Analyzing auction and bargaining mechanisms in e-procurement with supply quality risk

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\textbf{A R T I C L E I N F O}

Article history:
Received 15 March 2012
Received in revised form
30 March 2013
Accepted 1 April 2013
Available online 18 April 2013

Keywords:
Auctions
Bargaining
E-procurement
Mechanism design
Quality
Risk

\textbf{A B S T R A C T}

We compare two mechanisms from the buyer's perspective in multi-attribute supply procurement, with verifiable and unverifiable quality of the supplies and risk aversion to deviation from acceptable quality: an auction mechanism and a generalized Nash bargaining mechanism. We develop a model to represent the effects on the buyer's dominant strategy of bargaining and auction participation. The results suggest the conditions for which bargaining is preferred over the auction mechanism alone.

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\section{1. Introduction}

In auction theory, the auctioneer often is regarded as a monopolist who has the power to choose auction rules to maximize the expected revenues that will be received in the auction \cite{4,10}. The procurement process may consist of an auction phase, followed by a bargaining phase. The latter enables the buyer and seller to effectively share information to support the completion of a purchasing transaction. Auctions and bargaining may also be conducted separately, but bargaining most often occurs in procurement auctions after the supplier who makes the best bid is determined. The buyer also may play the role of an auctioneer, but will operate with no precise information about the cost of production for the suppliers. The suppliers act as bidders, and so the economic logic that they employ is to not make bids that are unprofitable for them relative to their production costs. Prior auction models have given full bargaining power to the buyer in the procurement market \cite{3,9}. The upshot is that the outcome of the procurement process is influenced by a variety of factors, such as the tactics employed in bilateral bargaining, the outside options of the buyer and the suppliers, and their risk profiles, among other issues.

Prior research has frequently used the generalized Nash bargaining model to account for these factors with one parameter to represent the buyer’s or the supplier’s bargaining power. In this research, we will evaluate the basic tradeoffs involved in auction and bargaining outcomes in procurement when the buyer exhibits some degree of risk aversion and the quality of the supplies offered by the supplier cannot be verified in advance of the buyer’s receipt. We also will assess some other issues, including the effects of the number of bidders on the buyer’s outcome, the effects of the buyer’s bargaining power on the outcome of bilateral bargaining, and the effects of various aspects of unverifiable pre-delivery quality on the choices that the buyer will make in the presence of each market mechanism.

The relationship between generalized Nash bargaining and procurement auctions is interesting for two reasons. First, it has implications for mechanism design choices in practice. Many wholesale buyers in industry prefer to bargain with existing suppliers whom they know well, and whose production costs have been revealed based on their previous procurement transactions. Other buyers prefer to use auctions to find new suppliers whose cost and supply quality may be more attractive. Second, auctions are simple, since they only involve asking suppliers to submit their
bid prices on their supplies, after identifying the targeted level of quality that the buyer needs to have. Auctions also are able to protect the buyer’s private information about how supply quality and price are evaluated, and they are relatively easy to implement effectively by both sides.

Several works in the prior literature have compared auctions and bargaining in procurement. Harstad and Rothkopf [5] presented a common-value auction model, in which the winner is allowed to withdraw a bid after all of the other bids in the auction have been revealed. Waehrer [12] modeled bidders who are permitted to obtain more information after the auction phase, in order to support bargaining between the buyer and the supplier. These two approaches give the winner of the procurement auction the right to withdraw from the auction, or to bargain over the price and quality outcome of the supplies that are received. This prior work found that the option to withdraw bids increases the value of the mechanism design from the buyer’s viewpoint as the auctioneer. Bajari [1] also studied bargaining in procurement auctions. The model he proposed is similar to a first-price sealed-bid auction, but it also permits a follow-on bargaining process. He concluded that negotiable prices convey important information to the buyer about a firm’s production costs, which will be helpful when both firms enter the bargaining phase.

Bulow and Klemperer [2] have examined the issues with auction and post-auction bargaining also. They assessed whether it is more profitable for the buyer to acquire supplies via an English auction without post-auction bargaining, or via an English auction with one less bidder and post-auction bargaining. They reported that an auction with one more than a base number of suppliers typically produces greater expected value for the buyer than a procurement auction with follow-on bargaining involving only the base number of suppliers. Their results suggest that a buyer should put greater emphasis on encouraging competition among suppliers than on its post-auction bargaining capabilities.

Thomas and Wilson [11] used experimental methods to compare first-price auctions to an exchange process involving multilateral bargaining. Contrary to Bulow and Klemperer [2], who argued that auctions are more efficient than bargaining, the authors found that observed transaction prices are statistically indistinguishable under the auction and bargaining mechanisms. They also found that the bargaining mechanism is more efficient than auctions when there are more sellers, and that prices tend to be higher with bargaining than in auctions. They further determined that the two mechanisms are equally efficient when there are just two sellers. Another article by Manelli and Vincent [7] compared second-price auctions with a take-it-or-leave-it mechanism. They also combined features of the two to construct a hybrid mechanism that has useful properties under certain conditions.

There is also empirical research that has been conducted which compares the procurement auction and bargaining mechanisms. Based on data from 216 contracts between buyers and suppliers of medical and surgical articles, Kjerstad [6] tested whether auctions and bargaining resulted in significantly different prices. The main results are that auctions do not result in significantly lower prices compared with bargaining. More recently, Bajari et al. [1] examined a comprehensive data set of private sector building contracts awarded in northern California during the years 1995–2000. They found that auctions perform poorly when projects are complex, the contract design is incomplete, and there are relatively few bidders.

The findings we have discussed illustrate that our understanding of how and why auctions outperform bargaining in procurement is not complete, especially when the nature of the procurement transaction is complex. Bajari et al.’s [1] results showed that the number of bidders and the inability of the buyer to verify the quality of the supplier’s goods affect which is likely to be preferred. In this article, we analyze the inclusion of several important determinants of this decision in a model of multi–attribute procurement decision-making. We stress the inability of the buyer to specify quality fully prior to receipt of the supplies. We also show how each of the mechanisms, bargaining-based and auction-based exchange, can dominate the other if certain conditions are met. We do this by comparing generalized Nash bargaining with a second-price auction mechanism. In the two mechanisms, the buyer must take a risk related to the quality of the supplies that are made available by the supplier.

In this article, we will study a circumstance in which a buyer must decide whether to bargain with an existing supplier—one that it has used previously, and for which it knows something, if not everything about its quality of supplies, or to initiate an auction that will involve other suppliers whose supply quality is unknown. If the buyer decides to bargain with its existing supplier, the primary benefit is that the buyer will have relatively complete information about the supplier’s production costs, and, we will assume, at least some knowledge of the variability of the supplier’s supply quality. Suppliers who participate in an auction, in contrast, might permit the buyer to achieve a lower cost in the purchase of new supplies. But there will inevitably be risks involved, and it is entirely possible that the outcome of an auction for the buyer is that its supply costs will be higher and the quality of its supplies will be lower than what the buyer has experienced in the past. We will compare these alternative mechanisms to determine the conditions under which the performance of one dominates the other.

Our modeling approach is different from the approaches of Bulow and Klemperer [2] and Wang [13] in several respects. First, we compare a procurement auction with bargaining related to bargaining power and the number of participating bidders. Our approach contrasts with the model of Bulow and Klemperer [2]. They compared the performance of an auction that has some number of suppliers with another auction that has one fewer supplier but permits post-auction negotiation. Second, in our model, the buyer’s valuation is private information; Bulow and Klemperer [2] regarded it as common knowledge, however. Third, in our procurement setting, we view the verification of the quality of supplies as a decision variable for the buyer. Fourth, we consider both the risk preferences and the possibility that the buyer cannot specify the quality before the receipt of the supplies. This is an attractive assumption to make for the mechanism that we are testing, since it is more representative of real-world procurement operations. Our model and analysis also can be applied in circumstances in which a risk-averse decision-maker must choose between two mutually-exclusive mechanisms in the presence of supplies whose quality cannot be verified.

2. The basic model

Suppose that a risk-averse buyer needs to procure supplies for a project that it must complete. A single-valued variable for quality can represent all of the non-monetary characteristics of the supplies, which can be verified by the buyer. We call this verifiable quality (VQ). (Refer to the Appendix at the end for a table of notation and definitions.) Further assume that there is a risk-neutral existing supplier (which we refer to as an old supplier, with the notation Old), who has sold supplies to the buyer in the past. The result is that the buyer knows the existing supplier’s production cost, $C_{\text{Old}}$, based on the verifiable quality of past supplies. In addition, suppose that there are $N$ other risk-neutral new suppliers (New) that are willing to bid. The new suppliers have production costs, $C_{\text{New}}$, that are their private information. Further suppose that each supplier’s production cost, $C_{\text{New}}$, is drawn independently from a uniform distribution.
Each supplier’s production capability can be stated in terms of the verifiable level of quality \( VQ \) that it can produce using a cost function with the well-accepted convex functional form, \( C_{\text{new}}(VQ) = C_{\text{new}}VQ^\alpha \), with \( \alpha \geq 1 \). The existing supplier with which the buyer is familiar will have a similar cost function, \( C_{\text{old}}(VQ) = C_{\text{old}}VQ^\alpha \), again with \( \alpha \geq 1 \). The buyer’s benefits or revenues \( R \) for carrying out a project with supplies of verifiable quality are determined by the concave function \( R(VQ) = aVQ^\omega \) with \( 0 < \omega \leq 1 \). The parameters \( a > 2 \) and \( \omega \) here are the buyer’s private information. The existing suppliers know the value of these parameters, since they have previously made procurement transactions with the buyer. At the same time, there is unverifiable quality (\( UQ \)) in procurement, as any procurement contract between a buyer and a supplier cannot exactly specify all aspects of quality.

When the buyer employs the procured supplies of the existing supplier in its production, its associated revenue with the unverifiable quality \( UQ \) and the outcome the buyer can achieve. Not made procurement transactions. We are interested in evaluating the effect that the unverifiable quality of supplies will have on the outcome the buyer can achieve.

In the development of our model, we will assume that \( R_{\text{old}}^{\text{UQ}}, R_{\text{new}}^{\text{UQ}} \) and \( C_{\text{old}} \) and \( C_{\text{new}} \) are all independent of one another. We note, however, that procuring supplies is riskier for the buyer when a new supplier is involved; we represent this via the standard deviation for quality, \( \sigma_{\text{old}} \leq \sigma_{\text{new}} \). The buyer’s utility for profit \( \pi \) is specified as a negative exponential function, a standard representation of risk aversion, with \( U(\pi) = -e^{-\pi} \). Here, \( r > 0 \) is the Arrow–Pratt coefficient of absolute risk aversion, which is defined as \( r = U''(\pi)/U'(\pi) \) for all different levels of profit. Mas-Colell et al. [8] have suggested that different levels of risk aversion can be readily captured. A greater level of risk aversion is represented by \( U_1(\pi) = -e^{-r_1\pi} \), and a lesser degree of risk aversion by \( U_2(\pi) = -e^{-r_2\pi} \) with \( r_1 \geq r_2 \). The profit earned by the buyer is the revenue that can be achieved from the acquisition of supplies of verifiable and unverifiable quality, \( R(VQ) + R_{\text{old}}^{\text{UQ}} \) or \( R(VQ) + R_{\text{new}}^{\text{UQ}} \), minus the payment, \( P \), for supplies that must be made to the selected supplier.

The buyer has two options. First, the buyer may select to source its supplies from the existing supplier, by bargaining with the supplier on the basis of what it knows about the existing supplier’s supply cost. In this case, imagine that the buyer’s existing supplier has an outside option to offer its supplies to another buyer, and that the Nash bargaining solution codifies how a transaction outcome will be reached. Second, the buyer may select the supplier that submits the lowest bid for some previously fixed level of verifiable quality through a second-price sealed-bid procurement auction. The payments associated with the procurement of supplies with verifiable quality based on these two options will be different. In addition, recall that the buyer’s existing supplier and the winner of the second-price procurement auction will also have different revenue levels associated with them for supplies of unverifiable quality. The result is that the bargaining and auction outcomes will yield different levels of profit and utility.

3. Case 1: Supplier and buyer quality verification with \( \theta = \omega = 1 \)

Case 1 occurs when \( \theta = \omega = 1 \). For this case, it is easy to verify that the buyer is always better off by requiring supplies of higher verifiable quality \( VQ_H \) rather than lower verifiable quality \( VQ_L \), for any values of \( VQ_L < VQ_H \). This result is obtained because, in a linear utility-cost system, supplies of higher verifiable quality will yield greater value than supplies of lower verifiable quality. Thus, the buyer’s dominant strategy will be to specify a level of verifiable quality that is as high as possible for the existing supplier and the supplier that wins the procurement auction. The buyer and the existing supplier will agree on a price for supplies under generalized Nash bargaining that is determined by the function:

\[
\psi(VQ) = \arg \max (aVQ - P)^\alpha (P - C_{\text{old}}VQ)^{1-\alpha}, \quad 0 < \alpha < 1. \quad (1)
\]

The generalized Nash bargaining solution splits the surplus such that the buyer receives a fraction \( \alpha \) of value. The reader should think of this as the influence of the buyer’s bargaining power. The buyer’s bargaining power also is closely related to the value of its outside option (Opt) with its existing supplier. By applying the outside option principle from bargaining theory, we can define the Buyer’s bargaining power as follows: \( \alpha = 1 - \text{Opt}/(a - C_{\text{old}}VQ_H) \), based on the existing supplier’s cost \( C_{\text{old}} \) of supplying high quality supplies. This permits us to compute the price for bargaining (\( B \)) from the first-order condition of Eq. (1), by replacing \( VQ \) with \( VQ_H \), to yield:

\[
P_B = (C_{\text{old}} - \alpha \sigma_{\text{old}}) VQ_H. \quad (2)
\]

The certainty equivalent of \( R_{\text{old}}^{\text{UQ}} \), \( \text{Cert}[R_{\text{old}}^{\text{UQ}}] = \mu_{\text{old}} - r\sigma_{\text{old}}^2/2 \) is the amount of profit that provides the buyer with the following level of utility, \( E[-e^{-r^2\sigma_{\text{old}}^2}/2] \). Thus, the buyer’s expected utility from bargaining, \( E[U_{\text{old}}] \), is given by:

\[
E[-e^{-r(\alpha - C_{\text{old}}) VQ_H + \sigma_{\text{old}}^2/2}] = -e^{-r(\alpha - C_{\text{old}}) VQ_H + (\mu_{\text{old}} - 1/2 \sigma_{\text{old}}^2)}. \quad (3)
\]

Since the procurement auction is a modified second-price auction, the competing suppliers’ optimal bidding strategies to provide supplies to the buyer will involve telling the truth about their underlying cost of production. The payment to the winning supplier in the auction (\( A \)) will be a random variable:

\[
P_A = \text{Bid}(2)VQ_H \quad (4)
\]

where \( \text{Bid}(2) \) is the second lowest bid among the new participating suppliers, reflecting our use of a second price auction mechanism here. In addition, the buyer’s expected utility from the procurement auction, \( E[U_{\text{old}}] \), is:

\[
E[-e^{-r(\alpha - C_{\text{old}}) VQ_H + \sigma_{\text{old}}^2/2}] = -e^{-r(\alpha - C_{\text{old}}) VQ_H + (\mu_{\text{old}} - 1/2 \sigma_{\text{old}}^2)} \cdot E[e^{\text{Bid}(2)VQ_H}]. \quad (5)
\]

We define a function related to the Arrow–Pratt coefficient of absolute risk aversion for Case 1 as:

\[
\gamma_1 = (N + 1)/(N + 1) - (1 - \alpha \sigma_{\text{old}} VQ_H + (\mu_{\text{old}} - \mu_{\text{new}})) / \sigma_{\text{old}}^2 - \sigma_{\text{new}}^2. \quad (6)
\]

Based on this, we can state the first proposition:

**Proposition 1 (Buyer’s Choice to Bargain).** If the Arrow–Pratt coefficient of absolute risk aversion \( r \geq \gamma_1 \), the buyer will choose to bargain under the Case 1 conditions.

**Proof.** Note that Eqs. (3) and (5) have negative values. By Jensen’s inequality, we can use the expression \( E[e^{\text{Bid}(2)VQ_H}] \geq e^{\text{Bid}(2)VQ_H} \)
to compare these equations to create a sufficient condition for bargaining. This is given by:

$$
\frac{E[U_b]}{E[U_a]} \leq e^{(1 + \alpha) \sigma_{Old}^2 + \mu_{Old}^2 - N + 1 + (\mu_{Old} - \mu_{New}) + \frac{1}{2} \sigma_{Old}^2} \leq 1.
$$

(7)

Eq. (7) is similar to $r \geq r_1(N, \sigma_{Old}, \alpha, \mu_{New}, \sigma_{New}, \mu_{Old}, \sigma_{Old})$. □

The Arrow–Pratt coefficient of absolute risk aversion is widely used to represent risk aversion, and is included in this model as $r$ for the buyer relative to the verifiable quality of supplies. Proposition 1 and Eq. (6) together show that there are utility trade-offs that have to be evaluated when the bargaining and auction mechanisms are under consideration. When the number of suppliers $N$ increases, since $\sigma_{Old} \leq \sigma_{New}$, the value of $(N + 3)/(N + 1)$ in Eq. (6) motivates a buyer with any degree of risk aversion to initiate a procurement auction. Moreover, since $\sigma > \sigma_{Old}$, stronger bargaining power $\alpha$ and a larger difference between the mean of the profit from unverifiable quality of supplies from the different suppliers, $\mu_{Old} - \mu_{New}$, will motivate the buyer to prefer bargaining over a procurement auction.

Also, the greater the difference between the variances of the different unverifiable quality levels from the existing suppliers and the new one, $\sigma_{Old}^2 - \sigma_{Old}^2$, the greater will be the tendency for bargaining to be desirable. The alternative of using bargaining versus using an auction needs to be considered based on the factors we have discussed, and the level of risk aversion is the main consideration relative to the risk of the unverifiable quality of supplies. Thus, the Buyer’s Choice to Bargain Proposition (P1) is intended to direct the analyst’s attention to the form of $r \geq r_1$. We would like to see if we can hold all of the parameters fixed, except for the coefficient of risk aversion $\alpha$ and then allow the buyer to decide whether to choose to use an auction or engage in bargaining. Proposition 1 suggests that a more risk-averse buyer will find that is likely to be risk-averse in excess of the level suggested by the value $r_1$. As a result, the buyer will choose to bargain with an existing supplier to gain greater assurance for the quality of the supplies to be purchased.

Moreover, the terms $\mu_{Old} - \mu_{New}$ and $\sigma_{Old}^2 - \sigma_{Old}^2$ are useful to compare the trusted existing supplier and the new supplier with respect to the variability of unverifiable quality. An economic explanation of the coefficient of risk aversion is that, in addition to the buyer’s risk profile, the ratio between the incremental profit associated with unverifiable quality and the total profit that can be obtained by the buyer. The intuition is that the higher the ratio is, the greater will be the likelihood for the buyer to choose to bargain over an auction.

### 4. Case 2: Supplier and buyer quality verification with $\theta > 1$ and $\omega \leq 1$

In this case, the buyer is unable to achieve maximum social surplus by procuring supplies of the highest possible verifiable quality, such as $VQ_H$ in the first case, via bargaining or via a procurement auction. In contrast in this case, now we will show that there are the optimal verifiable quality standards that apply to both mechanisms. In bargaining, payment to the existing supplier under verifiable quality $VQ$ is also generated by Eq. (1), and is of a similar form as Eq. (2), that is, $P_b = C_{Old} \alpha \sigma VQ^{\prime} + (1 - \alpha) \sigma VQ^{\prime}$. By substituting $P_b$ into $E[U_b] \equiv E[-(e^{-r(\omega VQ^{\prime} - Bid_{2,2})^2})^{\alpha}]$, we can verify that $\frac{\partial E[U_b]}{\partial VQ^{\prime}} \leq 0$. Thus, the expected utility of bargaining is concave with respect to the verifiable quality of supplies, $VQ$. This permits us to find the optimal quality for bargaining based on the first-order condition:

$$
VQ^{\ast}(\omega) = (\omega \theta) / \theta(\sigma_{Old}^2) \omega \theta.
$$

(8)

The buyer’s expected utility from bargaining for the existing supplier is:

$$
E[U_b] = E[-e^{-r(\omega VQ^{\ast} - P_b + \sigma_{Old}^2) VQ^{\ast} + \omega \theta)}] = -e^{-r(\omega VQ^{\ast}(\omega) - C_{Old} VQ^{\ast}(\omega^2) + \omega \theta)}]
$$

(9)

For the auction, given verifiable quality $VQ$, the buyer’s expected utility is denoted by $E[U_a] = E[-e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}]$. We next will show the existence of optimal ex ante verifiable quality $VQ$ from the buyer’s point of view. The following proposition presents this property.

### Proposition 2 (Existence of Optimal Ex Ante Verifiable Quality), $E[U_a]$ is concave with respect to verifiable quality of supplies, $VQ$.

**Proof.** Since $Bid_{2,2}$ is independent of the revenue generated by unverifiable quality, $\sigma_{New}^2$ we only need to consider $E[-e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}]$ in $E[U_a]$. Note that:

$$
E[-e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}],
$$

(10)

where the second-order density function, $f_{Bid_{2,2}} = N(N - 1)(2 - x)N^{-2}$, is non-negative. Further note that $\omega VQ^{\ast} - Bid_{2,2}$ is concave with respect to $VQ$, with $\theta > 1$, or $\omega \leq 1$. It turns out that the second derivative, $\frac{\partial^2 E[-e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}]}{\partial VQ^{\ast^2}} < 0$ holds, which completes the proof. □

In fact, we also can see that the sign of $\frac{\partial E[-e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}]}{\partial VQ^{\ast^2}}$ changes from positive to negative. This means that $E[U_a]$ first increases and then decreases in verifiable quality of supplies, $VQ$. Thus, by the Existence of Optimal Ex Ante Verifiable Quality Proposition (P2), we can conclude that the unique optimal verifiable quality $VQ^{\ast}$ exists in the auction, as given by its solution in the first-order condition. However, the explicit form of $VQ^{\ast}$ is not easy to derive in general, and the corresponding maximum value of the buyer’s expected utility is similarly hard to present in explicit terms. Nevertheless, we can identify the least upper bound (or sup) for the buyer’s expected utility in the auction. We offer the following proposition for this purpose:

### Proposition 3 (Least Upper Bound on Buyer’s Expected Utility), $\operatorname{Max} E[U_a]$ decreases in $Bid_{2,2}$; and $\sup E[U_a] = -e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}$, where $VQ^{\ast} = (ao)(\omega \theta) \omega \theta.

**Proof.** Note that the random variable $Bid_{2,2}$ is also a general variable $Bid_{2} \in [1, 2]$, thus $\max_{VQ} E[U_a]$ is equivalent to the problem $\max_{VQ} -e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}$, with $Bid_{2} \in [1, 2]$. The first-order condition of the objective function produces a unique solution for this problem, since $-e^{-r(\omega VQ^{\ast} - Bid_{2,2})^2})^{\alpha}$ is concave with respect to the verifiable quality of supplies, $VQ$. Therefore, given a fixed value for $Bid_{2,2}$, $VQ^{\ast}(Bid_{2,2}) = (ao)(\omega \theta)(\omega \theta)$ $\omega \theta > 0$ is the optimal verifiable quality of supplies. By the envelope theorem, we can write:

$$
\frac{d}{dBid_{2,2}} \left(-e^{-r(\omega VQ^{\ast}(Bid_{2,2}))^2 - Bid_{2,2}(VQ^{\ast}(Bid_{2,2})^2)}\right) = \frac{\partial}{\partial Bid_{2,2}} \left(-e^{-r(\omega VQ^{\ast}(Bid_{2,2}))^2 - Bid_{2,2}(VQ^{\ast}(Bid_{2,2})^2)}\right) = -re^{-r(\omega VQ^{\ast}(Bid_{2,2}))^2 - Bid_{2,2}(VQ^{\ast}(Bid_{2,2})^2)}(VQ^{\ast}(Bid_{2,2})^2) \omega \theta < 0.
$$

Therefore, $\max_{VQ} E[U_a]$ decreases in $Bid_{2,2}$, and an upper bound of $E[U_a]$ is given by $Bid_{2,2} = 1$. This is associated with the verifiable
quality of supplies, $VQ'' = VQ^*(Bid_2) = (ao/\theta)^{1/\omega}$, and thus $\sup E[U_A] = -e^{-r(aVQ'')^\omega - (VQ'')^\omega + \mu_{\text{New}} - 1/2\sigma_{\text{New}}^2}$. This completes the proof.

The proof of the Least Upper Bound on Buyer’s Expected Utility Proposition (P3) shows that, if the decision-maker knows value of $Bid_{(2)}$ as $Bid_d$, then the optimal verifiable quality of the auction can be determined as $VQ^*(Bid_2) = (ao)^{1/\omega} \cdot (Bid_d/\theta)^{1/\omega}$. However, the second bid price $Bid_{(2)}$, will not be known before the auction finishes. Proposition 3 analyzes the relationship between $Bid_{(2)}$ and the expected revenue from the auction, which decreases in $Bid_{(2)}$, and the upper bound of the expected revenue is given by $\sup E[U_A]$.

The explicit form for verifiable quantity of supplies in an auction, $VQ'$, is hard to derive an as ex ante value, however, we can define an approximately optimal value of verifiable quality of supplies, $VQ'$, for practitioners. This will give them an idea of what they need to expect from the suppliers before they initiate an auction. This approximately optimal value is given by:

$$VQ' = (ao)^{1/\omega} \cdot (E(Bid_2/\theta)^{1/\omega})$$

Further, $E[U_A]$ with approximately optimal verifiable quality $VQ'$, denoted as $E[U_A(VQ')]$, is $E[U_A(VQ')] = -e^{-r(aVQ')^\omega - (VQ')^\omega + \mu_{\text{New}} - 1/2\sigma_{\text{New}}^2}$. Then, by comparing the values of $E[U_A(VQ')]$ with $\sup E[U_A]$, we can offer an additional proposition:

**Proposition 4 (Expected Value of Verifiable Quality of Supplies with Many Suppliers).** When the number of new suppliers $N$ in the procurement system becomes large, the value of $E[U_A(VQ')]$ will be equal to its least upper bound, $\sup E[U_A]$.

**Proof.** Note $\lim_{N \to \infty} VQ' = VQ''$, and $\lim_{N \to \infty} Bid_{(2)} = 1$, so that $\lim_{N \to \infty} E[U_A(VQ')] = \lim_{N \to \infty} e^{-r(aVQ')^\omega - (VQ')^\omega} \cdot E[\rho^{\text{Bid}_{(2)}}(\nu_{\text{New}})] = 1$. This completes the proof.

The Expected Value of Verifiable Quality of Supplies with Many Suppliers Proposition (P4) confirms that there is a useful approximation for an auction’s optimal revenue of $E[U_A(VQ')]$, and that it converges to the value of $\sup E[U_A]$ when the number of new suppliers becomes large. This is a useful supplement to the result that we obtained in Eq. (11).

Next, we will compare the bargaining mechanism with the auction mechanism that we have considered in Case 2, but with a more general setting. We will use non-linear functions for both the buyer’s utility function and the suppliers’ cost functions. We will state a decision criterion beyond which it is better for the buyer to choose bargaining over the auction mechanism.

**Proposition 5 (Risk Aversion Criterion for the Buyer to Choose Bargaining).** If the Arrow–Pratt coefficient of absolute risk aversion satisfies the equation given in Box 1, the buyer will choose bargaining over an auction mechanism.

**Proof.** The proof is similar to the proof of Proposition 1, and is not presented here.

In Case 1, the optimal verifiable qualities of supplies for the auction mechanism and the bargaining mechanism are the same. The reason behind this is that verifiable quality in the linear utility and linear cost procurement system acts as in production to create the basis for increasing returns to scale. However, when we use non-linear utility and non-linear cost functions, as in the setting of Case 2, the optimal verifiable quality of supplies for the bargaining mechanism is given by Eq. (8), and there is an ex ante optimal verifiable quality $VQ^*$ that exists for the auction mechanism, even though it cannot be written down with an explicit functional form.

We presented an approximately optimal verifiable quality $VQ'$, so the buyer has a criterion for decision-making for the verifiable quality of supplies that is required when utility and cost are both non-linear. Replacing $VQ'$ with $VQ^*$ is a more precise way to apply the Risk Aversion Criterion for the Buyer to Choose Bargaining Proposition (P5).

Other determinants related to the buyer’s selection of the most appropriate mechanism, such as $r$, $N$, $\alpha$, $\mu_{\text{Old}} - \mu_{\text{New}}$ and $\sigma_{\text{Old}}^2 - \sigma_{\text{New}}^2$, also impact the decision-making in Case 2 similar to what we learned from Proposition 1 for Case 1. Proposition 5 states that it is useful to gauge the buyer’s decision-making risk aversion level relative to some criterion value, for example $r \geq r_2$. Buyers who are more risk-averse are more likely to choose the bargaining mechanism to determine the supplies procurement decision outcomes in both scenarios. It is also possible for managers to focus on $N$ or $\alpha$ as a means to analyze tradeoffs other than for the magnitude of the buyer’s risk aversion.

5. Numerical examples

To show the tradeoffs that identify the different regions for using an auction or bargaining mechanism, we offer examples of the two cases with numerical analysis illustrations. We will use the following parameters: $\alpha = 2.2$, $C_{\text{Old}} = 1.10$, $\mu_{\text{New}} = \mu_{\text{Old}} = 1$, $\sigma_{\text{Old}} = 0.4$, $\sigma_{\text{New}} = 0.6$, and $VQ_H = 1.2$ for Fig. 1; these additional parameters $\theta = 1.2$ and $\omega = 0.8$ for Fig. 2; and $\alpha = 0.7$. $N = 10$ for Figs. 3 and 4.

When generalized Nash bargaining and an auction market mechanism are optimal, Figs. 1–4 all represent the tradeoffs stated in Propositions 1 and 5. In the region above the curved surface, the buyer should prefer bargaining; beneath the curved surface though, the buyer might wish to initiate an auction for new

$$r \geq r_2 = \frac{2\left[VQ''_{\text{Old}}(N+3)/(N+1) - aVQ''_{\text{Old}} + \alpha(aVQ''_{\text{Old}} - C_{\text{Old}}VQ''_{\text{Old}} - (C_{\text{Old}})) + (\mu_{\text{Old}} - \mu_{\text{New}})\right]}{\sigma_{\text{Old}}^2 - \sigma_{\text{New}}^2}$$
bargaining power $\alpha$ will lead the buyer to bargain with the existing supplier in the two cases. Figs. 3 and 4 show that the lower mean and higher standard deviation of the revenue generated by the new suppliers’ unverifiable quality will result in a bargaining decision in Case 2, which is similar in Case 1. Other than the parameters related to the existing and the new suppliers’ production effectiveness and reliability, the basic tradeoff between a bargaining and an auction mechanism arises in terms of two main forces. They are: (1) the intensity of market competitiveness, based on the number of potential suppliers; and (2) the buyer’s degree of risk aversion, in terms of the ratio that is influenced by unverifiable supply quality and the total revenue that is produced.

Acknowledgments

This research was supported by National Natural Science Foundation of China (71071171, 71002069), Program for New Century Excellent Talents in University (NCET-11-0550) and Fundamental Research Funds for the Central Universities (CQDXWL-2012-2018). Rob Kauffman appreciated support from the School of Information Systems and the Lee Kuan Yew Faculty Fellowship for Research Excellence at Singapore Management University.

Appendix. Notation and definitions

<p>| Notation Definition Comments |
|-----------------------------|--------------------------------------------------|
| VQ_H, VQ_L | Verifiable quality of supplies for the buyer (high = H, low = L) All of the non-monetary, observable and verifiable characteristics of supply are aggregated into this one-dimensional variable. |
| VQ’, VQ'' | An approximation of the verifiable quality of supplies; the associated verifiable quality for the upper bound on the buyer’s expected utility, $(a\omega/\theta)<em>{\theta}$ An approximate value for verifiable quality is useful because there is no closed-form solution for it that we can assert ex ante from the buyer’s involvement in procurement. |
| C</em>{New}, C_{Old} | New supplier’s and existing (old) suppliers’ cost for verifiable quality of supply offered, which functions like a bid price Buyer will know existing (old) supplier costs before procurement process begins, but will not know the new suppliers’ until a bid is made, since this is each new supplier’s private information. |
| Bid_{(2)} | The second lowest bid among the new participating suppliers Bid_{(2)} is a random variable for the second lowest of N bids to represent the clearing price in a second price auction. |</p>
<table>
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<th>Notation</th>
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<tr>
<td>( N )</td>
<td>Number of suppliers who make bids</td>
<td>New suppliers are willing to bid on the supplies.</td>
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<tr>
<td>( \theta, \omega )</td>
<td>Exponents in existing and new suppliers' cost functions</td>
<td>Case 1 is ( \theta = \omega = 1 ); Case 2 is ( \theta &gt; 1, \omega \leq 1 ).</td>
</tr>
<tr>
<td>( a )</td>
<td>Coefficient in buyer's revenue function</td>
<td>( a \in (2, \ldots) ) represents the buyer's knowledge of an existing suppliers' capability to produce supplies of verifiable quality, as well as lesser knowledge of new suppliers' abilities to deliver supplies of the same quality.</td>
</tr>
<tr>
<td>( R(V_Q) )</td>
<td>Buyer's benefits stream for the project using supplies of verifiable quality ( V_Q )</td>
<td>The buyer's benefit stream based on supplies of verifiable quality is what is desired, whereas the revenue is based on the quality of supplies can actually be procured.</td>
</tr>
<tr>
<td>( R(U_Q) \sim \text{Normal}(\mu, \sigma^2) )</td>
<td>Random variable for associated revenue the buyer obtains from production with supplies of unverifiable level of quality ( U_Q )</td>
<td>The buyer's revenue is normally distributed with mean ( \mu ) and standard deviation ( \sigma^2 ).</td>
</tr>
<tr>
<td>( U(\pi) )</td>
<td>Buyer's utility for profit ( \pi )</td>
<td>( U(\pi) = -e^{-\alpha \pi} ).</td>
</tr>
<tr>
<td>( r )</td>
<td>Absolute risk aversion for buyer for unverifiable quality of supply</td>
<td>( r = U''(\pi)/U'(\pi) ). We define ( r_1, r_2 ) as the coefficients of absolute risk aversion for the buyer in Cases 1 and 2.</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Buyer's bargaining power, with ( \alpha \in [0, 1] ). It also indicates the portion of the surplus the buyer receives in the Nash bargaining mechanism</td>
<td>Greater bargaining power should make the buyer prefer the bargaining mechanism over the auction mechanism.</td>
</tr>
<tr>
<td>( \text{Opt} )</td>
<td>Supplier's outside option to see the same goods as bid to supply</td>
<td>Option arises because the supplier can sell to others instead of the buyer.</td>
</tr>
<tr>
<td>( \text{Cert}(R_{U_Q}) )</td>
<td>The certainty equivalent of a random variable for revenue from the point of view of a risk averse-buyer</td>
<td>Certainty equivalent values can be constructed for existing and new suppliers' unverifiable quality of supplies.</td>
</tr>
<tr>
<td>( E(U_B), E(U_A) )</td>
<td>Expected utility from use of bargaining (B) or auction (A) for the buyer</td>
<td>The notation applies in Cases 1 and 2.</td>
</tr>
<tr>
<td>( P )</td>
<td>Payment made by the buyer to the supplier for supplies in Cases 1 and 2, as an equilibrium price under the auction mechanism, and as a random variable for payment under the auction mechanism</td>
<td>When money is paid to the winning supplier under the auction mechanism, this acts like the auction price. When it is paid to the supplier following bargaining, then it is a bargaining payment.</td>
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</table>

References
