tilde{\Gamma} + WT_{\delta} \tilde{m} \]

\[ + \tilde{W} T_{\delta} \tilde{S} - U_{s} + u^*(t). \] (56)
According to (21) and (56), the following equation can be obtained:

\[ u(t) = f_0(X, t) + G_n(X, t) + \eta(t) \]

Substituting (57) into (23) and using (34), the error dynamic equation can be represented as

\[ \dot{\tilde{x}}(t) = A_u(X, t) \tilde{x}(t) + B_u(X, t) u(t) \]

Substituting (57) into (23) and using (34), the error dynamic equation can be represented as

\[ \dot{\tilde{x}}(t) = A_u(X, t) \tilde{x}(t) + B_u(X, t) u(t) \]

Substituting (57) into (23) and using (34), the error dynamic equation can be represented as

\[ \dot{\tilde{x}}(t) = A_u(X, t) \tilde{x}(t) + B_u(X, t) u(t) \]

Substituting (57) into (23) and using (34), the error dynamic equation can be represented as

\[ \dot{\tilde{x}}(t) = A_u(X, t) \tilde{x}(t) + B_u(X, t) u(t) \]

Theorem 1: Consider the nonlinear motor-mechanism coupling system represented by (8), if the intelligent optimal control system is designed as (39); the optimal control law is designed as (34); the intelligent control law is designed as (49), in which the adaptation laws of the FNN controller are designed as (59)–(61) and the robust controller is designed as (62) with the adaptive bound estimation algorithm shown in (63), then asymptotic stability can be guaranteed

\[ \dot{W} = \eta_1 B_u^T P \dot{\tilde{x}} \dot{\tilde{x}}^T \]

Theorem 1: Consider the nonlinear motor-mechanism coupling system represented by (8), if the intelligent optimal control system is designed as (39); the optimal control law is designed as (34); the intelligent control law is designed as (49), in which the adaptation laws of the FNN controller are designed as (59)–(61) and the robust controller is designed as (62) with the adaptive bound estimation algorithm shown in (63), then asymptotic stability can be guaranteed

\[ \dot{W} = \eta_1 B_u^T P \dot{\tilde{x}} \dot{\tilde{x}}^T \]

Proof: Choose the Lyapunov function candidate as

\[ L_\alpha(\tilde{z}(t), \tilde{\psi}(t), \tilde{W}, \tilde{m}, \tilde{S}) = \frac{1}{2} \dot{\tilde{z}}^T(t) \tilde{P} \dot{\tilde{z}}(t) + \frac{1}{2 \eta_1} \text{tr}(\tilde{W} \tilde{W}^T) + \frac{1}{2 \eta_2} \tilde{m} \tilde{m}^T + \frac{1}{2 \eta_3} \tilde{S} \tilde{S}^T + \frac{1}{2 \eta_4} \tilde{\psi}(t)^2 \]  

where \( \eta_1, \eta_2, \eta_3 \) and \( \eta_4 \) are positive constants; \( \text{sgn}(\cdot) \) is a sign function; \( |\cdot| \) is the absolute value; \( \dot{\tilde{\psi}}(t) \) is the estimated value of the uncertain term bound \( \psi(t) \).

Substituting (58) into (65), one can obtain

\[ \dot{L}_\alpha = \frac{1}{2} \dot{\tilde{z}}^T(t) \tilde{P} \dot{\tilde{z}}(t) + \frac{1}{2} \dot{\tilde{z}}^T(t) \dot{\tilde{P}} \tilde{z}(t) - \frac{1}{\eta_1} \text{tr}(\tilde{W} \tilde{W}^T) \]

where \( \text{tr}(\cdot) \) is the trace operator and \( \tilde{\psi}(t) = \dot{\tilde{\psi}} - \dot{\tilde{\psi}}(t) \) denotes the estimated error. Differentiating (64) with respect to time, it can be obtained that

\[ \dot{L}_\alpha = \frac{1}{2} \dot{\tilde{z}}^T(t) \tilde{P} \dot{\tilde{z}}(t) + \frac{1}{2} \dot{\tilde{z}}^T(t) \dot{\tilde{P}} \tilde{z}(t) - \frac{1}{\eta_1} \text{tr}(\tilde{W} \tilde{W}^T) \]

Since \( L_\alpha(\tilde{z}(t), \tilde{\psi}(t), \tilde{W}, \tilde{m}, \tilde{S}) \leq 0 \), \( L_\alpha(\tilde{z}(t), \tilde{\psi}(t), \tilde{W}, \tilde{m}, \tilde{S}) \) is negative semi-definite, that is, \( L_\alpha(\tilde{z}(t), \tilde{\psi}(t), \tilde{W}, \tilde{m}, \tilde{S}) \leq L_\alpha(\tilde{z}(0), \tilde{\psi}(0), \tilde{W}, \tilde{m}, \tilde{S}) \), which implies \( \tilde{z}(t), \tilde{\psi}(t), \tilde{W}, \tilde{m}, \tilde{S} \) and are bounded. Let function \( \Xi(t) = \frac{1}{2} \tilde{\psi}(t) \text{sgn}(\tilde{\psi}) \leq \Xi(t) \leq 0 \), and integrate function \( \Xi(t) \) with respect to time

\[ \int_0^t \Xi(t) \, dt \leq -L_\alpha(\tilde{z}(0), \tilde{\psi}(0), \tilde{W}, \tilde{m}, \tilde{S}) \]

and

\[ -L_\alpha(\tilde{z}(0), \tilde{\psi}(0), \tilde{W}, \tilde{m}, \tilde{S}) \]
Because $L_{a}(\hat{z}(0), \hat{\psi}(0), \hat{\dot{W}}, \hat{\dot{m}}, \hat{\dot{S}})$ is bounded, and $L_{a}(\zeta(t), \psi^\prime(t), \hat{\dot{W}}, \hat{\dot{m}}, \hat{\dot{S}})$ is nonincreasing and bounded, the following result is obtained:

$$\lim_{t \to \infty} \int_{0}^{t} \Xi(\tau) \, d\tau < \infty. \quad (71)$$

Also, $\hat{\xi}(t)$ is bounded, so by Barbalat’s Lemma [25], [26], it can be shown that $\lim_{t \to \infty} \Xi(t) = 0$. That is, $\hat{\xi}(t) \to 0$ as $t \to \infty$. As a result, the intelligent optimal control system is asymptotically stable. Moreover, the tracking error of the control system, $\epsilon_{a}(t)$, will converge to zero according to $\hat{\xi}(t) \to 0$. Q.E.D.

Remark 2: The Lyapunov method is applied to derive the tuning algorithms for the parameters $(\hat{W}, \hat{\dot{m}}, \hat{\dot{S}})$ in the FNN. Since these adaptive learning algorithms are formulated from the stability analysis of the controlled system, the system performance can be guaranteed for closed-loop control. Moreover, the projection algorithm [9] also can be adopted to modify the learning algorithms shown in (59)–(61) for assuring the convergence property of the parameters in the FNN.

Remark 3: Selection of the upper bound of uncertain term $|E| < \psi$ has a significant effect on the control performance. If the bound is selected too large, the sign function of the robust controller will result in serious chattering phenomena in the control efforts. The undesired chattering control efforts will wear the bearing mechanism and might excite unstable system dynamics. On the other hand, if the bound is selected too small, the stability conditions may not be satisfied. It will cause the controlled system to be unstable. Therefore, an adaptive bound estimation algorithm shown in (63) is utilized in this paper to facilitate adaptive bound adjustment in real time for the intelligent optimal control system.

Remark 4: The motor-mechanism coupling systems in industrial manufacture are subjected to structured and/or unstructured uncertainties in practical applications. Thus, many works on optimal control, which are designed based on the rigorous mathematical model, are not suitable in the position control of the motor-mechanism coupling system [20], [21]. The intelligent optimizing feature of the proposed control scheme is suitable even without any knowledge of the system dynamics. It is emphasized that the FNN-weight values or the rules in the FNN may be initialized at zero, and stability will be maintained by the optimal auxiliary controller and the robust controller until the FNN controller learns according to the online learning algorithms. This means that there is no offline learning or trial and error phase to construct the “well-behaved” fuzzy neural network in this study.

Remark 5: The selection of rule number is not an easy task and corresponds to the usual model order determination problem. As the dimension and complexity of a system increase, the size of the rule base increases exponentially. The mapping capability of the FNN with less rule number is decreased, however, the improvement of mapping capability of the FNN with more rule number is limited and the computation burden for the CPU is significantly increased. In this paper, the robust controller designed in (62) can cope with the unsatisfactory mapping performance of the FNN due to the improper selection of rule number.

IV. NUMERICAL SIMULATION AND EXPERIMENTAL RESULTS

For numerical simulations, the parameters of the flexible robot arm are designed as follows:

$$L = 0.3 \text{ m}$$
$$b = 2.228 \times 10^{-2} \text{ m}$$
$$w = 2.36 \times 10^{-3} \text{ m}$$
$$A = 5.2581 \times 10^{-5} \text{ m}^2$$
$$c = 4.02 \times 10^{-3} \text{ m}$$
$$c = 1.498 \times 10^{-2} \text{ m}$$
$$\rho = 2700 \frac{\text{kg}}{\text{m}^3}$$
$$I_b = 9.7618 \times 10^{-11} \text{ m}^4$$
$$I = 9.7618 \times 10^{-11} \text{ m}^4$$

$$g_e = 1$$ \quad (72)

in which $b$ and $w$ are the width and height of the flexible arm. Moreover, the parameters of the motor system are

$$K_T = 0.6732 \text{ N m}$$
$$J_m = 1.32 \times 10^{-5} \text{ N m s}^{-2}/\text{rad}$$
$$B_m = 5.78 \times 10^{-3} \text{ N m s}^{-1}/\text{rad}$$ \quad (73)

In addition, the gains of the proposed intelligent optimal control system are given as

$$\lambda = 5.5$$
$$\eta_1 = 0.4$$
$$\eta_2 = 1$$
$$\eta_3 = 1$$
$$\eta_4 = 0.01$$
$$R = 0.08.$$ \quad (74)

All the gains in the control system are chosen to achieve the best transient control performance in both simulation and experimentation considering the requirement of stability. In addition, a second-order transfer function of the following form with rise time of 0.6 s is chosen as the reference model for the periodic step command:

$$\frac{u^n_m}{s^2 + 2\xi w_n s + w_n^2} = \frac{36}{s^2 + 12s + 36} \quad (75)$$

where $\xi$ and $w_n$ are the damping ratio (set at one for critical damping) and undamped natural frequency. On the other hand, when the command is a periodic sinusoidal reference trajectory, the reference model is set to be one. Furthermore, the most important parameter that affect the control performance of the motor-mechanism coupling system is the parameter variation of the tip mass, $m$. In this study, the flexible robot arm without tip mass ($m = 0 \text{ kg}$) is viewed as the nominal system, and the flexible robot arm with tip mass ($m = 0.1 \text{ kg}$) is taken as the perturbed system in the simulation and experimentation for verifying the effectiveness of the proposed intelligent optimal control system. Two cases due to periodic commands are addressed as follows:

Case 1: nominal case ($m = 0 \text{ kg}$) \quad (76)

Case 2: parameter variation case ($m = 0.1 \text{ kg}$). \quad (77)

To show the effectiveness of the FNN controller with small rule set, the FNN has two, six, nine, and one neurons at the input,
membership, rule, and output layers, respectively. It can be regarded that the associated fuzzy sets with Gaussian function for each input signal are divided into N (negative), Z (zero), and P (positive), i.e., $n = 2 \times 3 = 6$. Moreover, the rule can be represented in the form that “If $r$ is ~ and $r(1 - z^{-1})$ is ~, then $h_{FNN}$ is ~”, and the rule number is $l = (6/2)^2 = 9$. The interconnection weights in the input-to-membership and membership-to-rule layers are set to be unity to match the spirit of the fuzzy mechanism. Usually, some heuristics can be used to roughly initialize the parameters of the FNN for practical applications, e.g., the mean and standard deviation of the Gaussian functions can be determined according to the maximum variation of $r$ and $r(1 - z^{-1})$. The effect due to the inaccurate selection of the initialized parameters can be retrieved by the online training methodology. Thus, for simplicity, the means of the Gaussian functions are set at $-1, 0, 1$ for the N, Z, P neurons and the standard deviations of the Gaussian functions are set at 1.

The simulated results of the intelligent optimal control system due to periodic step command at Case 1 and Case 2 are depicted in Fig. 6. The tracking response, tracking error $e_p(t)$, control effort $U(t)$, FNN control $h_{FNN}$, bending vibration $\nu(x, t)$, and torsional vibration $\phi(x, t)$ at Case 1 are depicted in Fig. 6(a)–(f); the ones at Case 2 are depicted in Fig. 6(g)–(l). Observing the tracking errors shown in Fig. 6(b) and (h), favorable tracking responses can be obtained under all the simulated conditions. Moreover, there is no chattering phenomena existed in the control efforts owing to the online adjustment of the upper bound of uncertain term. In addition, the responses of the bending and torsional vibration displayed in Fig. 6(e), (f), (k), and (l) can be settled down. To verify the effectiveness of the proposed intelligent optimal control system with different reference trajectories, the simulated results due to periodic sinusoidal command at Case 1 and Case 2 are depicted in Fig. 7. The tracking response, tracking error, control error, FNN control, bending vibration and torsional vibration at Case 1 are depicted in Fig. 7(a)–(f); the ones at Case 2 are depicted in Fig. 7(g)–(l). From the simulated results, there is no chattering phenomena existed in the control efforts owing to the online adjustment of the upper bound of uncertain term, and perfect tracking responses can be obtained under the presence of parameter variation and different reference trajectory. It is worth noting that the transient tracking errors shown in Fig. 7 is large than the ones in Fig. 6 since the control gains of the intelligent optimal control system are chosen based on the periodic step command to achieve the best transient control performance.

In order to reveal the advantage of the proposed intelligent control, an optimal control system, i.e., the control system of Fig. 4 without FNN and robust controllers, is utilized to control the motor-mechanism coupling system for comparison. The simulated results of the optimal control system due to periodic step and sinusoidal commands are depicted in Figs. 8 and 9, respectively. The tracking response, tracking error and control effort at Case 1 are depicted in Figs. 8(a)–(c) and 9(a)–(c); the ones at Case 2 are depicted in Figs. 8(d)–(f) and 9(d)–(f). Comparing these results with Figs. 6 and 7, the absence of the intelligent control results in degenerate tracking responses. This problem may be solved by adjusting the optimal control gain, however, it cannot provide perfect control performance if the controlled plant is highly nonlinear and uncertain. Thus, the intelligent optimal control design method yields superior control performance without prior system knowledge and full state feedback.

Some experimental results are provided here to demonstrate the practicality of the proposed control system. A block diagram of the computer control system for the single-link flexible robot arm is depicted in Fig. 10. The control algorithms are realized in a Pentium computer with 2-ms sampling interval. Two conditions are tested here; one is the nominal condition, that is without a tip mass, and the other is the parameter variation condition, that is with a 0.1-kg tip mass. Fig. 11 depicts the experimental results of the intelligent optimal control system due to periodic step command at the nominal and parameter variation conditions. The tracking response, tracking error $e_p(t)$, and control effort at the nominal condition are depicted in Fig. 11(a)–(c); the ones at the parameter variation condition are depicted in Fig. 11(d)–(f). From the experimental results, good tracking responses can be obtained after one cycle of online training mechanism due to zero initial weights in the rule-to-output layer. Moreover, the chattering phenomena in the control efforts are much reduced owing to the online adjustment of the upper bound of uncertain term. To further examine the validity of the proposed intelligent optimal control system with different reference trajectories, the experimental results of the tracking response, tracking error and control effort due to periodic sinusoidal command at the nominal and parameter variation conditions are depicted in Fig. 12. From the experimental results, robust control performance can be obtained under the occurrence of parameter variation and different reference trajectory; moreover, the chattering phenomena are much reduced in the control efforts according to the online adjustment of the bound value in the robust controller.

V. Conclusion

This paper has successfully demonstrated the application of an intelligent optimal control system to the position control of a nonlinear mechanism system. The nonlinear mechanism used in this study is a flexible robot arm driven by a PM synchronous motor drive. The proposition of the control scheme is to achieve good tracking performance without the strict constraints and prior knowledge of the controlled system. The design procedure and the theoretical stability proof of the proposed intelligent optimal control system were described in detail. Moreover, simulation and experimentation were carried out using periodic reference trajectories to verify the effectiveness of the proposed control system. The major contributions of this paper are: 1) the successful development of an intelligent optimal control system, in which the proposed controller was made of an optimal controller which minimize a quadratic performance index, a FNN controller which learn a nonlinear function to implement the optimal controller, and a robust controller which compensate the approximation error of the FNN controller; 2) an adaptive bound estimation algorithm was proposed to estimate the bound of uncertain term avoiding the chattering phenomena; and 3) the successful application of the intelligent optimal control system to control the motor-mechanism coupling system considering the possible occurrence of uncertainties.
APPENDIX

In this Appendix, Hamilton’s principle is used to derive the nonlinear dynamic equations of the motor-mechanism coupling system. From Fig. 3, the displacement fields of the flexible arm are

\[
\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}
\]

(A1)

\[
\mathbf{u} = -yv(x,t)\mathbf{i} + v(x,t)\mathbf{j} + \phi(x,t)\mathbf{k}
\]

(A2)

where \(\mathbf{r}\) is the initial position vector of point \(A\); \(\mathbf{u}\) is the deviated vector from initial position; \(yv(x,t) = \partial v(x,t)/\partial x\) and \(\mathbf{i}, \mathbf{j}, \mathbf{k}\) are orthogonal units of a coordinate system that is rotating with angular velocity \(\omega(t) = \dot{\theta}(t)\). Then, the position vector of any point \(A'\) on the flexible arm after deformation can be represented as

\[
\mathbf{R}_{A'}(x,y,z,t) = \mathbf{R} + \mathbf{u} = [x - yv(x,t)]\mathbf{i} + [y + v(x,t) - \phi(x,t)]\mathbf{j} + [z + \phi(x,t)]\mathbf{k}.
\]

(A3)

The derivative of \(\mathbf{R}_{A'}\) with respect to time is

\[
\frac{d}{dt} \mathbf{R}_{A'} = \frac{d\mathbf{R}}{dt} + \omega \times \mathbf{R}_{A'}
\]

where\(\mathbf{r}\) is the initial position vector of point \(A\) and \(\mathbf{u}\) is the deviated vector from initial position. Then, the position vector of point \(H\) can be expressed as

\[
\mathbf{R}_H = (L + c)\mathbf{i} + c\mathbf{k}
\]

(A5)

\[
\mathbf{u}_H = [v(L,t) + cv_x(L,t)]\mathbf{j} + \phi(L,t)\mathbf{k}
\]

(A6)

where \(\mathbf{r}_H\) is the initial position vector of point \(H\) and \(\mathbf{u}_H\) is the deviated vector from initial position. Then, the position vector of point \(H\) can be expressed as

\[
\mathbf{R}_H(x,y,z,t) = \mathbf{R}_H + \mathbf{u}_H = (L + c)\mathbf{i} + yv(x,t)\mathbf{j} + \phi(x,t)\mathbf{k}
\]

(A7)

The derivative of \(\mathbf{R}_H\) with respect to time is

\[
\frac{d}{dt} \mathbf{R}_H = \frac{d\mathbf{R}_H}{dt} + \omega \times \mathbf{R}_H
\]

where \(\mathbf{r}\) is the initial position vector of point \(A\) and \(\mathbf{u}\) is the deviated vector from initial position. Then, the position vector of point \(H\) can be expressed as

\[
\frac{d}{dt} \mathbf{R}_H = \frac{d\mathbf{R}_H}{dt} + \omega \times \mathbf{R}_H
\]

(A8)

Thus, the total kinetic energy of the arm plus the tip body can be represented in the following form:

\[
T_K = \frac{1}{2} \int_0^L \rho A \left\{ \frac{d}{dt} \mathbf{R}_H \cdot \frac{d}{dt} \mathbf{R}_H \right\} dV
\]

(A9)

where \(V\) is volume per unit length and \(m\) is the attached rigid tip mass. The potential energy \(U_e\) including nonlinear terms has the following form:

\[
U_e = \frac{1}{2} \int_0^L \left[ EI v_{xx}^2 + GJ \phi_x^2 + \frac{1}{4} EA v_A^2 + \frac{1}{4} G A v_x^2 + E A v_x^2 + E I v_x^2 \right] dx
\]

(A10)

where \(E\) is Young’s modulus; \(G\) is shear modulus; \(EI\) is the uniform flexural rigidity; \(GJ\) is the uniform torsional rigidity; \(v_{xx}(x,t) = \partial^2 v(x,t)/\partial x^2\); \(\phi_x(x,t) = \partial \phi(x,t)/\partial x\). The virtual work done by the driving torque \(\tau_a\) applied on the arm is

\[
\delta W = \tau_a \delta \theta
\]

(A11)

By using Hamilton’s principle, it can be obtained that

\[
0 = \int_{t_1}^{t_2} \left\{ \delta (T_K - U_e) + \delta W \right\} dt
\]

(A12)

\[
0 = \int_{t_1}^{t_2} \left\{ \delta L_1 dx + \delta L_2 + \delta W \right\} dt
\]

where

\[
L_1 = \frac{1}{2} \rho A v_t^2 + 2cv \dot{\theta} + \dot{\theta}^2 + v^2 x^2 \dot{\theta}^2
\]

(A13)

\[
L_2 = \frac{1}{2} \rho A v_t^2 + 2cv \dot{\theta} + \dot{\theta}^2 + v^2 x^2 \dot{\theta}^2
\]

(A14)

Substituting (A9)–(A11) into (A12), the nonlinear governing equations of transverse vibration, torsional vibration and rigid-body motion can be concluded as follows:

\[
\rho A v_t + EI v_{xxx} - \frac{3}{2} \frac{EA v_x}{v_x} - \frac{1}{2} \frac{GA \phi_x}{\phi_x} - E A v_x \phi_x^2 - 2E A v_x \phi_x \phi_x^2 - 2E A v_x \phi_x^2
\]

(A15)

\[
\rho A \dot{v} - GJ v_{xx} - E R^2 v_{xx}^2 - 2E v_{xx} \phi_x^2 - \frac{1}{2} \frac{GA \phi_x}{\phi_x} + E A v_x \phi
\]

(A16)
where $I_g$ is the moment inertia of flexible arm; 
\[
\nu_{xxxx}(x,t) = \frac{\partial^4 v(x,t)}{\partial x^4} + \phi_{xxxx}(x,t) = \frac{\partial^2 \phi(x,t)}{\partial x^2}.
\]

ACKNOWLEDGMENT

The authors would like to express their gratitude to the referees and the Associate Editor for their useful comments and suggestions.

REFERENCES


