FSO links with diversity pointing errors and temporal broadening of the pulses over weak to strong atmospheric turbulence channels

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Free space optical (FSO) communication systems show a growing evolution in the commercial and the research field, due to the huge information capacity they provide along with the unlicensed and secure data transmission they can support. However, several destructive effects, such as the atmospheric turbulence and attenuation, the longitudinal pulse broadening due to the time dispersion, the spatial jitter which caused by the pointing errors between the trans-receivers, among others, mitigate their performance characteristics. Thus, for the improvement of the FSO communication systems’ performance, many techniques have been proposed, studied and used. In this work, we present a performance study and we derive closed form mathematical expressions for the estimation of the probability of fade for On–Off keying FSO communication systems which use Gaussian longitudinal pulses as information bit carriers, with reception diversity, over gamma gamma or gamma modelled, atmospheric turbulence channels taking into account the pointing errors and the group velocity dispersion (GVD) effect which, affects significantly the systems performance characteristics, especially for long link lengths and high data rate transmission. Finally, the derived mathematical expressions are used in order to present performance results, using common parameter values for channel’s and systems’ characteristics.

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1. Introduction

The terrestrial free space optical (FSO) communication systems have attracted significant research and commercial interest during the last years because of their low installation and operational cost, their very high data rate that they can transmit with high security level and the licence-free bandwidth use [1–10]. However, due to their nature, their performance depends strongly on the atmospheric conditions prevailing in the area were these systems are installed, the non–perfect alignment between the receiver and the transmitter of the optical link and the time dispersion effect which affects the form of the longitudinal pulses which transport the information signal [9–16].

More specifically, a very significant phenomenon which decreases the performance of the FSO links is the atmospheric turbulence which attenuates the propagating signal and also causes the so-called scintillation effect which results in random fluctuations of the irradiance which arrives at the receiver’s side, analogous to fading in RF systems. Due to this effect, the systems’ performance could be significantly degraded, especially for the case of deep signal fades [2,3,13,17]. Many statistical models have been proposed and studied, in order to model these irradiance fluctuations [6,10,12–30]. Here, we use the gamma gamma or the gamma distribution. The former, has been proven that models accurately, weak to strong turbulence conditions [18,20,28,29], while the latter provides a less complicated model which is accurate enough, especially for weak turbulence conditions cases [24,25,30].

The pointing errors effect, which describes the non–perfect alignment between the transmitter and the receiver of the FSO link, represents another mitigating factor for the performance of the FSO communication systems. In particular, the transmitter and the receiver of the communication system rely on structures which are not remain motionless, because of the building sway, strong wind, earthquakes, etc. Hence, due to the fact that the transverse radius of the optical beam at the receiver is relatively small, these small movements of the trans-receiver systems can cause significant pointing errors [4,9,14,18,31,32].

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Another effect which affects the performance of the FSO links, especially for the very high data rate, long link length, FSO communications systems which are using very narrow longitudinal pulses as information carriers, is the group velocity dispersion (GVD) due to the atmospheric propagation of the laser beam. In fact, the widening or narrowing of the shape of the propagating longitudinal pulse-envelope which entails the bit carrier, is caused by the GVD, and changes its characteristics, along the propagation path and thus affects the signal’s detection capability at the receiver. Consequently, its influence on the performance of the whole FSO system can be either detrimental or beneficial, under specific circumstances [11,12,15].

As mentioned above, due to the propagation of the optical signal through the atmosphere, there are many effects which degrade significantly the performance of the FSO links. In order to improve it, a technique which is very effective, concerns the reception diversity schemes which are very popular in wireless radio, as well [33–42]. In principle, the use of diversity refers to the consideration of multiple copies of the propagated signals in an attempt to overcome a poor transmission media state and enhance the communications systems’ reliability and performance. The reception diversity in FSO links can, mainly, be realized in space, in time or in wavelength [33–41]. For the spatial reception diversity scheme, the FSO system includes one transmitter and multiple receivers at different places. The transmitter is sending to each receiver, a copy of the same information signal, resulting to a decreased probability of error [33,34,41]. In time diversity schemes, the FSO link uses a single transmitter–receiver pair, but the copies of the information signal is retransmitting at different moments and this, results in decreasing the total effective bit rate of the link [38,39]. Finally, the wavelength diversity scheme uses a composite transmitter and the signal are transmitted at the same time at different wavelengths towards a number of receivers, each one of them detects the only the signal which has been transmitted at the suitable (for each receiver) wavelength [35–37,40].

In this work, we investigate the influence of the atmospheric turbulence, modelled with the gamma or the gamma gamma distribution, along with the GVD and the pointing errors, or spatial jitter, effects, at the availability of the FSO communication systems with reception diversity. More specifically, by assuming that the longitudinal pulses, which are the information bits carriers, are chirped Gaussian, we investigate the influence of each one of the above mentioned effects, at the probability of fade of the whole FSO link. Thus, we derive closed form mathematical expressions, for the estimation of this availability metric, for FSO links either with or without the reception diversity techniques, under the action of atmospheric turbulence, GVD and spatial jitter effects. Our mathematical expressions and the associated numerical results provide an efficient way for the estimation of the reliability of FSO links and therefore, the proposed analysis is useful to the system design engineer for performance evaluation purposes.

The remainder of this work is organized as follows. In Section 2, the channel model is presented while, in Section 3, the extraction of the performance metrics for the single point to point FSO pink is described. Next, in Section 4, the reception diversity scheme is studied and in Section 5, the numerical results are presented for common parameter values which are using in practical FSO links. Finally, Section 6 concludes with a summary of the main results.

2. The channel model

For the optical wireless link with diversity, we assume that the transmitter emits M copies of the same part of the information signal towards M receivers, for the cases of wavelength or spatial diversity schemes, or in M different time moments at the same receiver, for the time diversity configuration. Each copy arrives at the receiver(s) and remains at the buffer of the system until all the M copies been received. Thus, each copy of the information signal emitted from the transmitter, propagates through different or the same path(s) but with different channel’s characteristics and the total information signal at the receiver’s buffer is composed by M different versions of the same, initially, signal [35]. Consequently, the wireless optical communication system, under consideration, can be emulated as a single input multiple output (SIMO) scheme, with one transmitter and multiple receivers, for all the above mentioned diversity schemes [33–35].

Each copy of the optical information signal propagates through a turbulent channel with additive white Gaussian noise (AWGN) and pointing errors effect [14,31,32,39]. The channel, between the transmitter and each receiver is assumed to be memoryless, stationary and ergodic with independent and identically distributed (i.i.d) intensity fast fading statistics. Additionally, we assume that the transmitter of the intensity modulation direct detection (IM/DD) and On–Off Keying (OOK) modulation FSO system, emits longitudinal Gaussian optical pulses as information bit carriers [12,15,43,44]. Thus, the statistical channel model is given as [33–35]:

\[ y_m = \eta_m x_{r,m} + n, \quad m = 1, \ldots, M \]  

where \( y_m \) represents each of the M signals’ copy at the receiver, \( \eta_m \) is the effective photo-current conversion ratio of each receiver, \( l_{r,m} \) stands for the received irradiance of mth copy of the optical signal, \( x \) is the modulated signal which takes the binary values “1” or “0”, while \( n \) represents the additive white Gaussian noise (AGWN) with zero mean and variance equal to \( N_0/2 \) [29].

Assuming that the irradiance, \( l_{r,m} \), which arrives at the receiver’s side, fluctuates due to the atmospheric turbulence and the pointing errors effects, we conclude that it can be expressed as [45]:

\[ l_{r,m} = l_{a,m} l_{p,m} \]  

where \( l_{a,m} \) and \( l_{p,m} \) represent the dependence of the irradiance value due to the atmospheric turbulence and the pointing errors effects, respectively, for each one of the M signals’ copies.

Many statistical models have been proposed and accurately represent the irradiance fluctuations due to the atmospheric turbulence effect. Here, we present results using the gamma gamma or the gamma distribution. The former, has been proven that models accurately, the irradiance fluctuations for the cases of weak to strong turbulence conditions [18,20,22,28,35], while the latter provides a much less complicated model, compared to gamma gamma and additionally, it has been demonstrated that is very accurate, especially for weak turbulence conditions cases [24,25,30].

The probability density function (PDF), \( f_{G,G}(l_{a,m}, l_{p,m}) \), of the gamma gamma distribution as a function of \( l_{a,m} \), is given as [28]:

\[ f_{G,G}(l_{a,m}, l_{p,m}) = \frac{2(\alpha_m \beta_m)^{y_{l_{a,m}} + \beta_m/2}}{\Gamma(\alpha_m) \Gamma(\beta_m)} \left( \frac{\alpha_m \beta_m}{l_{a,m} \alpha_m + \beta_m} \right)^{1/2} \frac{l_{a,m}^{-\alpha_m - \beta_m}}{K_{\alpha_m - \beta_m}((2 \sqrt{\alpha_m \beta_m} l_{a,m}) \right)^{1/2}} \]  

where \( K_\nu(.) \) stands for the modified Bessel function of the second kind of order \( \nu \). \( \Gamma(.) \) is the gamma function while \( \alpha_m \) and \( \beta_m \) can be directly related to link’s parameters as [35]:

\[ \alpha_m = \exp \left[ \exp \left( \frac{0.49 \delta_{m}^{2}}{(1 + 0.18 d_{m}^{2} + 0.56 d_{m}^{12/5} - 0.7/6) \delta_{m}^{12/5}} \right) \right]^{-1} \]  

\[ \beta_m = \exp \left[ \exp \left( \frac{0.51 \delta_{m}^{2}}{(1 + 0.69 d_{m}^{2} - 0.56 d_{m}^{12/5} - 0.57/6) \delta_{m}^{2}} \right) \right]^{-1} \]
where \( d_m = 10D_m \sqrt{5 \pi_4^2 \lambda_m^2} \), \( D_m \) stands for the receiver's aperture diameter (in metres) while the parameters of wavelength, \( \lambda_m \), and link length, \( z_m \), are expressing in \( \mu \text{m} \) and \( \text{km} \), respectively [12, 15]. Additionally, the parameter \( \delta_m^2 \) stands for the Rytov variance which is given as [12, 15]:

\[
\delta_m^2 = 0.5C_n^2 \frac{z_m}{\lambda_m^2} \left[ \frac{z_m \times 10^3}{1} \right]^{1/6}
\]

with \( \lambda_m = 2 \times 10^6 \pi / \lambda_m \) being the optical wave number and \( C_n^2 \) is a parameter which is proportional to the atmospheric turbulence strength and varies between \( 10^{-17} \text{m}^{-2/3} \) and \( 10^{-13} \text{m}^{-2/3} \) for weak to strong atmospheric turbulence conditions, respectively [13].

Next, the PDF, \( f_{\xi_m}(x_m) \), of the much less complex, gamma distribution as a function of \( x_m \), is given as [24, 25, 30]:

\[
f_{\xi_m}(x_m) = \frac{x_m^{\xi_m-1} \exp(-x_m)}{\Gamma(\xi_m)}
\]

where \( \xi_m \) stands for the parameter of the gamma distribution and is connected with the parameters of the FSO link and the \( C_n^2 \), as [24, 30]:

\[
\xi_m = \left[ \frac{1}{\alpha_m} + \frac{1}{\beta_m} + \frac{1}{\alpha_m \beta_m} \right]^{-1}
\]

In order to investigate the behaviour of all the parameters of Eq. (2), the irradiance variations due to the pointing errors should be modelled. Thus the PDF for the \( I_{p,m} \) fluctuations due to the spatial jitter effect, is given as [31, 32, 45, 46]:

\[
f_{I_{p,m}}(I_{p,m}) = \frac{\xi_m^{I_{p,m}-1} \exp(-\xi_m)}{\Gamma(\xi_m)}
\]

with \( 0 \leq I_{p,m} \leq A_{0,m} \) (8)

where \( \xi_m \) stands for the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver and is given as \( \xi_m = \frac{W_{c,eq,m}}{2 \sigma_m} \), \( \sigma_m \) is the pointing error displacement at the receiver, the \( W_{c,eq,m} \) is the equivalent beam width and is given as \( W_{c,eq,m} = \sqrt{\pi \text{erf}(v_m)W_{c,m}/2v_m \exp(-v_m^2)} \), \( A_{0,m} = \text{erf}(v_m)^2 \), \( v_m = \sqrt{\pi/2} \lambda_m \sqrt{2W_{c,m}} \), and \( \text{erf}(.) \) is the error function while \( v_m = \text{dm}/2 \) and \( W_{c,m} \) represents the waist of the beam's Gaussian spatial intensity profile [32, 45, 46].

As mentioned above, the atmospheric turbulence and the spatial jitter affects affect simultaneously the irradiance which arrives at the receiver of the FSO link. Thus, the combined PDF of the \( f_{\text{comb}}(r_m) \), as a function of the \( r_m \), which includes both effects is given through the following integral [32, 45]:

\[
f_{\text{comb}}(r_m) = \int f_{I_{p,m}}(I_{p,m}) f_{\xi_m}(x_m) I_{p,m} dI_{p,m}
\]

where \( f_{\text{comb}}(r_m) \) stands for the conditional probability given \( I_{p,m} \) and is obtained from Eq. (8) [32, 45, 46]. Thus, by substituting (3) and (8) into (9), we conclude to the following integral for the estimation of the combined PDF for gamma gamma modelled turbulence along with pointing errors effect [32]:

\[
f_{\text{comb}}(r_m) = \frac{2(\alpha_m \beta_m)^{2(\alpha_m + \beta_m)/2} \xi_m^{\xi_m-1} \exp(-\xi_m)}{\Gamma(\xi_m) \Gamma(\xi_m) A_{0,m}^{\xi_m} \Gamma(\xi_m)}
\]

while, the corresponding integral for PDF estimation of the gamma modelled turbulence with pointing errors, is given as:

\[
f_{\text{comb}}(r_m, I_{p,m}) = \frac{2(\alpha_m \beta_m)^{2(\alpha_m + \beta_m)/2} \xi_m^{\xi_m-1} \exp(-\xi_m)}{\Gamma(\xi_m) \Gamma(\xi_m) A_{0,m}^{\xi_m} \Gamma(\xi_m)}
\]

By substituting the functions \( \text{exp(.)} \) and \( K_0(.) \), into the integral (10), with the appropriate Meijer functions [47], and using the expressions presented in Ref. [47], we result in the following closed form mathematical expression for the combined PDF for the case of the gamma gamma modelled turbulence [32]:

\[
f_{\text{comb}}(r_m, I_{p,m}) = \frac{\alpha_m \beta_m^{2 \alpha_m} \xi_m^{\xi_m-1}}{\Gamma(\xi_m) \Gamma(\xi_m) A_{0,m}^{\xi_m} \Gamma(\xi_m)}
\]

while the corresponding combined PDF for the gamma modelled turbulence is given as:

\[
f_{\text{comb}}(r_m, I_{p,m}) = \frac{\xi_m^{\xi_m-1}}{\Gamma(\xi_m) \Gamma(\xi_m) A_{0,m}^{\xi_m} \Gamma(\xi_m)}
\]

where \( \xi_m = \alpha_m \beta_m^{2 \alpha_m} \xi_m^{\xi_m-1} \exp(-\xi_m) \) represents the Meijer function that is a standard built in function which can be evaluated with most of the well known mathematical software packages and additionally, can be transformed to the more familiar hypergeometric functions [47].

As mentioned above, due to the fact that the optical pulses, which are carrying the information signal, propagate through the atmosphere, their longitudinal form can be affected by the GVD effect, especially for the cases where very narrow pulses are used, i.e. high data rate FSO links, and the propagation distance is relatively long [11, 12, 15]. Therefore, by taking into account that the constitution of the atmosphere does not remain invariable, for low altitudes, the atmospheric temperature and pressure can be accurately expressed as [11, 15, 48]:

\[
\begin{bmatrix}
\Theta(h) \\
P(h)
\end{bmatrix} = 
\begin{bmatrix}
288.19 - 6.49h \times 10^{-3} \\
2.23 \times 10^{-6}(44.41 - h \times 10^{-3})^{5.256}
\end{bmatrix}
\]

where \( \Theta(h) \) and \( P(h) \) stand for the altitude (in metres), the atmospheric temperature (in Kelvin degrees) and the atmospheric pressure (in millibars), respectively. These variations, clearly affect the atmosphere's refractive index, \( n(\lambda) \), and the GVD parameter, \( \beta_2 \) [11, 15, 49], which is given as [15]:

\[
\beta_2 = \frac{3.5 \Pi(m^2)}{2 \pi^2 \lambda_c^2} \frac{10^{15} + 135.8 \left( \frac{P(h)}{\epsilon(\Theta(h))} \right)^2}{\pi \lambda_c^2} + \frac{255.3}{\pi \lambda_c^2} \left( \frac{\omega P(h)}{n(\lambda)} \right)^2 \times 10^{-6}
\]

where \( \beta_2 \) is measured in \( \text{ps}^2/\text{km} \), while \( \nu \) stands for the propagation speed (in m/sec) of each spectral component of the signal, \( c \) represents the speed of light in vacuum and \( \omega \) is the angular frequency which is connected with the operational wavelength through the expression, \( \omega = 2 \pi n/\lambda \) [11, 12].

Using the expressions (14) and (15), the influence of the GVD effect at the longitudinal shape of the optical pulse, can be estimated according the specific characteristics of each optical link installation. Here, we investigate FSO links which are using chirped Gaussian longitudinal pulses as information carriers [11, 12, 43, 44].
In this case, the irradiance $I_{r,m}$, for each one of the $M$ transmissions of the diversity scheme, at the receiver’s side is given as [12,15]:

$$I_{r,m} = \frac{|u(z, T)|_m^2}{|U(z, T)|_m^2}$$  (16)

with $T$ being the retarded time (measured in ps) and is defined through the usual way as in many areas of electromagnetic propagation and physics, as well [49–52]. Additionally, the expected and the instantaneous dimensionless intensities of the longitudinal pulses at the receiver are obtained through the norms $|U(z, T)|_m^2$ and $|u(z, T)|_m^2$, and the value of the former, which depends on the GVD effect, for a longitudinal chirped Gaussian pulse envelope, is given as [12]:

$$|U(z, T)|_m^2 = \frac{\exp \left( -\frac{T_0^2 + 2\beta_2 z_m C + T_0^4 \beta_2^2 z_m^2 (C^2 + 1)}{1 + 2T_0^2 \beta_2 z_m C + T_0^4 \beta_2^2 z_m^2 (C^2 + 1)} \right)}{\sqrt{1 + 2T_0^2 \beta_2 z_m C + T_0^4 \beta_2^2 z_m^2 (C^2 + 1)}}$$  (17)

with $T_0$ and $C$ being the half width at $e^{-1}$ of maximum intensity and the chirp parameter, respectively [12,49]. From Eq. (17), it can be seen that the quantity which is inside the square root is almost equal to one for small values of the GVD parameter, $\beta_2$, and link length, $z_m$, and large values of pulsewidth, $T_0$. Moreover, from (17), it concludes that for chirped pulses with $C > 0$, i.e. upshifted chirped pulses, the value of $|U(z, T)|_m^2$, for specific value of $T$, decreases with the propagation distance, while, for $C < 0$, i.e. downshifted chirped pulses, increases with the propagation distance, up to a specific distance $z_{max}$, and its maximum value is achieved at $z_m = z_{0}$. The values of these critical distances are depending on the atmospheric and pulses characteristics [12].

The value of $|U(z, T)|_m^2$, in (17), depends on the time parameter, $T$, which determines its amplitude for different time moments. Thus, in order to obtain better reliability characteristics, the pulse’s detection should be done at the centre of the chirped Gaussian longitudinal pulse, i.e. for $T = 0$ ps, where its amplitude is larger [12]. Hence, by substituting (17) into (16), the normalized irradiance detecting at $T = 0$ ps at the receiver of the FSO link is given as [15]:

$$I_{r,m} = \frac{|u(z, 0)|_m^2}{|U(z, 0)|_m^2} \sqrt{1 + 2T_0^2 \beta_2 z_m C + T_0^4 \beta_2^2 z_m^2 (C^2 + 1)}$$  (18)

3. The performance metrics for the single point to point FSO link

A significant performance metric concerning the availability of the FSO links is the probability of fade, $P_{f,m}$, as a function of the irradiance, $I_{r,m}$, at the receiver’s input. More specifically, it shows the probability that the $I_{r,m}$ value falls below each receiver’s threshold, $I_{r,m,th}$, and is given through the following mathematical expression [12,53]:

$$P_{f,m}(I_{r,m,th}) = \frac{\int_{I_{r,m,th}}^{\infty} f_{I_{r,m}}(I_{r,m,th}) \ dI_{r,m}}{F_{I_{r,m}}(I_{r,m,th})}$$  (19)

where $F_{I_{r,m}}(I_{r,m,th})$ stands for the cumulative distribution function (CDF) which can be estimated by integrating the corresponding PDF. Thus, in order to estimate the probability of fade for the gamma gamma modelled turbulence with pointing errors we should estimate the combined CDF of the PDF of (12), through the following integral:

$$P_{f,m,comb,Gc}(I_{r,m,th}) = \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)} \times G_{2,4}^{3,1} \left( \frac{\alpha_m \beta_m}{\xi_m^2}, \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)}, 1, \xi_m^2 + 1 \right)$$  (20)

Using the mathematical expressions of Ref. [47], we conclude to the following expression for the probability of fade for the gamma gamma modelled turbulence with pointing errors:

$$P_{f,m,comb,GC}(I_{r,m,th}) = \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)} \times G_{2,4}^{3,1} \left( \frac{\alpha_m \beta_m}{\xi_m^2}, \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)}, 1, \xi_m^2 + 1 \right) \times G_{2,4}^{3,1} \left( \frac{\alpha_m \beta_m}{\xi_m^2}, \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)}, 1, \xi_m^2 + 1 \right)$$  (21)

Similarly, for the estimation of the probability of fade for the gamma modelled turbulence with pointing errors, we substitute the combined PDF of (13) into (19) and using the mathematical expressions of Ref. [47], we obtain the following expression:

$$P_{f,m,comb,Gc}(u(z, 0)|_{m,th}) = \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)} \times G_{2,4}^{3,1} \left( \frac{\alpha_m \beta_m}{\xi_m^2}, \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)}, 1, \xi_m^2 + 1 \right)$$  (22)

Next, in order to take into account the influence of the GVD effect at the performance of the single link, we substitute the equation (18) into (21), for the case of the gamma gamma turbulence, and into (22), for the gamma distribution. Thus, the total probability of fade, for each one of the $M$ copies of the FSO diversity scheme, under gamma gamma modelled turbulence with pointing errors, as a function of the threshold of the instantaneous intensity at the receiver’s side, is given through the following closed form mathematical expression:

$$P_{f,m,comb,G}(u(z, 0)|_{m,th}) = \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)} \times G_{2,4}^{3,1} \left( \frac{\alpha_m \beta_m}{\xi_m^2}, \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)}, 1, \xi_m^2 + 1 \right)$$  (23)

with $\Sigma_m = \sqrt{\Psi_m^2 + (1 + \Psi_m)^2}$ and $\Psi_m = T_0^2 \beta_2 z_m$.

Furthermore, following the procedure presented above for the gamma gamma distribution model, we conclude to the following mathematical expression for the estimation of the probability of fade for each one of the $M$ information signals, under the action of GVD, gamma modelled atmospheric turbulence and pointing errors:

$$P_{f,m,comb,G}(u(z, 0)|_{m,th}) = \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)} \times G_{2,4}^{3,1} \left( \frac{\alpha_m \beta_m}{\xi_m^2}, \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)}, 1, \xi_m^2 + 1 \right)$$  (24)
4. The reception diversity scheme

The closed form mathematical expressions (23) and (24) which have been obtained above are presenting the probability of fade of the optical link, without any diversity scheme, under various turbulence conditions and pointing errors. However, as mentioned before, the diversity schemes could improve the system's availability, taking into account that the information signal is sent in multiple copies in different moments which correspond in different channels states. More specifically, the probability of fade for the diversity scheme which is sending each part of the information signal in M copies, is given as [35]:

\[
P_{F,\text{total}} = \prod_{m=1}^{M} P_{F,m,\text{comb},X} = \prod_{m=1}^{M} Pr(I_{r,m} \leq I_{r,m,\text{th}}) = \prod_{m=1}^{M} f_{I_{r,m}}(I_{r,m,\text{th}})
\]

(25)

where the parameter X which appears at the subscript of the probability of fade, corresponds to the specific distribution which has been used in order to model the atmospheric turbulence effect, i.e. the gamma gamma or the gamma distribution.

By substituting (23) into (25), is obtained the close form mathematical expression that follows, for the estimation of the probability of fade of the whole FSO link with reception diversity, GVD, gamma gamma turbulence and pointing errors:

\[
P_{F,GG,\text{total}} = \prod_{m=1}^{M} P_{F,m,\text{comb},GG} = \prod_{m=1}^{M} \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)}
\]

\[\times \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)} \left( \frac{\alpha_m \beta_m m!}{\alpha_m \beta_m m!} \right)^\frac{1}{\alpha_m \beta_m} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \right]
\]

(26)

For the specific case where the time diversity scheme is used, the parameters of (26), are the same for all the M-copies of the information signal, i.e. \(\alpha = \alpha_1, \ldots, \alpha_M, \beta = \beta_1, \ldots, \beta_M, \xi = \xi_1, \ldots, \xi_M \), \(\xi_m = \xi_1, \ldots, \xi_M \), \(\Sigma_m = \Sigma_1, \ldots, \Sigma_M \), \(\lambda = \lambda_1, \ldots, \lambda_M \), \(\xi_m = \xi_1, \ldots, \xi_M \), \(\Sigma_m = \Sigma_1, \ldots, \Sigma_M \), \(\xi_m = \xi_1, \ldots, \xi_M \), and consequently the above expression, is taking the following form:

\[
P_{F,GG,\text{total}} = \left[ \frac{\xi_m^2}{\Gamma(\alpha)\Gamma(\beta)} \right]^{M} \left[ \frac{\alpha_m \beta_m m!}{\alpha_m \beta_m m!} \right]^{\frac{1}{\alpha_m \beta_m}} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \right]
\]

(27)

Additionally, the probability of fade of the FSO with reception diversity, taking into account the GVD, the gamma modelled turbulence and the pointing errors, has the following form and is derived by the Eqs. (24) and (25):

\[
P_{F,\text{G,total}} = \prod_{m=1}^{M} P_{F,m,\text{comb},G} = \prod_{m=1}^{M} \frac{\xi_m^2}{\Gamma(\alpha_m)\Gamma(\beta_m)} \left( \frac{\alpha_m \beta_m m!}{\alpha_m \beta_m m!} \right)^\frac{1}{\alpha_m \beta_m} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \right]
\]

(28)

Similarly to the previous case, for the limiting case of time diversity schemes, the parameters of the expression (28) are equal for all the M copies of the information signal, and the probability of fade is given as:

\[
P_{F,G,\text{total}} = \left[ \frac{\xi_m^2}{\Gamma(\alpha)\Gamma(\beta)} \right]^{M} \left[ \frac{\alpha_m \beta_m m!}{\alpha_m \beta_m m!} \right]^{\frac{1}{\alpha_m \beta_m}} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \left( \frac{\xi_m^2 + 1}{\xi_m^2 + 1} \right)^{\frac{1}{\alpha_m \beta_m}} \right]
\]

(29)

where \(\xi = \xi_1, \ldots, \xi_M\).

5. Numerical results

In this section we present numerical results which are illustrate the influence of GVD, atmospheric turbulence and spatial jitter effects at the performance of the FSO links, with or without reception diversity schemes, by means of the probability of fade estimation. More specifically, using the derived above expressions (23), (24) and (26)–(29), we investigate the system’s performance for common parameter values. Taking into account that the gamma gamma distribution is suitable for weak to strong turbulence conditions, while the gamma, for weak, the results that we present below have been obtained using the former distribution model for larger values of \(C_n^2\), i.e. \(1 \times 10^{-15} \text{m}^{-2/3}\) and \(1 \times 10^{-13} \text{m}^{-2/3}\), while the latter has been used for weak turbulence conditions, i.e. \(1 \times 10^{-17} \text{m}^{-2/3}\). Moreover, the results that we present here have been obtained only for a time diversity scheme with M = 2 and 3, link length 6 km and the whole system is assumed to be established at \(h = 30\) m above the sea’s surface. Moreover, the operational wavelength of the system, \(\lambda\), has been fixed at 1.55 \(\mu\text{m}\) while the receiver’s aperture diameter, D, is set at 0.01 m. It is clear that using the expressions (27) and (29), one, can easily obtain numerical results for the availability of the FSO system for others reception diversity schemes, e.g. wavelength, spatial diversity, etc., according the requirements of each FSO communication system.

In order to investigate the influence of the GVD and the pointing errors effects, we have used two values for the longitudinal pulsedwidth of the chirped Gaussian pulses, i.e. \(T_0 = 4\) ps and 7 ps, two values for the chirp parameter \(C\), i.e. -20 and +20, while the ratio \(\sigma_r/r\) is set at 0.5 and the \(W_{\text{r2}}/r\) is taking the values 1.5 and 4, for weak to strong pointing errors effects, respectively. Finally, the receiver’s dimensionless intensity threshold is varying between the values 0.01 and 0.1 [15].

Thus, in Fig. 1, we present the availability results for the case of weak turbulence conditions, i.e. using the gamma distribution.
model, for longitudinal upshifted and downshifted Gaussian pulses with pulsewidth equal to 4 ps, for weak and strong pointing errors effect with and without time diversity scheme, i.e. $M = 1, 2$ and 3. The positive influence of the negative chirp is clearly visible in this figure, as well the better performance results which are obtained by increasing the time diversity value $M$. However, it should be mentioned here that the larger value of $M$ increases significantly the system's availability but, on the other hand, decreases its data rate due to the fact that the same information signal should be sent (and received) by the same transmitter (and receiver) more than once is a specific time domain [39].

In Fig. 2, we present the corresponding results with those of Fig. 1, but with larger pulsewidth for the longitudinal Gaussian pulses. By comparing the results of Fig. 1 with those of Fig. 2, it is visible that we obtain the same qualitatively results but, the influence of the positive or the negative chirp value is much more weak. This conclusion was somehow expected because, as mentioned above, the influence of GVD effect is strong enough for narrow longitudinal pulses and is getting weaker as their pulsewidth increases.

Next, in Figs. 3 and 4, we present outcomes with similar parameters with those of Figs. 1 and 2 but, for moderate atmospheric turbulence conditions which are modelled with the gamma-gamma distribution model. Here, the stronger atmospheric turbulence results in worse availability results but, similarly to the previous results, the diversity scheme improves significantly the performance. Additionally, as in the previous cases, it is clear that the negative chirp upgrades the performance, but its influence is significant only for relatively narrow longitudinal Gaussian pulses which are used for the cases of communications systems with very high data rates.

Finally, in Figs. 5 and 6, we present the corresponding results with the previous cases but for strong turbulence conditions. By observing them, it is clear that the use of a diversity scheme, the time diversity here, is necessary due to the fact that the performance of the FSO link is very low without such a technique, even for the cases of narrow longitudinal pulses where the negative chirp can improve, slightly the system's availability.
6. Conclusions

In this work, we studied the influence of the atmospheric turbulence, modelled either with the gamma gamma or the gamma distribution, the GVD and the spatial jitter, effects at the performance of an FSO link with reception diversity. We extracted closed form mathematical expressions for the estimation of this communication system’s performance and availability, by means of its probability of fade, either with or without reception diversity. Using the extracted expressions we presented numerical results which are shown the influence of each effect at the system’s performance, depending on the system’s parameter values.

References


